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edited by M. Conversi
Director of the Course

VARENNA ON LAKE COMO
VILLA MONASTERO
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XXVI CORSO

a cura di M. CONVERSI
Direttore del Corso

VARENNA SUL LAGO DI COMO
VILLA MONASTERO
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Argomenti scelti sulla fisica delle particelle elementari

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SOCIETÀ ITALIANA DI FISICA

SCUOLA INTERNAZIONALE DI FISICA « E. FERMI »

XXVI CORSO - VARENNA SUL LAGO DI COMO - VILLA MONASTERO - 23 Luglio - 4 Agosto 1962

Introduction.

M. Conversi

Director of the Course

The present book contains the lectures and some of the seminars delivered at the XXVI Course of the «Enrico Fermi» International School of Physics at Varenna (Lake Como) during the summer 1962. This is not the first and will certainly not be the last Varenna Course concerned with the physics of elementary particles. This field is indeed growing so rapidly that it appears now adequate to have each year one course on some selected topics where significant steps have recently been accomplished.

It is perhaps worth recalling that the Varenna International School of Physics was born just a few years after the artificial production of some elementary unstable particles had become a reality. The first course dealt with elementary particles as observed in the natural source of cosmic radiation. The second course, in 1954, was mainly dedicated to elementary particles as artificially produced at the large accelerators. In the few years which have elapsed since then, a great amount of work has been carried out both on experimental and theoretical grounds. However, our understanding of the field is still far from being satisfactory; our information appears much too scanty (or pehaps our intelligence much too inadequate) to achieve such an understanding; and in accordance with a sort of general law of Science, every problem for which occasionally we thought we had found a solution dragged in its wake new questions to be answered. The present overall picture of the field is extremely complex and the old naive idea of a few fundamental particles as the ingredients of which nature is made up, appears today definitely obsolete. In spite of this unsatisfactory situation regarding our deep understanding of the field, we should not underestimate, however, the considerable progress which has been made, nor should we forget that the description of the elementary particles and of their interactions represents a formidable challenge and is probably the most crucial problem of contemporary physics.

The selection of the topics for the present Course aimed not only to present some of the most interesting results recently obtained, but also to fill certain gaps

with respect to the arguments chosen for previous Courses on elementary particles. Thus, lectures on strongly interacting particles and the new resonant states, as well as on the form factors of the elementary particles, appeared highly desirable for this Course.

The existence of resonant states for the pion-nucleon system has been known since the discovery of the isobar state in 1953; but only in the past two years has it been shown that the occurrence of similar resonances is a somewhat general characteristic of the strong interactions. Most of these resonant states have been established in bubble chamber experiments partly carried out at the « E. O. Lawrence » Radiation Laboratory in Berkeley. Professor A. H. ROSENFELD, from this Laboratory, presents in a first group of lectures the phenomenological aspects of the strongly interacting particles and resonances. The theoretical counterpart of the subject is developed in a second group of lectures by Professor J. J. SAKURAI, of the « Enrico Fermi » Institute for Nuclear Studies in Chicago.

It is not only a matter of historical interest to point out the existence of a link between this first and the third part of the Course (form factors of elementary particles) through the suggestion by NAMBU, in 1957, that excited states might exist for pionic systems, yielding a possible explanation for the nucleon form factors in the electron scattering experiments at Stanford. The series of lectures on the form factors of elementary particles, by Professor S. D. DRELL of the University of Stanford, yields, in the third part of this book, a self-consistent presentation of this important subject.

The second part of the Course is made up of two distinct groups of lectures, both of a phenomenological character, on the physics of the strange particles. The « weak » decays of these particles are discussed in the lectures delivered by Professor F. S. CRAWFORD, of the Radiation Laboratory at Berkeley. The other group of lectures, by Professor H. K. TICHO, of the University of California at Los Angeles, are devoted to the strange particle resonances.

In addition to the groups of lectures quoted above, which aimed to give a more or less self-consistent account of the corresponding topics, it was felt worthwhile presenting in isolated seminars by various speakers (or exceptionally in a group of seminars by the same speaker) other arguments of miscellaneous nature related to the field of elementary particles. Subjects of great present interest, such as the existence of two types of neutrinos and the Regge poles, where among those discussed in these seminars. However, only seminars containing unpublished material have been included in this book (and collected in the last part of it). Titles and speakers names of the remaining seminars are listed at the end of the book together with references to the corresponding relevant literature.

It was a fortunate coincidence that the President of the Italian Physical Society, Professor GILBERTO BERNARDINI, is also an expert in the field of

elementary particles. I am deeply indebted to him for profitable discussions and suggestions in the early stage of preparation of the programme.

I also wish to express my sincere thanks to the lecturers and speakers who contributed to make the Course pleasant and successful.

Dr. ANTONIO ZICHICHI acted as a Scientific Secreary at the Course. I gratefully acknowledge his close co-operation in preparaing the programme and his continuous efforts during this Course to make it alive by stimulating and contributing to the discussions.

Strongly Interacting Particles and Resonances.

A. H. ROSENFELD

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University of California - Berkeley, Cal.

Introduction.

I want to start with a few words about terminology. I will use the word « particle » to include both stable particles and « resonant states » which can decay rapidly via the strong interaction into other particles. Hence a precise but less conventional title for this course would be « Strongly Interacting Particles: Bound and Unbound ». Notice that I have altogether avoided the word « elementary ».

The most familiar example of an unbound state is the $I=\frac{3}{2}$, $J=\frac{3}{2}$ pion-nucleon *resonance*. In this case there is only one decay channel, and we can show that the pion-nucleon scattering phase shift goes through 90° at the resonance; if there were more than one channel we could still show that the scattering amplitude becomes pure imaginary at the resonance.

An example of a *slightly bound* system is the deuteron. Precisely because it is slightly bound its properties tend to be those of the sum of its constituents, and we tend to think of it as a « composite » system.

An example of a *tightly bound* system is the pion, considered as a bound state of a nucleon and an antinucleon. Its binding energy is so great ($m_\pi \ll 2m_{\mathcal{N}}$) that the new system has properties completely different from its constituents. Since at this moment in history we cannot calculate these properties, we tend to think of the pion as an « elementary » particle.

It is best to classify a particle by properties other than its decay via any particular channel; thus it would be an incomplete statement to say that the Y^0_* of mass 1520 MeV is a $\Sigma\pi$ state, because this neglects all other possible final states. In fact, the Y^*_0 (1520) decays into $\Sigma\pi$ (about 60 %), $\overline{K}\mathcal{N}$ (about 30 %), and $\Lambda\pi\pi$ (about 10 %).

I. - The Four Basic Interactions and Their Quantum Numbers.

Since there are many particles and only four interactions it is better to discuss first the interactions. Anyway I feel that eventually the interactions will explain the particles rather than vice versa.

In nature only four fundamental interactions exist: gravitational, weak, electromagnetic, and strong. It is generally assumed that all four interactions obey the following symmetries and conservations laws [1], even if they are not experimentally tested for all the interactions:

a) Conservation of four-momentum and angular momentum.

b) Conservation of electric charge.

c) Conservation of baryon number B and lepton number L.

d) invariance under CPT.

The operator C when applied to a single particle in its own rest frame transforms the particle into the corresponding antiparticle. (Thus we assume that every particle has an antiparticle which may or may not be distinct.) For some purposes it is convenient to think of an antiparticle as a hole in a negative-energy particle sea. The C concept is useful mainly for a system of nonstrange neutral mesons, discussed more fully later.

e) Invariance under T (or CP, because of *d*)).

The notion of a moving picture offers a simple physical model of the time-reversal operation. The time-inverted situation is obtained by running the movie backward. Time-reversal invariance requires that to an observer who does not know the initial conditions the inverted situation makes sense.

The time-reversal operation is meaningful only for microscopic systems, not for the large ensembles that are governed by statistical as well as microscopic mechanics.

Additional conservation laws are obeyed by some but not all of the four interactions. These we take up next.

1. – Strong interaction.

1·1. *Orders of magnitude.* – The strong (nuclear) interaction has the following characteristics:

a) The range is short. Its order of magnitude is given by the pion Compton wavelength,

$$\lambda\!\!\!^{-}_\pi = \frac{\hbar}{m_\pi c} = 1.4 \cdot 10^{-13}\,\text{cm} \equiv 1.4\ \text{fermi}\,. \tag{1}$$

b) The energy is large. For example, nuclear bindings run in tens of MeV and the production of mesons in hundreds of MeV.

c) The natural unit of time for strong interactions is given by the time it takes for a light signal to cross a distance equal to the range of nuclear forces:

$$\tau = \frac{\hbar}{m_\pi c^2} \approx \frac{1}{2}\cdot 10^{-23}\,\text{s}\,. \tag{2}$$

d) If a reaction takes place in a time τ the corresponding full width at half-maximum Γ of its Fourier transform is

$$\Gamma = \frac{\hbar}{\tau} = \frac{\frac{2}{3}\cdot 10^{-21}\,\text{MeV s}}{\tau}\,.$$

A more useful form is

$$\Gamma = \frac{\hbar c}{\tau c} = \frac{197\,\text{MeV fermi}}{\tau c}\,. \tag{3}$$

If we take, for example, $c\tau = 1$ fermi, we get $\Gamma = 200$ MeV. (Notice that if the particle has mass m and is produced with momentum $p \equiv \eta mc$, then it actually goes an average distance $\eta c\tau$ from its point of production. If η is large, this factor can help the resonance get out of the range of nuclear forces before it decays.) The $\frac{3}{2}$, $\frac{3}{2}$ pion-nucleon resonance Δ has $\Gamma_\Delta \approx 100$ MeV (*i.e.*, $c\tau \sim 2$ fermi). Similarly the ρ-meson has $\Gamma_\rho \approx 100$ MeV. But the ω-meson has $\Gamma_\omega \lesssim 15$ MeV and probably $\Gamma_\omega \approx 1$ MeV. This means a corresponding $c\tau \simeq 200$ fermi, which is « outer space » in comparison with the dimensions involved in nuclear interactions.

1·2. *Additional conservation laws for the strong interactions*. – The student who needs further basic information on these conservation laws will find a helpful exposition in a review article by WICK [2].

1·2.1. Conservation of isotopic spin I (both $|I|$ and I_z). Figure 1 shows the particles that are stable against decay via the strong interaction. Most of them have been known for many years, although the η-meson was not discovered until 1961, and its quantum numbers were sorted out only in 1962. The particles without any strong interaction (photons and leptons) are shown as thin bars; thick bars represent the strongly interacting particles (mesons and baryons), whose grouping into multiplets is evident. This grouping suggested that all members of the multiplet shared a new quantum number I, called isotopic spin. For the rest of this text we refer to it for short as isospin, a conserved vector in « ispace ». The projection of I along the « charge axis »

was called I_z, and gave the electric charge in units of $|e|$.

$$Q = \frac{Y}{2} + I_z\,. \tag{4}$$

The constant Y is called the « hypercharge », since it measures the « center of charge » of a multiplet. For mesons the hypercharge and the « strangeness » are the same thing. For the nucleon doublet, though, we have

$$Q = \tfrac{1}{2} + I_z\,,$$

i.e., $Y=1$. However, since nucleons are familiar, we like to say they have zero strangeness, *i.e.*, we invent the relation

$$Y = S + B\,,$$

where S is the strangeness and B the baryon number.

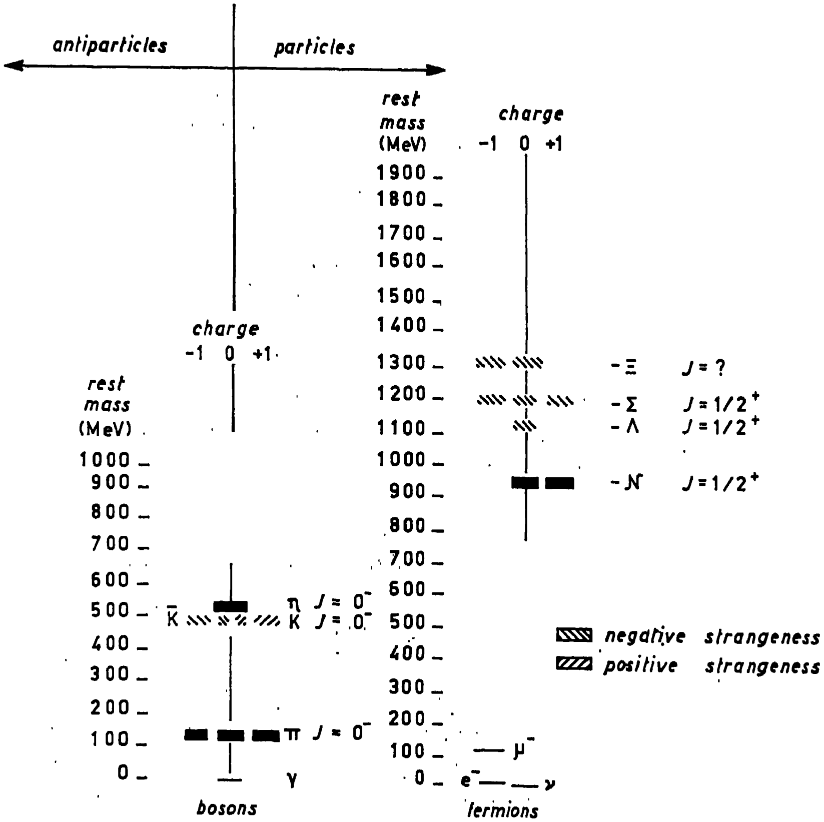

Fig. 1. – Particles stable against strong decay.

In summary, both Y and S are used to describe the position of the center of a multiplet, but Y is used to relate its displacement from zero charge, and S is used to relate its displacement from the center of charge of the « nonstrange » particles.

1.2.2. Separate invariance under C and P; G parity. We define *parity* P as the operation that reflects space co-ordinates. Then the parity of a wave function of orbital angular momentum l is $(-1)^l$. As discussed in any text on particle physics each particle has also an « intrinsic » parity, which for both the π-meson and the K-meson has been determined experimentally to be odd [1b]:

$$P|\pi\rangle = -|\pi\rangle\,, \tag{5a}$$

$$P|K\rangle = -|K\rangle\,. \tag{5b}$$

We discuss the parity of fermion-antifermion pairs in connection with eq. (18). We define *charge conjugation*, C, as the operation that transforms particles into antiparticles and vice versa; thus electrons e^- transform into positrons e^+, while π^+ transforms into π^-, and π^0 into itself. As W. S. C. WILLIAMS has pointed out [1a] a more satisfactory name for this operation would be particle-antiparticle conjugation, as there is not always a change in electric charge.

We want the photon to have the same behavior under C as electric or magnetic fields and currents, namely

$$C|\gamma\rangle = -|\gamma\rangle\,. \tag{6}$$

Since π^0-decays electromagnetically into two identical particles (two γ-rays) and since C is conserved by the electromagnetic (em) interaction as well as by the strong interaction, we have

$$C|\pi^0\rangle = +|\pi^0\rangle\,. \tag{6a}$$

Since charged particles are not in eigenstates of C, we can get no selection rules by applying C alone (although we can get other useful relations). Therefore for most charged particles we do not bother with C except to note that $C^2=+1$, and that $[C, H]=0$, where H is the strong or electromagnetic Hamiltonian. But in the case of charged pions we want to adopt the convention

$$C|\pi^\pm\rangle = +|\pi^\mp\rangle\,, \tag{6b}$$

in analogy with (6*a*). This choice is arbitary; we could have chosen

$$C|\pi^{\pm}\rangle = \exp[\pm i\delta]|\pi^{\mp}\rangle ,$$

but (6*b*) is simpler. We will use (6*b*) in defining G, which we take up next.

G parity. Next we wish to take advantage of simultaneous conservation of C and I to derive a new conserved quantity G, first introduced by LEE and YANG [3] and MICHEL [4]. They defined G as $C\exp[i\pi I_y]$, *i.e.*, a charge conjugation and a 180° rotation in ispace around I_y. Actually, with C defined as it is above, we must redefine G as

$$G = C\exp[i\pi I_x] . \tag{7}$$

The reason for introducing G is readily seen. Given a multiplet with $B = S = 0$, only the neutral component can be in an eigenstate of C, since C reverses charges. However, the rotation about I_x or I_y again reverses charges, so that the whole multiplet can be an eigenstate of G. Notice that C performs a reflection in ispace, and $\exp[i\pi I_x]$ is a rotation. Hence G has the properties of a parity operation in ispace—and WICK calls it «isotopic parity» [2].

Next we wish to prove eq. (8), *i.e.*, that for any system of n_π pions, $G=(-1)^{n_\pi}$:

To do this it is convenient to represent the pion (which has unit angular momentum in ispace) by the spherical harmonics in ispace, *i.e.*,

$$\begin{aligned} |\pi^0\rangle &= Y_1^0 \ \propto \quad \cos\theta = z , \\ |\pi^+\rangle &= Y_1^1 \ \propto -\sin\theta\exp[i\varphi] = -x - iy . \\ |\pi^-\rangle &= Y_1^{-1} \propto +\sin\theta\exp[-i\varphi] = +x - iy . \end{aligned}$$

Using the C parity of π from (6*a*) and (6*b*), we then have

$$G|\pi^0\rangle = C\exp[i\pi I_x]|z\rangle = C|-z\rangle = C|-\pi^0\rangle = -|\pi^0\rangle ,$$

and

$$G|\pi^{\pm}\rangle = C\exp[i\pi I_x]|\mp x - iy\rangle = C|\mp x + iy\rangle = C|-\pi^{\mp}\rangle = -|\pi^{\pm}\rangle ;$$

thus G changes the sign of each pion, and consequently

$$G = (-1)^{n_\pi} . \tag{8}$$

q.e.d.

Since C and I are conserved in strong interactions, so is G; consequently an even number of pions cannot transform into an odd number (and vice versa), therefore pion vertices in Feynman diagrams must consist of an even number

of pions. We can now repeat the above discussion of the effect of operating with $\exp[i\pi I_x]$ on $Y_l(z)$ to obtain a different result for *any* particle in an eigenstate of C (*i.e.*, any nonstrange neutral meson with arbitrary isospin I). The symmetry of $Y_I^0(z)$ is $(-1)^I$, *i.e.*, $\exp[i\pi I_x]\,Y_I^0 = (-1)^I\,Y_I^0$; thus we have the result

$$G = C(-1)^I \tag{9}$$

for neutral nonstrange mesons. Once established for the neutral member of a multiplet, G applies for the whole multiplet.

1'2.3. Particle-antiparticle systems: the rule « $CPX_s = +1$ ».

(i) *Case* I. *Boson-antiboson pairs.* For two identical spinless bosons such as $2\pi^0$ we all know that the wave function $\psi(r)$ must by symmetric under the operator X, which exchanges these two particles; *i.e.*,

$$X = +1\,. \tag{10}$$

If the bosons are charged, with charge Q, and have spins, we can write a generalized wave function of three variables

$$\psi = \psi(r)\,\psi(Q)\,\psi(S)\,, \tag{11}$$

where the diparticle spin $\boldsymbol{S}$ is given by the vector sum $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$.

$$X\psi = X_r\psi(r)\,X_Q\psi(Q)\,X_S\psi(S) = P\psi(r)\,C\psi(Q)\,X_S\psi(S)\,,$$

i.e.,

$$X = PCX_S\,.$$

This generalized ψ must again be symmetric under X, since boson field operators commute. Therefore

$$X = PCX_S = +1\,. \tag{12}$$

If $\psi(r)$ is an l wave,

$$P = (-1)^l\,. \tag{13}$$

Questions of intrinsic parity and C do not arise, since we have *two* bosons, and $P^2 = C^2 = +1$.

The symmetry of $\psi(S)$ is $(-1)^{S_1+S_2-S} = (-1)^S$, so

$$X_S = (-1)^S; \tag{14}$$

(12) then becomes

(15) $$C=(-1)^{l+s}.$$

We can use (15), for example, to calculate C for the ρ^0-meson, which decays into $\pi^+ + \pi^-$ in a p-wave. For spinless boson-antiboson pairs (15) is

(15a) $$C=(-1)^l,$$

i.e., $C_{\rho^0}=-1$.

Application of $C=(-1)^l$ *to* K_1K_1 *vs.* K_1K_2 *pairs.* Consider a nonstrange $K^0\bar{K}^0$ pair produced with relative angular momentum l. The K-meson is spinless, therefore (12) becomes

(16) $$PC=+1.$$

Now, it is well known that neutral K-mesons decay via the weak interaction not as K^0 or $\bar{K}^0$ but in eigenstates of CP called K_1 and K_2. Thus

$K_1 \to 2\pi$ (in an s-wave), therefore by (16), $CP=+1$;

K_2 cannot $\to 2\pi$ and has $CP=-1$.

If CP is applied to $|K_1K_1\rangle$ or $|K_1K_2\rangle$ we have

(17a) $$CP\psi_1(r)|K_1K_1\rangle = (-1)^l CP|K_1\rangle CP|K_1\rangle = +(-1)^l,$$

(17b) $$CP\psi_1(r)|K_1K_2\rangle = (-1)^l CP|K_1\rangle CP|K_2\rangle = -(-1)^l.$$

Combining (17) and (16), we see, for $K^0\bar{K}^0$ systems,

even $P(=$ even $C)$ requires decay via K_1K_1 and K_2K_2,

odd $P(=$ odd $C)$ requires decay via K_1K_2:

For further discussion, see ref. [5].

(ii) *Case* II. *Fermion-antifermion* ($f\bar{f}$) *pairs.* It so happens that eqs. (12) and (15) also apply to $f\bar{f}$ pairs. To explain this we must remind the reader of an important minus sign that enters in the parity of $f\bar{f}$ pairs. For $f\bar{f}$ pairs in an orbital l-wave, the parity is

(18) $$P=-(-1)^l.$$

There are two ways that we can understand this minus sign:

(*a*) A modern theoretical explanation is given by STAPP [6].

(*b*) Purely experimentally, we can note that when positronium annihilates from the ${}^1S_0$ state into two photons, they are linearly polarized perpendicular to each other [7]. We shall now show from this fact that ${}^1S_0$ must be a pseudo-

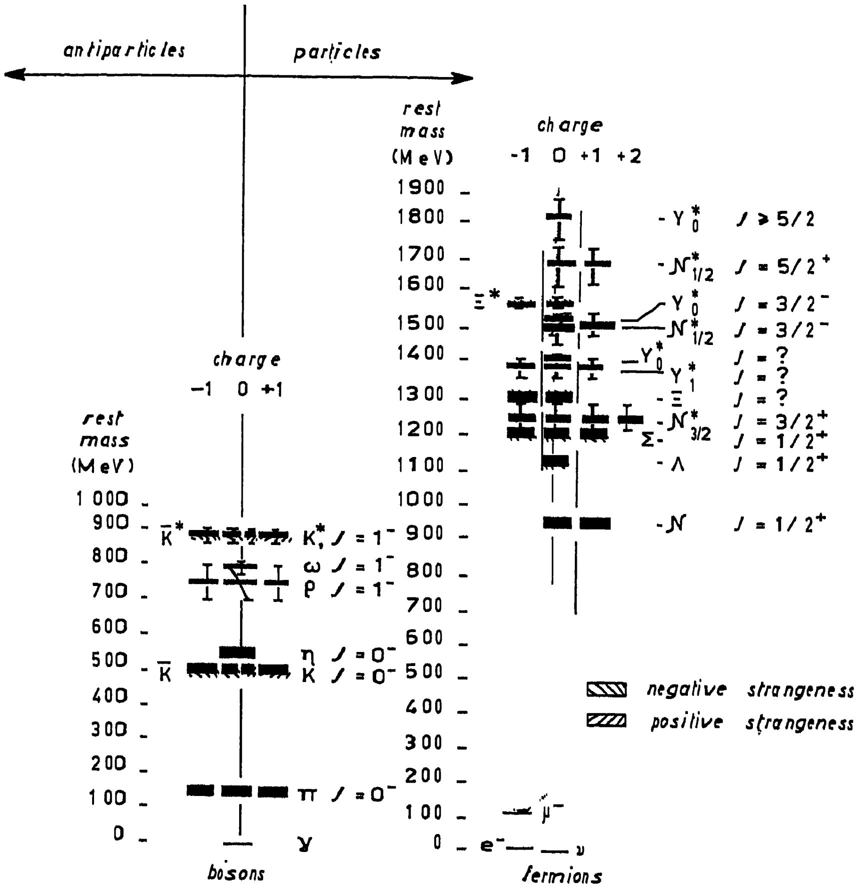

Fig. 2. – Particles and resonances. The data and references correspond to Table II. Strongly interacting particles are colored as follows: red for strangeness $S = 0$; blue for $S = \pm 1$; green for $S = -2$. To display antiparticles, one reflects the whole diagram about the heavy central line labeled « antiparticles – particles ».

scalar (0^-). We shall use this sort of argument many times. It runs as follows. The matrix element M for the process must involve linearly a vector for each photon created, and a scalar or pseudoscalar $0^\pm$ representing the annihilated $^1S_0$ state. For the two photon vectors we choose their polarization directions $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$. The matrix element M must be a scalar quantity. In addition to $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$, $0^\pm$, it can contain the photon momentum $\boldsymbol{k}$ to any power. If experimentally $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ are perpendicular to each other, M must contain $(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2)0^\pm$, where $\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2$ is an axial vector. To make a scalar quantity we must write

$$M \propto (\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2)\cdot \boldsymbol{k}0^-, \tag{19}$$

i.e., the parity of $^1S_0$ positronium must be negative, q.e.d.

It can further be shown that (18) holds not only for e^+e^- but also for all $f\bar{f}$ pairs [8].

Equation (18) comes up particularly often because antiprotons coming to rest in hydrogen are captured in s states. Hence we know that they are captured from negative parity states. The prediction that π^-, K^- and $\bar{p}$ should be captured from s states was first made by DAY, SNOW and SUCHER, in 1959 [9]. It was tested experimentally for π^- in 1960 by FIELDS *et al.* [10] and more recently by HILDEBRAND [11]. In 1961 M. SCHWARTZ [12] pointed out that one could use the constraint of s-wave capture to calculate the spin of K^* (888) when sufficient data became available. In 1962 ARMENTEROS *et al.* [13] confirmed the predicted s-wave capture and gathered enough data to suggest strongly that K^* (888) has spin $S = 0$. This experiment was well covered here at Varenna in the lectures by H. K. TICHO. G. A. SNOW [14] has recently published a table of other particles whose quantum numbers might be determined in similar experiments.

Having established (18) we can derive (12) exactly as for boson-antiboson pairs, except that for two fermions we want $X = -1$. Here X is still $X_r C X_S$, but this time $X_r = (-1)^l$ is *not* P, but rather $-P$, so we still have

$$PCX_S = +1, \tag{20}$$

which is identical with (15). This time, since spins are half-integral,

$$X_S = (-1)^S. \tag{21}$$

Since $P = -(-1)^l$, (20) and (21) combine to give

$$C = (-1)^{l+S}, \tag{22}$$

which happens to be identical with (15), q.e.d.

(iii) *Summary of particle-antiparticle systems.* Note that in this discussion of particle-antiparticle systems we have not yet used the concept of ispin. In fact, for Case II, $f\bar{f}$, we used for our example e^+e^-, for which ispin is not defined. If I, and therefore G, is defined, we can combine (15) or (22) with (9) to form

$$G = (-1)^{l+S+I}. \tag{23}$$

This applies for both boson-antiboson and fermion-antifermion pairs. For pairs of pions ($\pi^+\pi^-$, $\pi^\pm\pi^0$) in a pure istate, we can write $\psi = \psi(r)\,\psi(\text{ispin})$. The symmetry of $\psi(\text{ispin})$ is $(-1)^I$. For these two bosons we can as usual require

$$X = (-1)^l X_I = +1; \tag{24}$$

i.e., for dipions we have

$$l + I = \text{even}. \tag{25}$$

Thus the strong p-wave decay $\rho \Rightarrow \pi\pi$ shows that ρ has unit ispin.

It is not easy to generalize the approach above to other particles because one has to introduce both ispin *and* charge co-ordinates. Thus the $\bar{K}K$ istate $\uparrow\downarrow + \downarrow\uparrow$ could describe either K^+K^- or $K^0\bar{K}^0$.

It is interesting, and probably significant, to note the following relationship, illustrated in Table I: established so far are four nonstrange mesons—two

TABLE I. – *Nonstrange mesons.*

I spin	Pseudoscalar mesons				Vector mesons			
	Particle	J^{PG}	C	$\bar{\mathcal{N}}\mathcal{N}$	Particle	J^{PG}	C	$\bar{\mathcal{N}}\mathcal{N}$
0	$\eta \to \gamma\gamma,\ \pi^+\pi^-\pi^0$	0^{-+}	+	$^1S_0$	$\omega \Rightarrow \pi^+\pi^-\pi^0$	1^{--}	—	$^3S_1$, $^3D_1$
1	$\pi^0 \to \gamma\gamma$	0^{--}	+	$^1S_0$	$\rho \Rightarrow \pi\pi$	1^{-+}	—	$^3S_1$, $^3D_1$

The column labeled $\bar{\mathcal{N}}\mathcal{N}$ lists the $\bar{\mathcal{N}}\mathcal{N}$ states having same quantum numbers as the mesons. A single arrow indicates electromagnetic decay, a double arrow indicates strong decay. References are given in Table II.

pseudoscalars (one each with $I=0$ and 1) and two vectors (again with $I=0$ and 1). The pseudoscalar mesons both have $C=+1$, as illustrated by the fact that they both decay into two γ-rays; the vector mesons both have $C=-1$, as we shall-see in Sections 4 and 5 when we discuss their decay modes. The relationship is that these are precisely the characteristics of the possible nucleon-antinucleon states as summarized in (22) and (23).

Finally we may note that K and K^* (888) are *also* 0^- and 1^-, respectively, and could have S-wave $\bar{\Lambda}\mathcal{N}$ dissociation products.

2. - Electromagnetic interaction.

2·1. *Order of magnitude.* – The electromagnetic (em) interactions have the following characteristics:

a) The range is defined by the $1/r$ dependence of the Coulomb potential $U=e/r$.

b) The strength of the interaction is given by the fine-structure constant $\alpha=e^2/\hbar c=1/137$. In the same units the strong-interaction coupling constant is of the order unity. Therefore the em interaction is only about one-hundredth as strong as the strong interactions. Another way of comparing the em interaction with the nuclear one is computing the potential energy U at the range of the nuclear forces:

$$U=\frac{e^2}{1.4\ \text{fermi}}=1\ \text{MeV}\,. \tag{26}$$

2·2. *Conservation laws and selection rules.*

a) The em interaction has long been known to conserve C and P separately.

b) Early in the study of strange particles it was observed that photon emission conserved strangeness, *i.e.*, decays such as $\Lambda\rightarrow n\gamma$ did not occur.

c) By use of the postulate of « minimal electromagnetic interaction » we can show that a single photon can carry away only zero or one unit of ispin. The « minimal interaction » assumption is that the photon is coupled only to electrical currents, whose time component is the Q of eq. (14), which

we rewrite as

$$Q = \boldsymbol{I}\cdot \boldsymbol{e}_z + \tfrac{1}{2} Y \qquad (\boldsymbol{e} = \text{unit vector})\,. \tag{27}$$

The first term on the right-hand side contains an ispin vector: the second term is scalar. Hence the photon's ispin transformation properties cannot be more complicated that those of a scalar and a vector quantity; thus we have proved that, for a photon, $\Delta I = 0$ or 1.

d) When a single photon is emitted by a strongly interacting system either G changes *or* I changes by one unit: *i.e.*, either $\Delta G =$ yes, *or* $\Delta |I| = 1$. To see this, call the initial state $|i\rangle$, the final $|f\rangle$ and write the reaction as

$$|i\rangle \to \; |f\rangle + |\gamma\rangle\,,$$

where

$$C|i\rangle = C|f\rangle C|\gamma\rangle = -C|f\rangle\,. \tag{18}$$

For the initial state, (9) becomes

$$G|i\rangle = C|i\rangle(-1)^{I_i} = -C|f\rangle(-1)^{I_i}\,. \tag{28}$$

Dividing (28) by $G|f\rangle = C|f\rangle(-1)^{I_f}$, we have

$$\frac{G|i\rangle}{G|f\rangle} = -(-1)^{(I_f - I_i)}\,, \tag{29}$$

where according to (27), $(I_i - I_f) = \Delta |I|$ can be only 0 or 1, $(I_f - I_i) = 0$ or $= 1$.

In this sense a photon behaves either like a ρ-meson (γ_ρ) (*i.e.*, it « carries off » $I = 1$, and does not change G), or like an ω-meson ($I = 0$, $G = -1$) which can be written γ_ω. In our discussion of the η-meson we use this reasoning extended to emission and reabsorption of a virtual photon to derive eq. (47). We next want to prove two other important selection rules:

First, $(J = 0) \nleftrightarrow (J = 0) + \gamma$. Thus, for example,

$$\eta \nleftrightarrow \pi^0 + \gamma\,, \tag{30}$$

because a massless γ can exist only in the substate $J_z = \pm 1$.

Second, $(J=1) \nrightarrow 2\gamma$. Thus, for example,

$$\omega \nrightarrow \gamma+\gamma\,. \tag{31}$$

Proof. The two γ can have a total $J_z=0$ or ± 2. Since the $(J=1)$ system cannot have $J_z=\pm 2$, only the states in which the photons have $J_z=0$ need be examined further. There are two such states, one with both photons polarized left-handedly, and one with both right-handedly. A rotation of either state through 180° about the x axis merely interchanges the two photons which multiplies the state by $+1$ because the photons obey Bose-Einstein statistics. However, the same rotation of a $(J=1,\ J_z=0)$ system must multiply it by -1, as may be seen by considering $Y_1^0(z) \propto \cos\theta$. Thus the two γ's do not have the same angular momentum properties as a $J=1$ system, and the decay is forbidden, q.e.d.

3. – Weak interaction.

The weak interaction is so short-ranged and weak that it binds nothing, so it is responsible only for decays (π-decay, Λ-decay, β-decay, etc.). Its exact range is not yet known.

For processes of comparable momentum, weak interaction rates are only 10^{-14} as fast as strong rates. Thus where the strong decay

$$\mathrm{K}^*\,(888) \to \mathrm{K}\pi \quad \text{(with a momentum of 286 MeV/}c\text{)}\,,$$

takes 10^{-23} s, the weak decay

$$\mathrm{K}_1^0 \to \pi^+\pi^- \quad \text{(with a comparable momentum of 206 MeV/}c\text{)}\,,$$

takes about 10^{-10} s.

The weak interaction is well known to violate C and P separately. Its characteristics are discussed in the lectures by Professor F. S. CRAWFORD.

4. – Gravity.

Gravitational attraction is so weak as to have almost no applications to particle physics, therefore we ignore it.

II. - Nineteen multiplets.

This section is limited to presenting and commenting on Fig. 2 and Table II, which display data on the currently known mesons and baryons.

Figure 2 shows the great increase since 1960 in the number of particles known. Thick colored bars represent those particles which are stable against strong decay; those which decay strongly are represented by thin colored bars, and their width Γ is shown as a vertical « flag ».

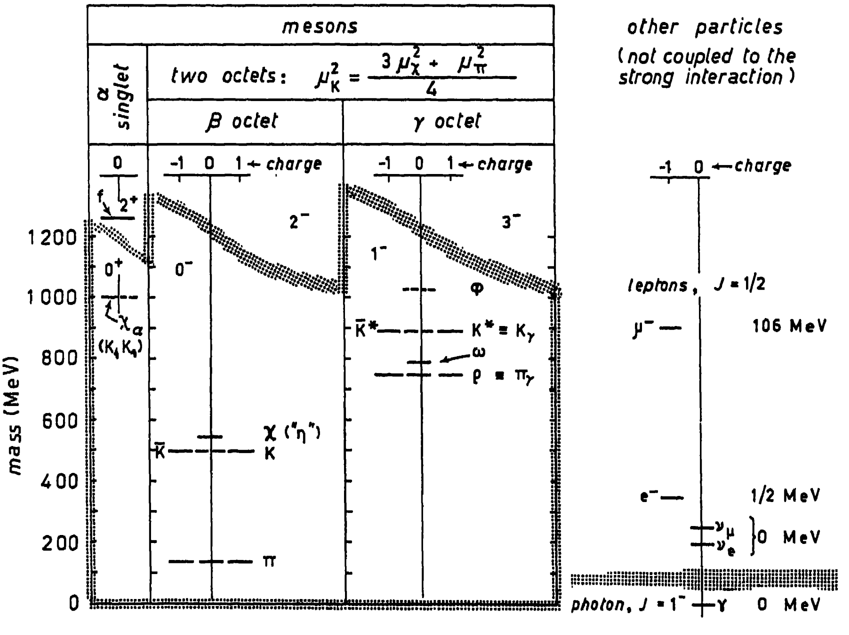

Fig. 2*a*. – Mesons. The meson unitary multiplets include a β (pseudoscalar) octet and a γ (vector) octet. There are *two* observed χ_γ states (*i.e.* vector isotopic singlets) called ω and φ — one linear combination of these is presumably the eighth member of the γ octet, the orthogonal linear combination is assigned to a unitary singlet. φ is seen as a K_1K_2 enhancement at 1030 MeV (see L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, L. Gray, J. Leitner, S. Lichtman and J. Westgard: *Phys. Rev. Lett.*, **9**, 180 (1962), and J. J. Sakurai: *Phys. Rev. Lett.*, **9**, 472 (1962)). In addition there appear to be two singlets; and a $J^{PG} = 2^{++}$ pion-pion resonance at 1250 MeV called f, and a 0^{++} K_1K_1 interaction near $\bar{K}K$ threshold. For more complete references see Barkas and Rosenfeld: Lawrence Radiation Laboratory Report UCRL-8030 Rev., Feb. 1963. All the meson states have charge-conjugation properties such that they may couple to baryon-antibaryon states, *e.g.* π^0, and have $C = +1$ (and decay into two photons), while φ^0 and ω have $C = -1$ (and couple to a single photon).

The mass scale of fig. 2 is so small that the mass differences *within* multiplets cannot be illustrated. There is a handy rule to help us remember which components are heavier—namely, that with the single exception of π, all multiplets in Fig. 2 (for which the mass differences are known) slope up to the left; *i.e.*, the more negative the charge the greater the mass. Thus

$$m(\mathrm{K}^0) > m(\mathrm{K}), \quad m(\mathrm{n}) > m(\mathrm{p}), \quad m(\Sigma^-) > m(\Sigma^0) > m(\Sigma^+), \quad m(\Xi^-) > m(\Xi^0).$$

Not included in Fig. 2 are two particles not yet firmly established $\mathrm{K}^*_{\frac12}$ (725), and Y^*_1(1660)—which are listed in Table II.

The notation of Table II for particle names (outlined in the notes to the table) is used henceforward.

We have displayed the mesons and baryons in a more systematic way in Fig. 2*a* and 2*b*, which are taken from A. H. ROSENFELD and S. L. GLASHOW; *Phys. Rev. Lett.*, **10**, 192 (1963). States with the same J and P have been gruped into « supermultiplets » as suggested by the Unitary Symmetry scheme of M. GELL-MANN and Y. NE'EMAN.

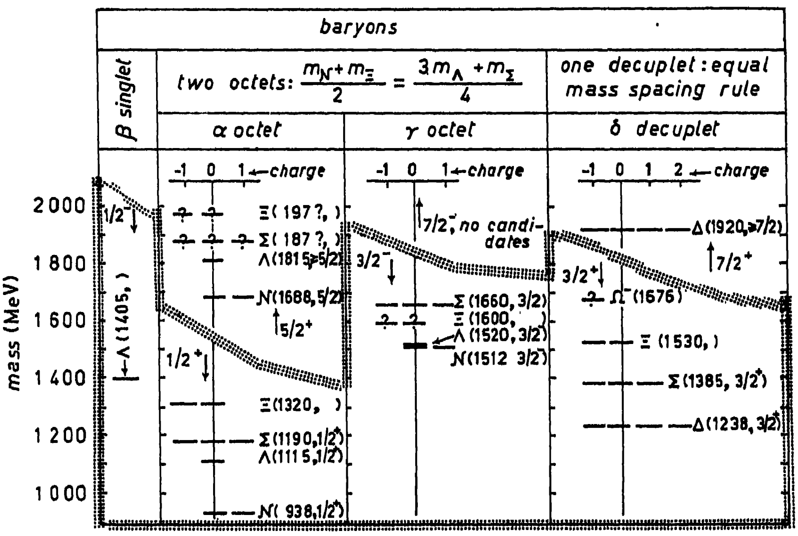

Fig. 2*b*. -- Baryons. The four unitary multiplets and their Regge recurrences. Spin and parity assignments J^P are written beside each particle if they are supported by any experimental evidence; if not, J^P have been conjectured by assigning one known resonance to each set of quantum numbers. The notation was introduced in the *Proc. of the 1962 Intern. Conf. on High-Energy Physics at CERN*, p. 783 and 325. Observe that the families so defined coincide with the unitary multiplets of the eightfold way. Heavy bars show stable or metastable particles; light lines show resonances. States predicted by the eightfold way but not yet seen are indicated by question marks. The masses of Ξ_γ and Ω_δ follow from the mass formulae alone; those of the $\frac32^+$ Σ_α and Ξ_α also require the assumption of nearly parallel Regge trajectories.

TABLE II. – *Tentative data on* [illegible] *roughly* [illegible]

Particle	Established quantum no. $I(J^{PG})$	Possible assignment: Quantum no. $I(J^{PG})$	Possible assignment: REGGE (¹) trajectory	Mass (MeV)
K_1K_1	$0\,(J^{++}_{\text{even}})$	$0\,(0^{+}{}^{+})$	$+\omega_\alpha$	$\sim 2m_K$
f = vacuum?	$0(>2^{++})$	$0\,(2^{++})$	$+\omega_\alpha$	1250
η	$0(0^{-+})$		$+\omega_\beta$	548
ω	$0(1^{--})$		$-\omega_\gamma$	782
φ	$0\,(J^{--}_{\text{odd}})$	$0(1^{--})$	$-\omega_\gamma$	1020
π $\begin{cases}\pi^0\\ \pi^\pm\end{cases}$	$1(0^{--})$		$-\pi_\beta$	135 140
ρ	$1\,(1^{-+})$		$+\pi_\gamma$	750
K $\begin{cases}K^0\\ K^\pm\end{cases}$	$\frac{1}{2}(0^-)$		$\varkappa_\beta$	498 494
$K^*_{\frac{1}{2}}$ (888)	$\frac{1}{2}(1^-)$		$\varkappa_\gamma$	888
$K^*_{\frac{1}{2}}$ (725)	$\frac{1}{2}$ (?)	?	?	725

(?) Means data that either I have not seen, or of which I am not yet convinced.

(¹) The reader can use the data without reference to this shorthand notation. The first [illegible] perhaps the only useful) contraction comes in choosing a single symbol to denote baryon number B, [illegible] geness S, and I-spin I. Thus for the $S=0$ meson with $I=0$ (like ω) we chose ω. For the S [illegible] 0 meson [illegible] $I=1$ (like π, ρ) we chose π. For K and $K^*_{\frac{1}{2}}$ we chose a Greek ϰ. Suggestive names ($\mathcal{N}$, Λ, Σ, Ξ) [illegible] for the baryons with $I=\frac{1}{2}$, 0, and 1. For $I=\frac{3}{2}$ [*e.g.*, the $\mathcal{N}^*_{\frac{3}{2}}$ ($\frac{3}{2}^+$, 1238) and $\mathcal{N}^*_{\frac{3}{2}}$ (1922) isobars], we [illegible] symbol Δ; if $\Xi^*_{\frac{3}{2}}$ shows up, we suggest O (omicron). One shock is that Λ ($I=0$) now stands for some [illegible] that can break up into Σπ, but is forbidden by conservation of I to break up into Λ and a sin[illegible]

The symbols above are useful *independent* of the idea of a Regge trajectory. *In addition*, the [illegible] conjecture suggests that particles (*e.g.*, ω, $\mathcal{N}$, Δ, etc.) having the same parity, but J-values differing [illegible] can lie in the same trajectory. To emphasize this point, and to further condense the notation, we su[illegible] the following *subscripts* to denote parity and a string of J's differing by 2:

Subscript	*For mesons*	*For baryons*
α	$0^+, 2^+, \ldots$ (*e.g.*, vacuum or ABC)	$\frac{1}{2}^+, \frac{5}{2}^+, \ldots$ (thus $p = \mathcal{N}_\alpha$)
β	$0^-, 2^-, \ldots$ (*e.g.*, π-meson)	$\frac{1}{2}^-, \frac{5}{2}^-, \ldots$
γ	$1^-, 3^-, \ldots$ (γ for « vector »)	$\frac{3}{2}^-, \frac{7}{2}^-, \ldots$ (*e.g.*, $D_{\frac{3}{2}}$ Kp resonance Y^*_0 (1520))
δ	$1^+, 3^+, \ldots$ (none known)	$\frac{3}{2}^+, \frac{7}{2}^+, \ldots$ (*e.g.*, the $\frac{3}{2}$, $\frac{3}{2}$ isobar Δ_δ)

interacting states (April 1963, A. H. ROSENFELD).

Mass² (GeV)²	Dominant decays: Mode	%	Q (4) (MeV)	p or p_{max} (MeV/c)
	Even number of pions $K\bar{K}(K_1K_1,\ K_2K_2$ not $K_1K_2)$		<0	<0
1.56	2π 4π $K\bar{K}(K_1K_1,\ K_2K_2$ not $K_1K_2)$	large < 30 ?	980 710 256	690 550 380
0.30	$\pi^+\pi^-\pi^0$ $\pi^0\pi^0\pi^0$ (3) $\pi^+\pi^-\gamma$ $\gamma\gamma$	23 39 7 31	134 143 269 548	174 182 235 274
0.62	$\pi^+\pi^-\pi^0$ (3,5) $\pi^0\gamma$ $\pi^+\pi^-$	84 12±4 4	368 647 503	326 379 364
1.04	$K\bar{K}(K_1K_2$ not $K_1K_1,\ K_2K_2)$ Odd number of pions		24	111
0.018 0.02	$\pi^0 \to \gamma\gamma$ (6) $\pi^\pm \to \mu\nu$	100 100	135 34	67 30
0.56	$\pi\pi$ (3) (p-wave)	100	471	348
0.24	$K^0 \to \pi^+\pi^-$ (6) $K^\pm \to \mu\nu$	$\frac{2}{3}K_1$ 58	219 388	206 236
0.78	$K\pi$ (p-wave)	100	251 ($K^0\pi^-$)	283
0.53	$K\pi$	?	101 ($K^-\pi^0$)	161

G parity is written as a prescript (this avoids confusion with the charge of a particle). In the past it has been conventional to use an asterisk to indicate an excited state; instead we use a roman superscript to indicate a rotational recurrence. Thus the α-baryons are written $\mathcal{N}_\alpha$ for the proton ($J^P = \frac{1}{2}^+$), and $\mathcal{N}_\alpha^{II}$ (1688) for the 900 MeV $\pi\mathcal{N}$ resonance, which is known to have $J = \frac{5}{2}$ and which we guess has positive parity and is the «second occurrence» of $\mathcal{N}_\alpha$. Where its properties are essentially unknown, a particle has been given the simplest possible assignment merely because it had to be listed somewhere. This notation was evolved in conversations with G.F. CHEW and M. GELL-MANN.

(2) Γ = empirical full width at half-maximum with background subtracted.

(3) For analysis of possible neutral decay modes, see Tables II and III in G of R. LYNCH: *Proc. Phys. Soc.* (*London*), **80**, 46 (1962).

(4) Q values apply to decays to neutral particles (unless that mode is forbidden).

(5) See notes below on this particle.

(6) Common electromagnetic or weak decays are listed for convenience. The masses come from Table I, except for $m(\Xi^-)$ for which see note on Ξ^- below.

TABLE II (*continued*)

Particle	Established quantum no. $I(S^{PG})$	Possible assignment: Quantum no. $I(J^{PG})$	Possible assignment: REGGE [1] trajectory	Mass (MeV)		Γ [2] (MeV)
$\mathcal{N}\begin{cases}\text{n}\\ \text{p}\end{cases}$	$\frac{1}{2}(\frac{1}{2}^+)$		$\mathcal{N}_\alpha$	940 938		0
$\mathcal{N}^*_{\frac{1}{2}}(1688)$ = « 900 MeV πp »		$\frac{1}{2}(\frac{5}{2}^+)$	$\mathcal{N}_\alpha^{\text{II}}$	1688		100
$\mathcal{N}^{*}_{\frac{1}{2}}(1512)$ = « 600 MeV πp »		$\frac{1}{2}(\frac{3}{2}^-)$	$\mathcal{N}_\gamma$	1512		100
$\mathcal{N}^*_{\frac{3}{2}}(1238)$ = « Isobar »		$\frac{3}{2}(\frac{3}{2}^+)$	Δ_δ	1238		100
$\mathcal{N}^*_{\frac{3}{2}}(1920)$	$\frac{3}{2}(\frac{7}{2})$	$\frac{3}{2}(\frac{7}{2}^+)$	$\Delta_\delta^{\text{II}}$	1920		$\sim$200
Λ	$0(\frac{1}{2}^+)$		$\Lambda_\alpha^{\text{II}}$	1115		0
$Y_0^*(1815)$	$0(J>\frac{1}{2})$	$0(\frac{5}{2}^+)$	Λ_α	1815		120
$Y_0^*(1405)$	0(?)	$0(\frac{1}{2}^-)$	Λ_β	1405		50 (

CERN means: *Proc. of the 1962 Intern. Conf. on High-Energy Physics* (New York, 1962). [For … mplete bibliography to 11-7-1961, see M. LYNN STEVENSON: *Bibliography on pion-pion interaction*, … wrence Radiation Laboratory Report UCRL-9999, November 7, 1961 (unpublished)].

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ABC? See A. ABASHIAN, N. E. BOOTH and K. M. CROWE: *Phys. Rev. Lett.*, **7**, 35 (1961).

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Mass² (GeV)²	Dominant decays			
	Mode	%	Q (4) (MeV)	p or p_{max} (MeV/c)
0.88	$e^- \bar{\nu} p$ (6) —	100 —	0.78 —	1.2 —
2.84	$\mathcal{N}\pi$ (f-wave) ΛK (f-wave)	80 <2	610 76	572 235
2.28	$\mathcal{N}\pi$ (d-wave)	80	434 ($\pi^- p$)	450
1.53	$\mathcal{N}\pi$ (p-wave)	100	160 ($\pi^- p$)	233
3.69	$\mathcal{N}\pi$ ΣK	30 <4	842 ($\pi^- p$) 233	722 425
1.24	$\pi^- p$ (6)	67	38	100
3.29	$\bar{K}\mathcal{N}$ $\Sigma\pi$	60 <33	383 490	541 504
1.97	$\Sigma\pi$ $\Lambda 2\pi$	{100}	69 ($\Sigma^- \pi^+$) 10 ($\Lambda\pi^+\pi^-$)	144 69

M. Block, A. Engler, R. Gessaroli and C. Meltzer: *Phys. Rev. Lett.*, **7**, 421 (1961); M. L. Stevenson, L. W. Alvarez, B. C. Maglić and A. H. Rosenfeld: *Phys. Rev.*, **125**, 687 (1962); W. H. Xuong and G. R. Lynch: *Phys. Rev. Lett.*, **7**, 327 (1961); Neutral mode from CERN, p. 712. The $\pi^+\pi^-$ decay mode is a private communication from Murray *et al.*

φ L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, L. Gray, J. Leitner, S. Lichtman and J. Westgard: CERN, p. 297 and *Phys. Rev. Lett.*, **9**, 180 (1962).

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ρ See summary by M. L. Stevenson: UCRL-9999, and CERN (1962).

$\mathcal{N}^*$ For reviews see P. Falk-Vairant and G. Valladas: *Rev. Mod. Phys.*, **33**, 362 (1961); B. J. Moyer: *Rev. Mod. Phys.*, **33**, 367 (1961). For recent data, see J. Helland: *Phys. Rev. Lett.*, **10**, 27 (1963), and CERN, p. 4. The πp phase shift for $\mathcal{N}^*_{\frac{3}{2}}$ (1238) goes through 90° at 1238 MeV, but because of a $\pi\lambda^2$ factor, the πp cross-section reaches its maximum at 1225 MeV; see F. de Hoffman, N. Metropolis, E. F. Alei and H. A. Bethe: *Phys. Rev.*, **95**, 1586 (1954); and N. P. Klepikov, M. G. Meschcheryakov and S. Sokolev: JINR-D-584 (1960). The established quantum numbers of the 600 and 900 MeV πp states $\mathcal{N}_{\frac{1}{2}}$ (1512) and $\mathcal{N}_{\frac{1}{2}}$ (1688) are not given for lack of space; they are $\frac{3}{2}$? and $\frac{5}{2}$? At 1640 MeV invariant mass in $I = \frac{1}{2}$ there is another shoulder, probably not a pure resonance.

1. – Some comments on widths Γ.

The widths listed in Table II are completely empirical half-widths at half-maximum, even though the resonance may have $J > 0$ and not be described by a simple resonance curve. However, background has been subtracted and experimental resolution unfolded.

For all the well-established particles, except φ and Ξ^*, the widths seem to be rather reasonable. At first it may seem surprising that $Y_0^*(1520)$ has Γ only 15 MeV, but it should be remembered that before it can break up it has to penetrate a *d*-wave barrier. I do not understand, why φ has $\Gamma < 5$ MeV and Ξ^* has $\Gamma < 7$ MeV.

2. – K^0K^0 attractions at about 1 GeV.

Also not listed in Fig. 2 are: (*a*) the $K_1^0K_1^0$ attraction [J and C both even, probably $I=0$, *e.g.* $0(0^{++})$] reported recently (as indicated in Table II) by ERWIN *et al.*, ALEXANDER *et al.*, and BIGI *et al.*; (*b*) the $K_1^0K_2^0$ attraction [J and C both odd, probably $I=0$, *e.g.*, $0(1^{--})$], reported by BERTANZA *et al.* (and now called the φ) and $K_{\frac{1}{2}}^*$ (888) because we do not know where to put it.

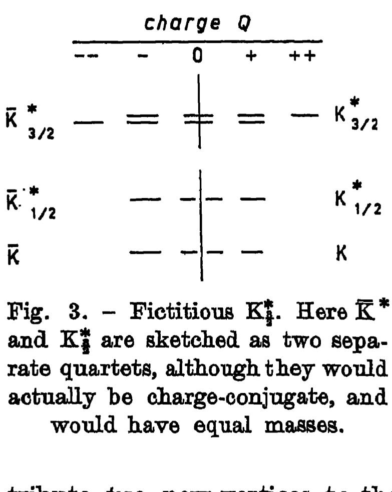

Fig. 3. – Fictitious $K_{\frac{3}{2}}^*$. Here $\bar{K}^*$ and $K_{\frac{3}{2}}^*$ are sketched as two separate quartets, although they would actually be charge-conjugate, and would have equal masses.

In concluding our discussion of displays like that of Fig. 2, we would like to sketch, in Fig. 3, a fictitious quartet, $K_{\frac{3}{2}}^*$, and its antiquartet, $\bar{K}_{\frac{3}{2}}^*$. Although these particles do not exist (or, if they exist, they have certainly not been found), they help us visualize both new vertices which Crawford has to introduce in his Puppi tetrahedron as extended to allow for $\Delta|I| = \frac{3}{2}$, with $\Delta S = \pm\Delta Q$ (see Section 5 of Crawford's lectures).

Note that the $K_{\frac{3}{2}}^*$, $\bar{K}_{\frac{3}{2}}^*$-mesons would contribute *two* new vertices to the Puppi tetrahedron with charge $Q=+1$. $K_{\frac{3}{2}}^{*+}$ corresponds to the one which, like the real $K_{\frac{1}{2}}^{*+}$, decays into leptons with $\Delta S = \Delta Q$; but $\bar{K}_{\frac{3}{2}}^{*+}$, with $S=-1$, must decay with $\Delta S = -\Delta Q$.

3. – Determination of J.

J for most of the particles has been established by data on angular distributions, but, for a few particles, lower limits have been set merely by general considerations of the total cross-section; we now discuss this point.

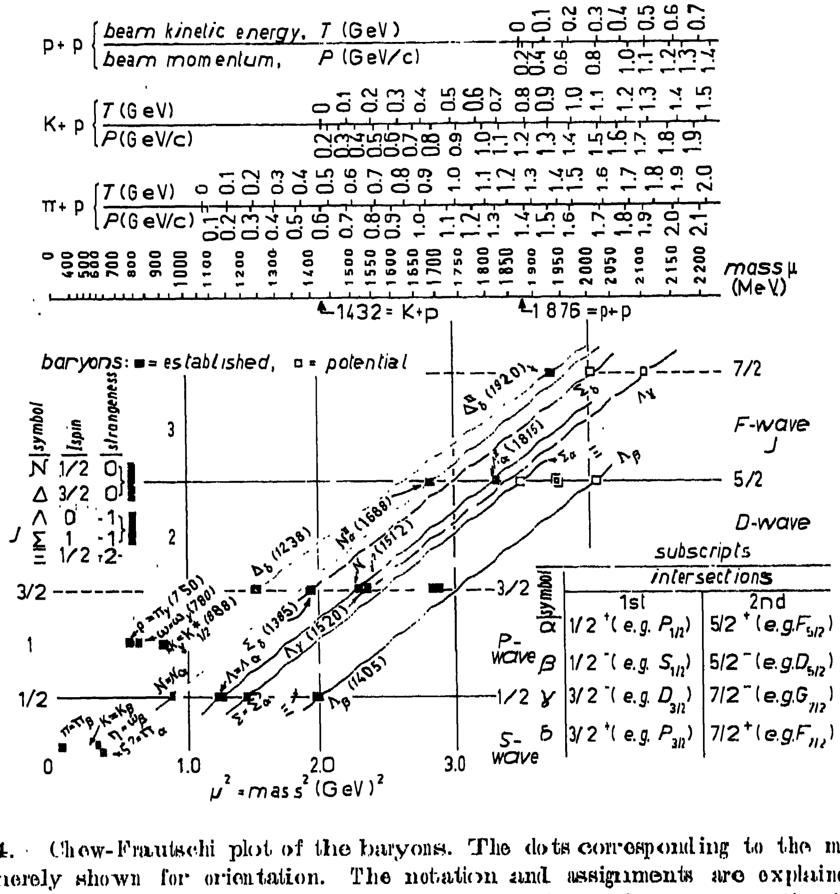

Fig. 4. – Chew-Frautschi plot of the baryons. The dots corresponding to the mesons are merely shown for orientation. The notation and assignments are explained in Table II and its references. The solid lines correspond to «signature $\frac{1}{2}$»; the dashed lines to «signature $\frac{3}{2}$»: *i.e.*, Regge trajectories of signature $\frac{1}{2}$ can generate particles of spin $\frac{1}{2}+2n$ (namely, $\frac{1}{2}$, $\frac{5}{2}$, etc.). These values of J are shown as solid black lines, joined by solid colored trajectories; dashed lines indicate the signature-$\frac{3}{2}$ set.

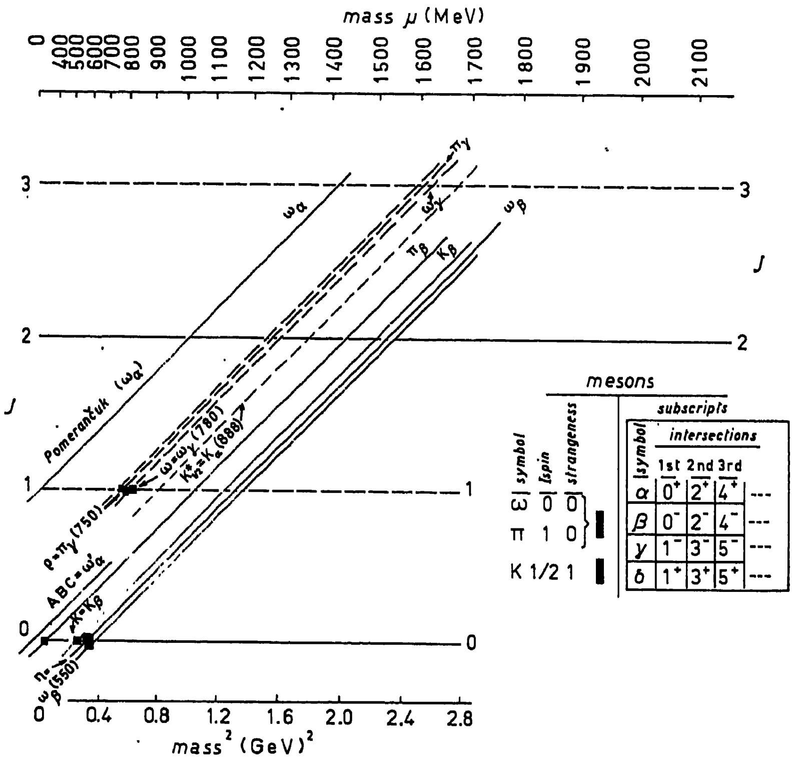

Fig. 5. – Chew-Frautschi plot of the mesons.

Consider elastic scattering,

$$\mathrm{A}+\mathrm{B}\to\mathrm{A}+\mathrm{B}\,,$$

where the target B can be a nucleon or a peripheral pion.

If the A and B are spinless, then

$$\sigma\leqslant 4\pi\lambda\!\!\!^{-2}(2J+1)\sin^2\delta\,, \tag{32}$$

where $\delta=$ phase shift, $J=$ total angular momentum of the state. (The equality sign refers to the case in which the elastic scattering channel is the only channel.)

For a resonant state, when the phase shift is 90°,

$$\sigma\leqslant 4\pi\lambda\!\!\!^{-2}(2J+1)\,. \tag{33}$$

In the general case of particles with spin [15] but no isospin

$$\sigma\leqslant\frac{1}{(2S_1+1)(2S_2+1)}\,4\pi\lambda\!\!\!^{-2}(2J+1)\sin^2\delta\,,$$

where S_1 and S_2 are the spins of the two incoming particles.

The appropriate Clebsch-Gordan coefficients must still be applied for taking isospin into consideration.

The case in which the target is a π leads us to a discussion of the ρ-meson in Section 4.

4. – Mnemonics for quantum numbers of nonstrange baryons.

For a given πp orbital angular momentum l like the p-wave 1^+, we can make $J^P=\frac{1}{2}^+$ or $\frac{3}{2}^+$. Ispin $\frac{1}{2}$ chooses to form a particle (the $\frac{1}{2}^+$ nucleon) which has the lower value of J; $I=\frac{3}{2}$ forms a resonance (the $\frac{3}{2}^+\Delta$) which has the higher value. This same association of lower I with lower J, higher I with higher J, seems to be general. J. A. Helland of Lawrence Radiation Laboratory illustrates it as shown in Table III. For a more recent treatment of Sections 3 and 4 see reference [15a].

TABLE III. – *Nonstrange baryons.*

Symbol	State $J^P=(l-\frac{1}{2})^P$	Angular momentum and parity	State $J^P=(l+\frac{1}{2})^P$	Symbol
No particle		s-wave $=0^-$	$\frac{1}{2}^-$	No particle
$\mathcal{N}_\alpha$ (938)	$\frac{1}{2}^+$	p-wave $=1^+$	$\frac{3}{2}^+$	Δ_δ (1238)
$\mathcal{N}_\gamma$ (1512)	$\frac{3}{2}^-$	d-wave $=2^-$	$\frac{5}{2}^-$	No particle
$\mathcal{N}_\alpha$ (1688)	$\frac{5}{2}^+$	f-wave $=3^+$	$\frac{7}{2}^+$	Δ_δ (1920)

III. - Sixteen Regge Trajectories.

Figure 4 and 5 are Chew-Frautschi plots (of all the particles) as described by Drell elsewhere in this volume. The notation is that explained in the notes to Table II, and the optimistic assignments correspond to the « possible assignment » columns of that table. It should be pointed out that the Regge trajectories really are somewhat S-shaped curves about which little is known, so our straight lines represent only absence of information.

For the baryons (Fig. 4) there are three possible trajectories that may each join two established particles.

For the mesons (Fig. 5) the situation is still a very sorry one—no two known mesons lie on the same trajectory.

The slopes of all trajectories (baryon and meson) correspond to a range $R \approx \frac{1}{2}(\hbar/m_\pi c)$, and agree with current information on the slope of the vacuum trajectory near $m^2=0$.

IV - Peripheral Collisions, the ρ-Meson.

1. – One-pion exchange.

Consider the reaction $\pi^+ p \to p\pi^+\pi^0$. It can be represented by the « one-pion-exchange » diagram of Fig. 6, where the target proton has a laboratory 3-momentum $\boldsymbol{P}=0$, and recoils with momentum $\boldsymbol{P}'$. In 4-vector notation, where the energy is called P_0 and the 3-momentum $\boldsymbol{P}$ has components P_1, P_2, P_3, we can then write

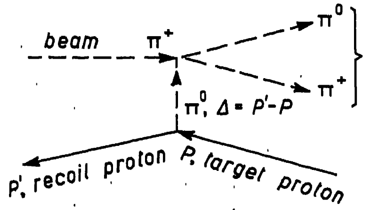

Fig. 6. – Production of a ρ meson by a peripheral collision.

$$P=\begin{pmatrix} M \\ 0 \end{pmatrix}, \qquad P'=\begin{pmatrix} P'_0 \\ \boldsymbol{P}' \end{pmatrix},$$

$$P'-P \equiv \Delta = \begin{pmatrix} P'_0 - M \\ \boldsymbol{P}' \end{pmatrix} \approx \begin{pmatrix} 0 \\ \boldsymbol{P}' \end{pmatrix},$$

note that $P^2 = M^2 - 0^2 = M^2$, $P'^2 = E'^2 - |\boldsymbol{P}'|^2 = M^2$, and $\Delta^2 \approx 0^2 - |\boldsymbol{P}'|^2$ which is negative.

Chew and Low [16] pointed out that the 4-pion vertex would represent real $\pi\pi$ scattering if it were possible (it is not) to choose Δ^2 (which represents the exchange π) to have the value $+m_\pi^2$.

At that point ($\Delta^2 = m_\pi^2$) Chew and Low tell us that the experimental cross-section for production of a π^0 leading to a ρ with total energy ω is

$$\frac{d^2\sigma}{d\Delta^2\, d\omega^2} = \frac{f^2(=0.08)}{2\pi}\left(\frac{\Delta/m_\pi}{\Delta^2 - m_\pi^2}\right)^2 \begin{pmatrix}\text{kinematic}\\ \text{factors}\end{pmatrix} \cdot \sigma_{\pi\pi}(\omega)\,. \tag{34}$$

The factors $f^2(\Delta/m_\pi)^2$ represent the probability of creating a p-wave pion at the πp vertex in Fig. 6. The factor $\sigma_{\pi\pi}$ represents the probability of real $\pi\pi$ scattering at the second vertex, and $1/(\Delta^2 - m_\pi^2)$ is the pion propagator.

The experimental importance of (34) is that it sometimes holds in the physical region (*i.e.*, $\Delta^2 < 0$) for small $|\Delta|$, say < 400 MeV/c.

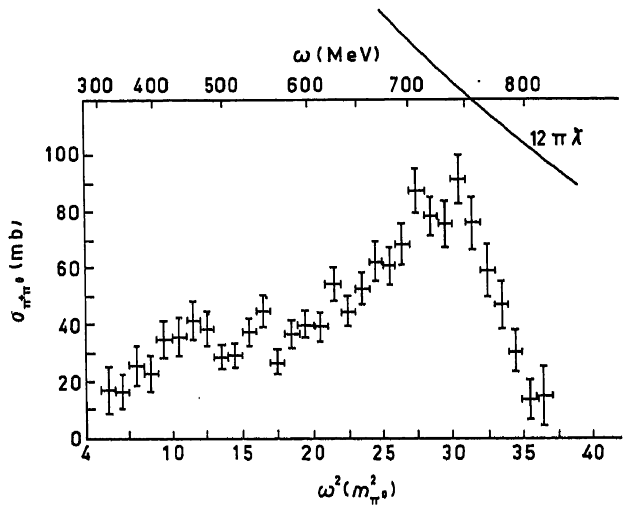

Fig. 7. – The total $\pi\pi$ cross-section $\sigma_{\pi\pi}$ as a function of the dipion total energy ω squared, as determined in the physical region from the reaction 1.255 GeV/c $\pi^+ + p \to p\pi^+\pi^0$. The line labeled $12\pi\lambda\!\!\!^-{}^2$ is the cross-section given by eq. (32) with $J=1$ and $\delta = 90°$. From D. D. Carmony, ref. [17].

Figure 7 displays ρ production by 1.255 GeV/c $\pi^+ + p$, with the recoil momentum $\boldsymbol{P}'$ selected to be < 400 MeV/c.

Both charged and neutral ρ have now been produced in many different reactions, yet ρ^{++} has never been seen so that it is clear that ρ has ispin $=1$. Hence by [25] $J=1$ or 3. The fact that $\sigma_{\pi\pi} \approx 12\pi\lambda\!\!\!^-{}^2$ as shown in Fig. 7 suggests strongly that $J=1$.

2. – Other tests on the peripheral production of ρ.

2·1. *Angular distribution test.* – Consider the $\pi\pi$ vertex in Fig. 6, in the $\pi\pi(=\rho)$ rest frame. If we measure an angle θ with respect to the beam direction, the ρ wave function must be $Y_J^0(\cos\theta)$; and if ρ has a spin $J=1$, then its amplitude must be $\cos\theta$. Hence the angular distribution of the decay of ρ in its own rest frame must be $\cos^2\theta$. For some experiments this test works beautifully [17, 18]; sometimes it does not work so well [19]. But $\cos^2\theta$ is always the dominant term, and this adds further to our conviction that ρ indeed has $J=1$.

2·2. *Treiman-Yang test.* – TREIMAN and YANG [20] have pointed out that if the two vertices of Fig. 6 are connected only by a spinless pion, then the overall reaction is azimuthally symmetric about the direction of the exchanged pion. This means that there must be no correlation between the azimuths of the plane of the ρ produced at the $\pi\pi$ vertex and the plane made by the target and the recoil proton. In the laboratory reference frame the latter plane is not defined, so it is conventional to choose for a reference frame either the rest frame of the ρ, or of the beam pion. We next want to give two examples of reactions which probably would introduce a correlation between these planes. *First*, suppose that there is a final-state interaction between the scattered π^+ and the recoil proton. This could be important if certain orientations of the $\pi\pi$ scattering permitted the formation of the Δ isobar. It is then easy to see that a correlation between planes would be introduced. *Second* suppose that only a single particle was exchanged, but that it had a nonzero spin; again a correlation could exist.

2·3. *Results of above tests on ρ production.* – PICKUP *et al.* [21] have applied tests 1 and 2 to about 100 ρ produced by 1.4 GeV/c $\pi^-p \rightarrow p\pi^-\pi^0$. They report that if they choose $\Delta < 80$ MeV/c ($\Delta/m_\pi < \frac{1}{2}$) and select the dipion mass ω to lie inside the ρ band, the Treiman-Yang test is satisfied within statistics (*i.e.* there is no correlation). But although the angular distribution of $\theta_{\pi\pi}$ is nearly $\cos^2\theta$, there is a definite linear term which seems to be related to a competing process, namely Δ formation, which cannot proceed via one-pion exchange (« OPE »).

CARMONY and VAN DE WALLE [19] and XUONG [21] tested the 1500 ρ produced by 1.2 GeV/c $\pi^+p \rightarrow p\pi^+\pi^0$ which are shown in Fig. 7. In contrast with PICKUP *et al.*, they can include data for Δ up to 400 MeV/c ($\Delta/m_\pi = 3$) and still get an impressive (but misleading) $\cos^2\theta_{\pi\pi}$ angular distribution. But then

when they apply the Treiman-Yang test it fails, and cannot be satisfied until the selection of events is further restricted to areas of the Dalitz plot far from the bands corresponding to Δ.

In conclusion, present experience with the ρ shows that the combination of tests 1 and 2 is a quite sensitive measure of OPE.

V. - Mesons (η, ω, K) that Decay into 3π.

We shall study the decay modes $M \to 3\pi$, where M stands for any of the mesons ω, η, K^+, K_1^0, K_2^0. We limit our considerations to mesons with I and J both $\leqslant 1$.

1. - Dalitz-Fabri variables.

Following Dalitz, the kinematics of the 3π decay may be analysed as shown in Fig. 8, where $\boldsymbol{q}$ denotes the relative π_2-π_3 momentum in the $(\pi_2\pi_3)$ rest frame, and $\boldsymbol{p}$ the momentum of π_1 in the rest frame of M.

In his lectures here at Varenna, H. K. TICHO discusses the general properties of a *Dalitz plot* and shows that unit area is proportional to Lorentz-invariant phase space and that collinear events lie on the boundary. Dalitz also invented the idea of displaying the symmetry of the pions by picking energy axes normal to the bases of an equilateral triangle (see GELL-MANN and ROSENFELD, Appendix C), as shown in Fig. 9. For every point P inside the triangle one has $PL+PM+PN=$ constant $=$ height. If the height is taken equal to the Q value of the reaction, then the geometrical relation that we have just written represents the energy conservation relation $T_1+T_2+T_3=Q$. Every point P thus may represent a decay event, provided momentum is conserved.

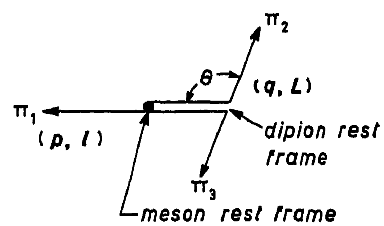

Fig. 8. – Dalitz-Fabri co-ordinates for three-body systems.

We shall soon discuss the population of the events on a Dalitz plot and express the wave function in terms of powers of p and q: p^l and q^L. It is worth noticing that one can qualitatively associate angular momenta $\boldsymbol{l}$ and $\boldsymbol{L}$ with these two exponents. To see this association, write down the l-wave eigenfunction ψ_l for a free pion as

$$\psi_l(r, \theta) \approx j_l(pr)\, Y_l(\cos\theta)\ ,$$

where $j_l(pr)$ is a Bessel function and Y_l is a spherical harmonic. In the small radius approximation $(pr \ll l)$ the leading terms of the spherical Bessel functions are proportional to p^l (or q^L), so that the total wave function for the 3π system is proportional to the leading terms $p^l q^L$.

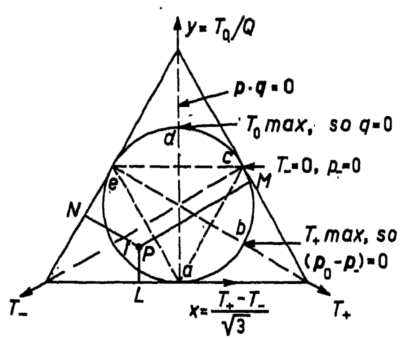

Fig. 9. – Triangular Dalitz plot. Conservation of momentum forces events to lie inside either the circle or the triangle *a-c-e* in the nonrelativistic or relativistic limits, respectively.

We conclude that if we see a Dalitz plot which behaves for small p like $|\psi|^2 \propto |p^l|^2 = p^{2l}$, then the angular momentum involved must be mainly l, with perhaps some admixture of $l+2$, $l+4$, etc., whose presence we cannot detect.

To form the spin J of the meson we simply write

$$J = l + L \tag{35}$$

(*i.e.*, there are no complications with intrinsic spins, since we deal here with three spinless pions).

The «orbital» parity of $\psi(p^l, q^L)$ is $(-1)^{l+L}$. However, the parity P of our meson M has an extra factor of $[P(\text{pion})]^3 = (-1)^3 = -1$, so

$$P\,(\text{meson}) = -(-1)^{l+L}. \tag{36}$$

Note that for a $J=0$ meson, l and L must be equal, so $l+L$ is even and $P(J=0 \text{ meson}) = -1$, *i.e.* a 0^+ meson simply cannot decay into three pions.

To take advantage of the fact that *neutral* mesons are in an eigenstate of C, we find it convenient to choose L so that the dipion is *neutral*, *i.e.*

$$\boldsymbol{q} = \boldsymbol{p}_{\pi^+} - \boldsymbol{p}_{\pi^-}, \qquad \boldsymbol{p} = \boldsymbol{p}_{\pi^0}.$$

Then C is $(-1)^L$.

For *charged* mesons we get a simplification if we let L describe the $\pi^+\pi^+$ (or $\pi^-\pi^-$) dipion. Then L can only be even.

2. – Ispin considerations.

Next we introduce ispin, *i.e.* we write the overall ψ as the product of $\psi(\boldsymbol{p}, \boldsymbol{q})$ or $\psi(l, L)$, times ψ_I (ispin):

$$\psi_{3\pi} = \psi(\boldsymbol{p}, \boldsymbol{q})\,\psi(I_1, I_2, I_3). \tag{37}$$

Bose statistics requires that ψ be symmetric under interchange of any of the two pions. But since we have factored ψ in (37) each ψ factor separately must have identical symmetries under interchange.

ψ_I can represent $I=3, 2, 1$, or 0; we discuss only the cases 1 and 0.

Case I: $I=0$ (*sextant symmetry*). We want to discuss the properties of $\psi_0(I_1, I_2, I_3)$. To simplify the notation, write $\boldsymbol{I}_1=\boldsymbol{a}$ (when I_1 is the ispin vector of the first π), $I_2=\boldsymbol{b}$, $I_3=\boldsymbol{c}$.

Then there is only one way to combine $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ to make $I=0$, namely,

$$\psi_{I=0}=\boldsymbol{a}\cdot\boldsymbol{b}\times\boldsymbol{c}\,, \tag{38}$$

which is totally antisymmetric, so $\mathrm{M}\to3\pi^0$ is forbidden. Since the ispin factor is antisymmetric in the exchange of any two pions, the spatial factor $\psi(\boldsymbol{p}, \boldsymbol{q})$ must have the same property, and its square, which is the population of events on a Dalitz plot, must be symmetric. This means that the Dalitz-plot population must be symmetric about every median; *i.e.* it must have « sextant symmetry ». It is conventional in discussing $I=0$ Dalitz plots to fold all the data until they fall into one sextant; this simplifies the statistical analysis.

This total antisymmetry can also be seen in a more familiar way. Take any of the three pions: it has $I=1$, therefore the remaining dipion must be in an $I=1$ state if the 3π final state has $I=0$. A dipion state with $I=0$ or 2 would be symmetric, but our $I=1$ state is antisymmetric. (The ρ-meson is a familiar example.) Since we have chosen an arbitrary pion to start with, we conclude that the $I=0$ three-pion state is *totally* antisymmetric.

Case II: $I=1$. We can have a totally symmetric isovector state

$$\psi^{\text{symm}}_{I=1}(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) = \boldsymbol{a}(\boldsymbol{b}\cdot\boldsymbol{c}) + \boldsymbol{b}(\boldsymbol{c}\cdot\boldsymbol{a}) + \boldsymbol{c}(\boldsymbol{a}\cdot\boldsymbol{b})\,. \tag{39}$$

But we can also form states of the nonsymmetric (nsym) form:

$$\psi^{\text{nsym}}_{I=1} = \boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) \text{ or } \boldsymbol{b}\times(\boldsymbol{c}\times\boldsymbol{a}) \text{ or } \boldsymbol{c}\times(\boldsymbol{a}\times\boldsymbol{b})\,. \tag{40}$$

By $\boldsymbol{b}\cdot\boldsymbol{c}$ in (39) we mean the iscalar combination of two vectors. Using the Clebsch-Gordan table included in Crawford's lectures, we have

$$\boldsymbol{b}\cdot\boldsymbol{c} = \frac{1}{\sqrt{3}}\,(b^+c^- - b^0c^0 + b^-c^+)\,. \tag{41}$$

Assuming $I_{3\pi}=1$, and assuming the symmetrical form (39), we now show with the help of Clebsch-Gordan coefficients, that the ratio $R_\eta \doteq \pi^+\pi^-\pi^0/\pi^0\pi^0\pi$ is proportional to $\frac{2}{3}$ whereas the ratio $R_{K^+}=\pi^+\pi^+\pi^-/\pi^+\pi^0\pi^0 \propto 4$. We say « proportional to » rather than « equal to » because there is still a phase-space factor which is slightly larger for $\pi^0\pi^0$ than for $\pi^+\pi^-$.

Proof. Combining (39) and (41), we have

$$(42)\quad \psi_1^{\text{symm}} = \sqrt{\tfrac{1}{3}}\,\{a(b^+c^- - b^0c^0 + b^-c^+) + b(a^+c^- - a^0c^0 + a^-c^+) + c(a^+b^- - a^0b^0 + a^-b^+)\}.$$

For the decay of a neutral meson like η we are interested in the *neutral* component of (42), so that we have

$$(43)\quad \psi_1^{\text{symm}} = \sqrt{\tfrac{1}{3}}\,\{a^0(b^+c^- - b^0c^0 + b^-c^+) + b^0(a^+c^- - a^0c^0 + a^-c^+) + c^0(a^+b^- - a^0b^0 + a^-b^+)\}.$$

The $\pi^+\pi^-\pi^0$ state appears six times, but with different (noninterfering) permutations of a, b, c, so that when one squares the amplitudes one gets $\frac{1}{3}(6\times 1^2)=\frac{6}{3}$. The $\pi^0\pi^0\pi^0$ state appears in three terms, and they clearly interfere, so squaring it, we get $\frac{1}{3}\,3^2=\frac{9}{3}$. Therefore the ratio

$$(44)\qquad R_1^{\text{symm}} = R_\eta = \frac{\pi^+\pi^-\pi^0}{3\pi^0} = \frac{6}{9} = \frac{2}{3}\,.$$

In the case of K^+-decay we show below that (39) again dominates over (40). For K^+ we are interested in the positive component of (39), *i.e.*,

$$(45)\quad \psi_1^{\text{symm}} = \sqrt{\tfrac{1}{3}}\,\{a^+(b^+c^- - b^0c^0 + b^-c^+) + b^+(a^+c^- - a^0c^0 + a^-c^+) + c^+(a^+b^- - a^0b^0 + a^-b^+)\}\,.$$

The $\pi^+\pi^+\pi^-$ state appears twice as $a^+b^+c^-$, twice as $a^+b^-c^+$, and twice as $a^-b^+c^+$, so that, squaring the amplitudes, we get $\frac{1}{3}(3\times 2^2)=4$. The $\pi^+\pi^0\pi^0$ state appears three times, always in a different permutation, so that $\pi^+\pi^0\pi^0 \propto \frac{1}{3}(3\times 1^2)=1$. Hence, for K^+, we have

$$(46)\qquad R_1^{\text{symm}} = R_{K^+} = \frac{\tau}{\tau'} = \frac{\pi^+\pi^+\pi^-}{\pi^+\pi^0\pi^0} = 4/1\,.$$

3. – Allowed *vs.* *G*-forbidden decays.

In Sections 4 and 5 below we discuss the properties of 3π states $\psi(3\pi)$ with $I=0$ and 1, respectively, but we do not want to imply that the meson producing these states necessarily has the same quantum numbers as $\psi(3\pi)$;

after all, K-mesons decay to 3π via the weak interaction, which does not conserve parity and we shall show that $\eta \to 3\pi$ only after emitting and reabsorbing a virtual photon.

This brings us to the topic of « G-forbidden » em decays. Consider the η, whose quantum numbers are $0(0^{-+})$. A 0^- meson cannot decay to 2π. It can decay to $\gamma\gamma$ and to $\gamma\pi^+\pi^-$, but these modes seem to be so slow that a 3π mode competes, even though it is G-forbidden. How can this be? We have already shown in (29) that a single photon changes C and hence must *either* change G or else change I by one unit. Suppose now that this photon is merely virtual, and is reabsorbed as sketched in Fig. 10. The first vertex changes C, and the second vertex changes it back again; *i.e.*, this juggling of a virtual photon preserves C. Since there are two em vertices, $\Delta|I|$ can be 0, 1, or 2. Our rule $G=C(-1)^I$, eq. (9), then says that if $\Delta|I|=1$, G changes; if $\Delta|I|=0$ or 2 then G cannot change and nothing has been accomplished in the way of producing an intermediate meson state that can then fall apart into 3π. However, we see that photon emission and reabsorption can change G and change I by one unit so as to permit the *isoscalar* η to decay into an *isovector* configuration of 3π. Of course each em vertex decreases the amplitude by a factor e (really $e/\sqrt{\hbar c}$), so that $\psi(3\pi)$ is down by e^2, and $|\psi|^2$ is down by e^4; *i.e.*, the rate is down by approximately 10^4 with respect to comparable G-allowed rates.

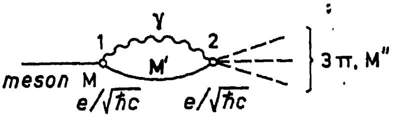

Fig. 10. – G-forbidden decay via emission and reabsorption of a virtual photon.

In summary, G-forbidden decays have the following properties, which have been pointed out by many authors [22-24]:

$$\Delta C = \text{No}\,, \qquad \Delta G = \text{Yes}\,, \qquad \Delta|I| = 1\,, \qquad \Gamma \propto e^4\,. \tag{47}$$

4. – Isoscalar $(C=-1)\,3\pi$ states.

4·1. *General properties*. – In this section we discuss the properties of $I=0$ 3π states, of which there are three: pseudoscalar, axial vector, and vector [eq. (36) rules out the scalar possibility]. Combining (8) and (9), we have

$$G = -1 = C(-1)^I, \tag{48}$$

therefore

$$C = -(-1)^I\,. \tag{48a}$$

So for $I=0$ all $\psi(3\pi)$ states have $C=-1$, and hence—by (15*a*)—L must be odd.

These states can arise from the allowed decay of isoscalar mesons with $G=-1$, or from the G-forbidden decay of isovector mesons with $G=+1$, which according to (9), also have $C=-1$.

In Table IV we have listed in the first column the three possibilities for J

TABLE IV. – $I=0$ 3π states, which necessarily have $C=L=-1$.

Meson properties	Properties of $\psi_{3\pi}(\boldsymbol{p}, \boldsymbol{q}, L, l)$				
Spin and parity	Spatial transformation	C and L	l	Leading terms $\to$ antisymmetrical form	Zeros on Dalitz plot
PS(0^{--})	$S(0^+)$	odd	odd	$\boldsymbol{q}\cdot\boldsymbol{p}\to(w_1-w_2)(w_2-w_3)(w_3-w_1)$	medians
A(1^{+-})	V(1^-)	odd	even	$\boldsymbol{q}\to(\boldsymbol{p}_1-\boldsymbol{p}_2)w_3+(\boldsymbol{p}_2-\boldsymbol{p}_3)w_1+$ $+(\boldsymbol{p}_3-\boldsymbol{p}_1)w_2$	where $q=0$ and center
V(1^{--})	A(1^+)	odd	odd	$\boldsymbol{q}\times\boldsymbol{p}\to(\boldsymbol{p}_1\times\boldsymbol{p}_2)+(\boldsymbol{p}_2\times\boldsymbol{p}_3)+$ $+(\boldsymbol{p}_3\times\boldsymbol{p}_1)$	boundary

and P. We now discuss each row in turn. Although the first row applies to a pseudoscalar *meson*, the $\psi(3\pi)$ must have scalar *spatial* transformation properties. Parity then forces l to be odd also. The simplest possibility is then q^1 and p^1, and the simplest scalar quantity is $\boldsymbol{q}\cdot\boldsymbol{p}=qp\cos\theta$, which vanishes when $\cos\theta=0$, *i.e.*, when $p_+=p_-$ or $E_+=E_-$ (see Fig. 11). Thus ψ vanishes along the vertical median of the Dalitz triangle. The antisymmetrized form is the simplest scalar expression that vanishes along all three medians. $|\psi\ (\text{antisymm})|^2$ is drawn in isometric projection in Fig. 12*B* (taken from ref. [25]).

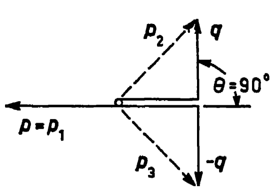

Fig. 11. – Configuration that lies on the vertical median of the Dalitz plot in Fig. 9.

The second row of Table IV is the axial meson (1^+) illustrated in Fig. 12*A*; $\psi_{3\pi}$ must then be a spatial vector (1^-), *i.e.*, l is even, and $\boldsymbol{p}$ need not enter. Then $\psi=\boldsymbol{q}$ is the simplest vector, and ψ must vanish when the dipion has $q=0$, *i.e.*, when π^+ and π^- « touch » in momentum space. To antisymmetrize ψ we first tried $(\boldsymbol{p}_1-\boldsymbol{p}_2)+(\boldsymbol{p}_2-\boldsymbol{p}_3)+(\boldsymbol{p}_3-\boldsymbol{p}_1)$, but because $\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3=0$ this has the unfortunate property of vanishing. The energy factors w_i preserve

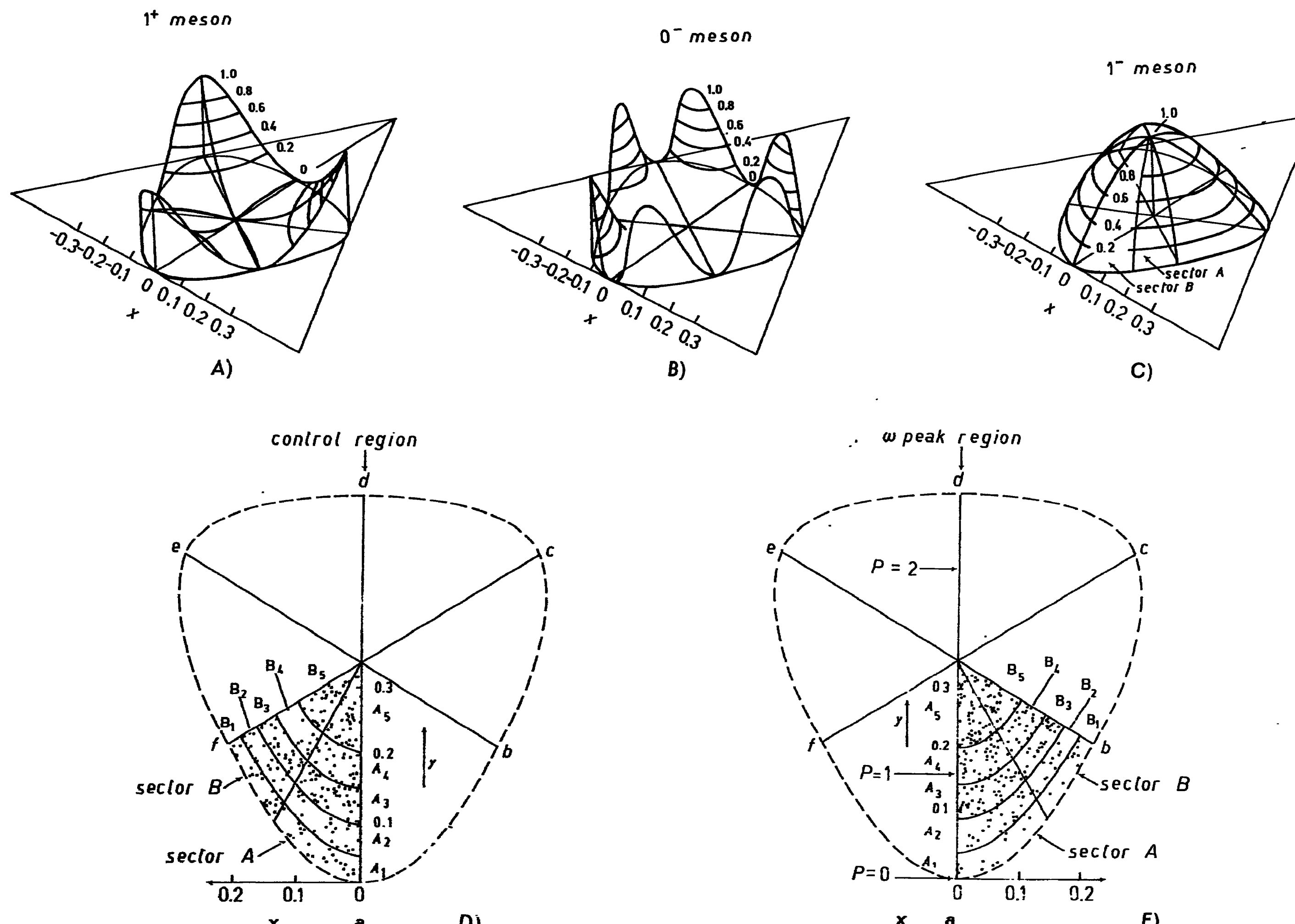

Fig. 12. – *A*), *B*), *C*) Isometric projections of $|\psi|^2$ from Table IV. *D*), *E*) Dalitz plot of ω decays and background (« control region ») events: (*D*) is 241 control events, (*E*) is 270 « ω-region » events. The contour lines are the projections of the contours of (*C*) for a 1^- meson. From STEVENSON *et al.*, ref. [25].

the antisymmetry and make ψ nonzero. Note that ψ (antisymm) $\to 0$ when *any* dipion has $q=0$. It also vanishes at the symmetry point where $w_1=w_2=w_3$. In ref. [1b] DALITZ gives a more general proof that ψ must in fact vanish at the symmetry point for $J=0^-$, 1^+ and 2^-.

For the third row of Table IV, ψ must be an axial vector (1^+), so l is odd and $\psi \propto \boldsymbol{q}\times\boldsymbol{p}$, which vanishes for q and p collinear. Collinear decays correspond to the boundary of the Dalitz plot, as illustrated in Fig. 12*C*. This correspondence is proved in Ticho's lectures; it can be made plausible by noting that both collinear decays and the boundary have one degree of freedom less than the normal configuration.

4·2. *The* ω*-meson.* – The Columbia-Rutgers data on 1100 ω-decays are presented in Fig. 13 [26]. It is clear that the data are perfectly consistent with 1^{--} and not with the other two $I=0$ possibilities listed in Table IV.

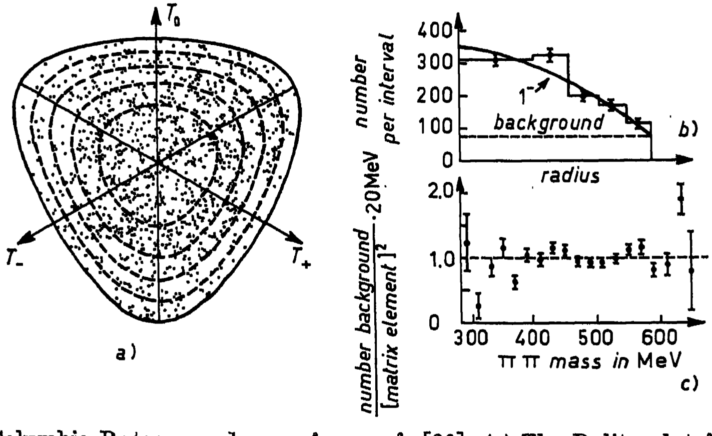

Fig. 13. – Columbia-Rutgers ω decays from ref. [26]. (*a*) The Dalitz plot for 1100 ω's (including a background of 375 nonresonant triplets). (*b*) The density of points on the Dalitz plot compared to the expected density for a 1^- ω plus a uniformly distributed background. (*c*) The dependence of the $\pi\pi$ interaction in the $T=1$ $J=1$ state as a function of energy. This was obtained by summing the $\pi^+\pi^-$, $\pi^+\pi^0$ and $\pi^-\pi^0$ mass spectra for pion pairs from the ω decays, subtracting a background, and dividing by the distribution expected for 1^- decay into $\pi^+\pi^-\pi^0$. Since two of the three mass combinations are independent, an error corresponding to $(\frac{3}{2}N)^{\frac{1}{2}}$, where N is the number of pairs per interval before background subtraction, was assigned to each point.

No charged mode of ω has ever been seen, so we conclude that ω has the quantum numbers $0(1^{--})$.

5. – Isovector ($C = +1$) 3π states.

5·1. *General properties.* – Table V summarizes the properties of the 3π states with $I=1$. Equation (48*a*) requires that the neutral states all have $C=+1$. As discussed in obtaining eq. (47), these 3π states can be reached in several different ways.

(*a*) Strong decay of a meson with any of the quantum numbers $1(0^{--})$, $1(1^{+-})$, or $1(1^{--})$, ... However, no such meson resonances are known, nor can the $\overline{\mathcal{N}}\mathcal{N}$ system have such quantum numbers. The simplest model would be a p-wave $\eta\pi$ resonance $1(1^{+-})$.

(*b*) G-forbidden decay of a $G=+1$ meson with $I=0$ or 2. The η is an example of the $I=0$ case.

(*c*) Weak decay of the K-mesons, as we shall see below.

TABLE V. – *Isovector 3π states, which necessarily have $C = L = +1$.*

Meson	$\psi_{3\pi}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{L}, \boldsymbol{l})$					
J^P	Spatial transformation	C and L	l	Momentum-dependence of leading terms	ψ(I spin)	Zeros on Dalitz plot
PS(0^-)	S(0^+)	even	even	$q^0 p^0 = \text{const}$	mainly symmetric	none
A(1^+)	V(1^-)	even	odd	$\boldsymbol{p}$	nonsymmetric	$p = 0$
V(1^-)	A(1^+)	even	even	$(\boldsymbol{q} \times \boldsymbol{p})(\boldsymbol{q}\cdot\boldsymbol{p})$	nonsymmetric	Boundary and vertical median

For neutral states, $C = +1$, so L is even if it applies to the dipion. For charged states like $\pi^+\pi^+\pi^-$, L must again be even if it applies to $\pi^+\pi^+$.

The reasoning used to construct Table V is identical with that used for Table IV; we take the first row as an example. Given J^P of the meson in column 1, we get spatial parity by changing P by $(-1)^3=-1$. We have just mentioned that (48*a*) requires all the neutral states to have $C=+1$ and hence L even. For the charged states we choose L to apply to the doubly charged diparticle, and then L must be even, by symmetry. We then choose l so as to satisfy the spatial parity. The simplest spatial *scalar* quantity with $L=l=$ even is a constant, so that the Dalitz plot should tend to be uniformly populated. Of course the constant can be multiplied by any scalar quantity,

e.g., $(1+q^2+p^2+p^2q^2+...)$; but η-decay and K-decay both have low Q values, so that barrier penetration suppresses the higher values of L and l, leaving q^0p^0 dominant. Pure q^0p^0 is of course symmetric, so ψ (ispin) is also symmetric and $\pi^0\pi^0\pi^0$ can be produced with $R_\eta = \frac{2}{3}$, as proved in (44). Terms like $(q^0p^0)p^2$ are nonsymmetric and cannot yield $\pi^0\pi^0\pi^0$.

For the remaining rows it is not hard to see that the simplest spatial *vector* with L even is $\boldsymbol{p}$. The simplest spatial *axial vector* is $\boldsymbol{q}\times\boldsymbol{p}$, but this had L odd, so we choose $(\boldsymbol{q}\times\boldsymbol{p})(\boldsymbol{q}\cdot\boldsymbol{p})$. Note that the $\boldsymbol{q}\times\boldsymbol{p}$ factor makes ψ vanish at the boundary of the Dalitz plot, and $\boldsymbol{q}\cdot\boldsymbol{p}$ makes it vanish at the vertical median.

5'2. *The η-meson.* – The η is a neutral meson with a mass of 548 MeV, $\Gamma < 10$ MeV (probably about 1 keV) which decays into several neutral modes $\frac{3}{4}$ of the time and into $\pi^+\pi^-\pi^0$ $\frac{1}{4}$ of the time. References to the data are given in Table II. Note that η has a mass of $4m_\pi + 8$ MeV, so that its Q value for 4π decay is limited.

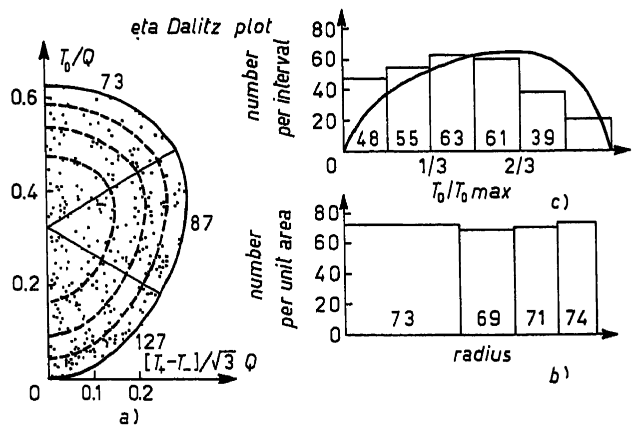

Fig. 14. – The Dalitz plot and projections for all published η decays into $\pi^+\pi^-\pi^0$. (*a*) Shows the distribution of points, (*b*) the radial density, and (*c*) the projection of the points on the π^0 axis.

Eta Dalitz plot.

BERKELEY: *Phys. Rev. Lett.*, **8**, 114 (1962)	23
U.C.R.L. 10219	17
U.C.R.L. 10237	46
B.N.L.: *Phys. Rev. Lett.*, **8**, 329 (1962)	42
Columbia-Rutgers	90
John Hopkins	25
Yale	44
Total as of August 17, 1962	287

The ispin of η is taken to be zero for two reasons: *first*, no evidence for any charged η has ever been observed; *secondly*, the η was discovered in the reaction

$$\pi^+ n \to p\eta^0 .$$

If η has $I=1$, then charge-independence sets a lower limit on the cross-sections $\sigma^\pm$ for the related reactions:

$$\pi^\pm p \to p\eta^\pm .$$

More precisely, there is a « triangle inequality » $\sqrt{\sigma^+}+\sqrt{\sigma^-}\geqslant\sqrt{2\sigma^0}$. But CARMONY *et al.* find no evidence for these reactions and report that the inequality is badly violated [27].

The Dalitz plot for 287 η published up to Aug. 1962 is shown in Fig. 14 (taken from ref. [26]). It has two salient features:

First: It does not show sextant symmetry. If symmetric, the 200 events in the top and bottom sectors would be distributed 100 ± 7 in each; but this is not true by 3.8 standard deviations. So we are not dealing with an $I=0$ state.

Second: Of the three $I=1J^P$ possibilities listed in Table V, none except 0^- comes anywhere near fitting the data. However, the population is not perfectly flat, but instead favors low T_0. A good fit is

$$(49)\qquad |\psi|^2 \propto 1-0.7z , \qquad \text{where } -1\leqslant z\leqslant+1,$$

i.e., $z=1-2(T_0/T_0\,\text{max})$. Therefore within the errors we can write

$$\psi \propto 1-0.35z .$$

The first term, 1, is of course totally symmetric; $0.35z$ is not. Using the method of eq. (40) of GELL-MANN and ROSENFELD [1d] we find that the nonsymmetric term $0.35z$ contributes only about $\frac{1}{4}$ of $(0.35)^2=3\,\%$ to the η-decay rate. Thus the symmetric state dominates η-decay, and we expect (44) to hold, *i.e.*, we expect

$$(50)\qquad \frac{\Gamma(\pi^0\pi^0\pi^0)}{\Gamma(\pi^+\pi^-\pi^0)} \approx \frac{3}{2}\cdot\text{phase space}\approx 1.7 .$$

Therefore we conclude that η is a $0(0^{-+})$ meson and that sometimes it decays slowly to 3π after emission and reabsorption of a virtual photon.

Table II tells us $\Gamma(\eta\to\text{neutrals})/\Gamma(\eta\to\pi^+\pi^-\pi^0)=3$. Combining this with eq. (50), we see that another neutral mode, presumably $\gamma\gamma$, is important;

i.e., assuming no other significant neutral modes, we estimate

$$\frac{\Gamma(\gamma\gamma)}{\Gamma(\pi^+\pi^-\pi^0)} = 3 - 1.7 = 1.3 \,. \tag{51}$$

This mode has recently been reported by the Cambridge bubble chamber group [28], with a branching fraction consistent with (51).

The remaining unsettled question about η is why the G-forbidden 3π mode ($\propto e^4$) seems to be enhanced so that it dominates the $\pi^+\pi^-\gamma$ mode ($\propto e^2 \times p$-wave barrier penetration). Experimentally $\pi^+\pi^-\gamma$/(all modes) seems to be $< 5\,\%$ [23]. Theoretically this has not been explained very satisfactorily, but it is intelligently discussed by Brown and Singer [29], and by Gell-Mann, Sharp and Wagner [30]. Brown and Singer estimate a width Γ about $\frac{1}{4}$ keV, giving a mean life of 3×10^{-18} s.

5·3. K-*meson decay into* 3π. – We wish to discuss very briefly the following decay modes of the spinless K-meson:

$$\tau \text{ and } \tau' \;(i.e.,\ K^+ \to \pi^+\pi^+\pi^- \text{ or } \pi^+\pi^0\pi^0)\,,$$

$$K_2 \to \pi^+\pi^-\pi^0 \text{ or } \pi^0\pi^0\pi^0,$$

$$K_1 \to \pi^+\pi^-\pi^0\,.$$

These modes are discussed by Gell-Mann and Rosenfeld [1*d*], but we want to show that all except K_1 are covered by Table V.

τ-*decay*. Let L refer to the $\pi^+\pi^+$ dipion. Then L is even. The $\Delta|I| = \frac{1}{2}$ rule favors $I=1$. This is then the 0^- row of Table V. The Dalitz plot should be almost uniformly populated. It is. The same remarks apply to τ'. Equation (46) relates τ and τ'.

K_2-*decay*. Tables IV and V remind us that the $J=0$ states of 3π have $P = -1$ (*i.e.*, 0^-). Since $CP|K_2\rangle = -|K_2\rangle$, C must be $+1$. Then $I=1$ is allowed, and is favored by $\Delta|I|=\frac{1}{2}$, so again we expect (and find) the properties of the first row of Table V. Equation (44) again predicts $\pi^0\pi^0\pi^0/\pi^+\pi^-\pi^0 \approx \frac{3}{2}$.

K_1-*decay*. This time, since $CP|K_1\rangle = +|K_1\rangle$, C must be -1, so $I=0$ or 2; $I=2$ is suppressed by $\Delta|I|=\frac{1}{2}$. $I=0$ is suppressed by the complicated $(w_1-w_2)(w_2-w_3)(w_3-w_1)$ form of the antisymmetrized wave function shown in Table IV and Fig. 12*B*. Since $3\pi^0$ can have only $C=+1$ it is forbidden.

* * *

I want to thank I. Derado and G. Giacomelli for their help in writing up these notes, and to acknowledge helpful discussion with R. W. Huff.

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New Mesons and Resonances in Strong-Interaction Physics. Theoretical.

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1. – Historical introduction.

In order to appreciate the roles of the newly discovered mesons and resonances in strong-interaction physics, it is appropriate to review the status of strong-interaction physics prior to 1961. Nonstrange particle phenomena were divided into two classes.

A) The class of phenomena that are understandable on the basis of the Yukawa concept:

$$\mathcal{N} \rightleftarrows \mathcal{N} + \pi ,$$

i.e., the nucleon can emit or absorb a pseudoscalar, charge-triplet pion in a charge-independent manner.

B) The class of phenomena that are completely mysterious on the basis of the Yukawa concept.

We first discuss class *A*) phenomena. On the basis of the Yukawa concept we can understand the tail end of nucleon-nucleon forces, which is given correctly by the famous one-pion-exchange potential. For istance, we can understand why ^3S and ^1S central forces at large distances are attractive, and why there exists a long-range tensor force. The sign of the tensor force determined from the deuteron quadrupole moment is correctly predicted by this kind of theory.

A lot of work has also been done within the framework of the Yukawa concept on pion-nucleon scattering. Especially we understand the so-called

3-3 resonance to some extent; if we try to derive an effective potential on the basis of the idea that the nucleon emits or absorbs the pseudoscalar pion in a charge-independent manner, then the only attractive p-state is the $T=\frac{3}{2}$, $J=\frac{3}{2}^+$ state. Furthermore, it is possible to understand along these lines why s-waves near threshold are important in the π^+ photoproduction

$$\gamma+\mathrm{p}\rightarrow\pi^+ +\mathrm{n}\,,$$

and why s-waves are negligible in the π^0 photoproduction

$$\gamma+\mathrm{p}\rightarrow\pi^0 +\mathrm{p}\,.$$

More quantitatively, CHEW and LOW [1] were able to show that the width of the 3-3 resonance can be related to the rationalized pseudoscalar-pseudovector coupling constant $f^2_{\pi\mathcal{N}\mathcal{N}}$ via

$$\Gamma_{33}=\frac{8}{3}\frac{f^2_{\pi\mathcal{N}\mathcal{N}}p^3_{\mathrm{res}}}{m^2_\pi}\,.$$

This constant can also be measured in five other independent ways—from $\pi\mathcal{N}$ dispersion relations, nuclear forces, photoproduction experiments, etc. All methods yield the value $f^2_{\pi\mathcal{N}\mathcal{N}}\approx 0.08$.

However, attempts to extend the Yukawa concept to other phenomena (class B) phenomena) were not successful.

i) The nature of nuclear forces at very short distances ($\leqslant (3m_\pi)^{-1}$) turns out to be completely mysterious. The existence of the hard core and the spin-orbit force as revealed by high-energy p-p scattering could hardly be understood on the basis of the ordinary pion theory based on the Yukawa concept.

ii) If we turn to low-energy $\pi\mathcal{N}$ scattering, we find that the isospin-dependent part of the s-wave scattering amplitude a_1-a_3 is large, which is quite contrary to the expectation based on the Yukawa theory.

iii) One finds that the average pion multiplicity in anti-proton annihilation processes

$$\mathrm{p}+\overline{\mathrm{p}}\rightarrow\text{many }\pi\text{'s}\,,$$

is larger than is expected on the basis of a statistical theory. Assuming a reasonable volume of interaction, one finds $\langle n\rangle_{\text{expected}}\approx 3$, whereas experiments indicate $\langle n\rangle\approx 5$. A similar tendency is also observed in multipion production in pion-nucleon collisions.

iv) The electron-proton and electron-deuteron scattering experiments done at Stanford and later at Cornell and Orsay show that the mean square

charge radius of the proton is

$$\langle r^2\rangle_p^{\frac{1}{2}} \approx 0.8 \text{ fermi},$$

whereas the charge radius of the neutron is

$$\langle r^2\rangle_n^{\frac{1}{2}} \approx 0.0 \text{ fermi}.$$

On the basis of the simple Yukawa theory based on the virtual processes

$$\mathrm{p} \to \mathrm{n} + \pi^+$$

$$\mathrm{n} \to \mathrm{p} + \pi^-,$$

we expect

$$\langle r^2\rangle_p^{\frac{1}{2}} \approx -\langle r^2\rangle_n^{\frac{1}{2}} > 0,$$

where the negative sign means that the sign of the charge cloud is on the average negative. More quantitative calculations have shown that even the magnitude of the isovector magnetic moment μ_p-μ_n is difficult to account for on the basis of the Yukawa concept.

Now let us turn our attention to strange-particle physics. There were again successful features. Various experiments done at Brookhaven and Berkeley conclusively established the existence of isospin multiplets as predicted by the strangeness scheme of Gell-Mann and Nishijima. Between 1953 and 1960 no new strange particles were discovered apart from Σ^0 and Ξ^0 which were predicted by the strangeness scheme. The number of particles actually decreased in 1957 because people used to think that $\tau \neq \theta$, but with the discovery of parity nonconservation it became clear that τ and θ are different decay modes of one and the same particle.

From a more theoretical point of view, it was natural to extend the Yukawa concept to strange-particle physics. We then have, in analogy with $\mathcal{N} \rightleftarrows \mathcal{N}+\pi$, the following « fundamental » interactions

$$\Sigma \rightleftarrows \Lambda+\pi, \qquad \Sigma \rightleftarrows \Sigma+\pi, \qquad \mathcal{N} \rightleftarrows \Lambda+\mathrm{K}, \qquad \mathcal{N} \rightleftarrows \Sigma+\mathrm{K},$$

$$\Xi \rightleftarrows \Xi+\pi, \qquad \Xi \rightleftarrows \Lambda+\overline{\mathrm{K}}, \qquad \Xi \rightleftarrows \Sigma+\overline{\mathrm{K}}.$$

We see that we must introduce 8 coupling constants. Strong coupling calculations with only one coupling constant are difficult enough; if we had 8, the situation would be hopeless. So people suggested that some of them be equal.

The model of global symmetry [2, 3] assumes the pion-baryon coupling constants to be all equal, but then the kaon-baryon couplings must necessarily be asymmetric; PAIS [4] was able to show that if the K coupling

constants were also equal, reactions such as

$$K^- + p \to \Sigma^+ + \pi^-,$$

would be forbidden. But these « forbidden » reactions are experimentally as frequent as $K^- + p \to \Sigma^- + \pi^+$, etc., which are fully allowed by the symmetry scheme. Thus, considerable amounts of arbitrariness (several adjustable parameters) must be introduced. If the K couplings were weak, the model might have made some sense, but the K couplings turn out to be rather strong in several cases. So calculations made along these lines were not too successful.

Furthermore, from a more fundamental point of view, there is no compelling reason to believe in the global symmetry model. The defects of the model were most clearly expressed by one of its originators, GELL-MANN himself [5] — « The global symmetry makes no direct connection between physical couplings and the currents of the conserved symmetry operators ». We shall later show how to construct a model of strong interactions which avoids these difficulties.

By the end of 1959 it had become evident, both from a purely phenomenological and from a more fundamental point of view, that a new approach to strong-interaction physics was needed. It was about that time that a number of theoreticians came to realize that many of the mysteries could be solved if there existed vector particles that decay via strong interactions into two and three pions.

NAMBU [6], as early as in 1957, suggested that $\langle r^2\rangle_n^{\frac{1}{2}} \approx 0$ might be explained if a $T=0$, $J=1^-$, $G=C=-1$ meson (now known as the ω-meson) existed. Such a meson must be coupled to the nucleon in the following way

$$\omega(\bar{p}p + \bar{n}n)\,.$$

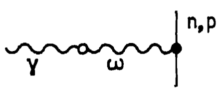

Fig. 1.

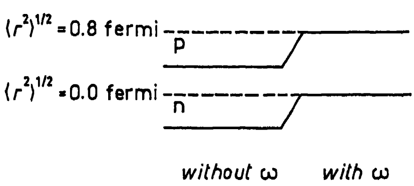

Fig. 2.

Then the graph shown in Fig. 1 gives contributions of the same sign and magnitude to both $\langle r^2\rangle_p^{\frac{1}{2}}$ and $\langle r^2\rangle_n^{\frac{1}{2}}$ so that the negative charge cloud of the neutron might conceivably be canceled by the ω contribution as shown in Fig. 2. Subsequently it was realized by BREIT [7] and myself [8] that such a vector meson would lead to a unified understanding of the hard core and spin-orbit forces in the nucleon-nucleon interaction.

Meanwhile FRAZER and FULCO [9] showed that an *isovector* vector meson ($T=1$, $J=1^-$, $G=+1$ meson, now called the ρ-meson) was needed to explain the isovector part of the anomalous moment form factor of the nucleon. Such an isovector meson would also throw light on the long-standing mystery of the isospin-dependence of the s-wave pion-nucleon scattering amplitude [8, 10], as we shall discuss in detail later.

If the conjectured mesons were to be useful for understanding nuclear forces, nucleon structure, pion-nucleon scattering, etc., they had to be coupled rather strongly to pions and nucleons. If so, they must be copiously produced in $p\bar{p}$ annihilations, $\pi\mathcal{N}$ collisions, etc. Since the conjectured mesons would instantly ($\sim 10^{-23}$ s) decay into two and three pions, the observed, unexpectedly large pion multiplicity in $p\bar{p}$ and $\pi\mathcal{N}$ reactions would no longer be mysterious [8].

To sum up, by the end of 1959, it had become evident that the most urgent task for experimentalists had to center around the search for these mesons (by plotting the effective mass distributions of the two-pion and the three-pion ($\pi^+\pi^-\pi^0$) systems [8]) which seemed to be the key to many of the mysteries in strong-interaction physics. Just around that time, quite apart from these purely phenomenological considerations, a new theory of strong interactions [8] was proposed which accomodated the conjectured strongly interacting vector mesons. This will be discussed in the next section.

2. – Vector theory of strong interactions.

The basic philosophy behind the vector theory (or gauge theory) of strong interactions can be summarized in the following way. It is essentially an attempt to construct a theory of strong interactions in analogy with electromagnetism. We know that quantum electrodynamics, from a certain point of view, is remarkably simple; the notions of conserved vector-current, universality and what we might call the notion of minimal electromagnetic couplings play important roles. Similarly in the realm of weak interactions, it has become apparent that the weak interactions are also vectorial, apart from parity nonconservation, and there are some speculations on the divergencelessness of the currents involved in the weak interactions, *i.e.*, on conserved currents or approximately conserved currents. We also know that the notion of universality has been applied successfully to some domains of weak interactions without strangeness changes. Finally, there are conjectures on the possible existence of spin-one particles (W-particles) that may mediate various weak processes.

If we now turn our attention to the strong interactions, the following questions arise very naturally. We have electromagnetic and weak interactions that are vectorial—why aren't the strong interactions also vectorial? Why

don't have a universal theory of strong interactions based on conserved currents? The vector theory of strong interactions is an attempt to answer these questions.

Now let us go back to some speculations made by WIGNER [11] many years ago. He noted that there are essentially two ways to determine the electric charge of a particle. First, « charge » is regarded as a pure number, a purely additive number, which is conserved in any reaction; *e.g.*, $\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$. If we know from some other experiments that the electric charge of the positron is plus one and the electric charges of the neutrinos are zero, then, by conservation of electric charge, the μ^+ charge is determined to be plus one. But the meaning of the electric charge is more than that—it has the meaning of coupling constant. The electric charge is not only « countable » but also « measurable », and it is only in the second sense that we know that the charge of the electron is equal in magnitude to the charge of the proton, to a fantastic degree of accuracy, something like to a few parts in 10^{19}. (This charge equality is one of the most remarkable equalities in modern physics. Quantum electrodynamics says that if the bare charges are equal, then the corresponding renormalized charges are also equal. Yet the equality of the bare charges nobody can explain.)

Now Wigner argues in the following way. Both electrons and protons are highly stable. The stability of the electron can be attributed to the conservation law of electric charge because the electron is the least massive particle with electric charge; similarly the stability of the proton can be attributed to the conservation law of what we might call « baryonic charge » because the proton is the least massive baryon. We known that the proton and the electron lifetimes are greater than 10^{27} years [12] and 10^{17} years [13] respectively; nobody knows the deep reason for the existence of the conservation laws of baryonic charges and electric charges, which seem to work so well. However, says Wigner, let us assume that the two conservation laws have similar causes and that they have similar consequences. With this in mind, what do we mean by « baryonic charge? ». Take, for instance, the reaction

$$\Lambda \to p + \pi^- .$$

If the baryonic charge of the proton is one and that of the pion is zero, then we argue that the baryonic charge of the Λ-particle is determined to be one. Note that we are using the notion that baryonic charge is countable, *i.e.*, it can be regarded as some additive number conserved in any reaction. In the ordinary theory there is nothing analogous to Wigner's second way of measuring « charge » *i.e.*, the notion of coupling constant is completely missing. So while the two conservation laws look similar in some sense, the meanings of the « charges » are very dissimilar.

Another way to see the asymmetry is as follows: In the electromagnetic case charge conservation is the immediate consequence of the Maxwell equations in the sense that the continuity equation $\partial_\mu j_\mu = 0$ follows from

$$\nabla \cdot \boldsymbol{E} = \varrho\,, \qquad \nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{j}\,.$$

In the baryonic case, there is nothing analogous to this elegant feature.

Wigner tried to remove this asymmetry by assuming that the pion is coupled universally to the various baryons. This is the origin of global symmetry [2, 3]. However, this analogy is rather superficial, and it cannot be pursued much further. The reason is that the quantity to which the photon field A_μ is coupled is a conserved density, whereas the quantity to which the pion field is coupled is a pseudoscalar density which is not conserved. A much more natural way is to assume that there is a vector meson coupled to the conserved baryonic current just as the photon is coupled to the conserved electric charge current:

$$\begin{aligned} &A_\mu \quad \text{coupled to } j_\mu^{(\text{electric})}, \\ &B_\mu \quad \text{coupled to } j_\mu^{(\text{baryonic})}. \end{aligned}$$

If the mass of the conjectured vector meson were zero, there would be a long-range anti-gravity effect between two macroscopic objects, as has been discussed by LEE and YANG [14]. Such an effect would have nothing to do with the strong interactions, which we know to be short-ranged. So we shall consider massive vector mesons.

We may naturally try to generalize this idea by associating a vector meson with each of the conserved currents of the strong interaction (isospin current, etc.). We postulate that for every conserved current of the strong interactions there exists a vector meson such that it is coupled linearly and universally to the corresponding conserved current. More precisely, by « universality » of our strong couplings we mean that the vector mesons are coupled with the same coupling constant to each of the terms of the corresponding currents

$$(2.1) \qquad \begin{cases} j_\mu^{(B)} = i\bar{p}\gamma_\mu p + i\gamma_\mu n + i\Lambda\gamma_\mu\Lambda + \dots, \\ j_\mu^{(T_3)} = \frac{1}{2}(i\bar{p}\gamma_\mu p - i\bar{n}\gamma_\mu n) + i\bar{\Sigma}^+\gamma_\mu\Sigma^+ - i\bar{\Sigma}^-\gamma_\mu\Sigma^- + \dots, \end{cases}$$

just as the photon is coupled universally to the electric charge current

$$j_\mu^{(EL)} = -ie^-\gamma_\mu e^- + i\bar{p}\gamma_\mu p + \dots\,.$$

Historically a number of people have tried to « derive » the vector theory of strong interactions from what we might call the gauge principle. The requirement that the gauge transformation associated with the conservation law of baryonic charge, etc., be local (space-time dependent) in character demands the existence of a vector field with zero bare mass coupled linearly to the baryon current etc. For a long time it has been generally believed that in such an approach the physical mass of the vector field must also be zero. However, recently SCHWINGER [15] was able to advance a remarkable argument that gauge invariance and bare mass = 0 do not necessarily imply that the physical mass = 0 for sufficiently strong couplings. The physical reason for this is that the polarization of the vacuum due to the presence of charge is so strong that the dynamical manifestation of the original charge plus the induced charge is short-ranged. We shall not discuss this very interesting and important question in this course. For details we refer to Schwinger's series of lectures at Trieste [16].

According to our lines of reasoning, given a symmetry of conserved operators in the strong interactions, the number and nature of the vector mesons are determined. So if we are just concerned with the exactly conserved currents of the strong interactions, we have only three currents; the baryon current $j_\mu^{(B)}$, the hypercharge current $j_\mu^{(Y)}$ and the isospin current $j_\mu^{(T_i)}$. With the baryon and the hypercharge currents we associate isoscalar, negative-G vector mesons while with the isospin current we associate an isovector, positive-G vector meson. To see this, we note that to give invariant interactions the vector fields must have the same symmetry properties as the corresponding currents. Using the $G=C(-1)^T$ rule for $T_3=0$ currents, we find that $j_\mu^{(B)}$ and $j_\mu^{(Y)}$ change sign under G while $j_\mu^{(T_i)}$ does not. (Note that $j_\mu^{(B)}$, $j_\mu^{(Y)}$ and $j_\mu^{(T_3)}$ are all odd under C; cf. eq. (2.1).) We have chosen hypercharge $Y=S+B$ rather than any other linear combination of B and S because the combination $B+S$ appears more naturally in a high-symmetry scheme.

Perhaps, in addition to baryon conservation, hypercharge conservation and isospin conservation, there may be hidden symmetries. By thys we mean that there may be conservation laws which are approximate. In the unitary symmetry model to be discussed in Section 5, there are isospinor ($T=\frac{1}{2}$), strangeness-changing currents which are conserved only in the limit where the mass differences between $\mathcal{N}$ and Λ etc., could be ignored. So we may conjecture the existence of $T=\frac{1}{2}$, $Y=\pm1$ vector mesons coupled to quasi-conserved strangeness-changing currents whose properties will be discussed later.

The first suggestion that there ought to be a $T=1$ vector meson coupled to the isospin current was given by YANG and MILLS [17]. FUJII [18] suggested within the framework of the Sakata model that there should be a strongly interacting vector meson coupled to the baryon current. Subsequently I formulated a theory in which three vector mesons (two $T=0$, one $T=1$), coupled

to the baryon current, the hypercharge current and the isospin current, play vital roles in strong-interaction physics [8]. With the currents generated by « gauge transformations » of unitary symmetry in the Sakata model, we may associate an octet of vector mesons, as shown by SALAM and WARD [19]. Now there is another version of unitary symmetry, the octet model, or the « eight-fold way », where we again have an octet of vector mesons plus a possible additional singlet vector meson. This was proposed by GELL-MANN [5, 20] and independently by NE'EMAN [21].

Now when these theories were proposed, there was no direct evidence for or against the existence of strongly interacting vector mesons. As discussed in Rosenfeld's course, there are two kinds of vector mesons with zero strangeness which have been discovered experimentally in the past year and a half—the ρ-meson with mass ≈ 750 MeV, $T=1$, $G=+1$ which decays strongly into two pions [22] and the ω-meson with mass ≈ 780 MeV, $T=0$, $G=-1$ which decays strongly into three $(\pi^+\pi^-\pi^0)$ mesons [23].

The ω-meson can be one of the candidates for the two $T=0$, $G=-1$ vector mesons proposed. If we subscribe to the philosophy that for each conserved current there should be a vector meson, then it would be better to have a second vector meson with the same quantum numbers as the ω-meson. In spite of their similarity the two vector mesons are quite distinct; for instance, the one coupled to the hypercharge current would not be emitted or absorbed by the Λ-hyperon since the Λ bears no hypercharge; on the other hand, the Λ can emit or absorb the vector meson coupled to the baryon current. From the practical point of view the most important questions is: Does there exist another vector meson with the above-mentioned quantum numbers? Perhaps it is relevant to mention that if the conjectured second vector meson has mass greater than $2m_K$, then it could decay into $K+\bar{K}$. Note that its decay into $K_1^0+K_1^0$ is forbidden by Bose statistics (and also by G invariance). The conjectured meson may be looked for in the reactions

$$\begin{aligned} K^-+p &\to K_1^0+K_2^0+\Lambda \\ &\to K^++K^-+\Lambda\,, \end{aligned}$$

which are now being studied in a Brookhaven-Syracuse collaborative experiment [24].

As mentioned earlier, in the unitary symmetry model there is room for a vector meson, called the M-meson, with $T=\frac{1}{2}$, $Y=\pm 1$, which may be the $K^*_{\frac{1}{2}}$ with mass ≈ 890 MeV [25]. There are some preliminary experimental indications in $p\bar{p}$ annihilations that the K^* spin is one (CERN-Paris group [26]) as discussed in Ticho's course.

To conclude this section, it is appropriate to summarize the prediction in the table I.

TABLE I.

Currents to which the meson is coupled	Isospin	G	Hyper-charge	Unitary symmetry classification
Baryon	0	—	0	singlet
Hypercharge	0	—	0	octet
Isospin	1	+	2	octet
S-changing	$\frac{1}{2}$	no meaning	± 1	octet

The existence of the ρ, ω and K^* is gratifying especially if we recall that when the theory was proposed, there was no evidence for or against any of the three mesons. There are, however, two predictions that have not yet been fulfilled:

i) If the K^* is the M-meson of unitary symmetry, then the spin of the K^* must be one (for which there is some evidence).

ii) There must exist another $T=0$, $J=1^-$, $G=-1$ meson whose major decay mode may well be $K_1^0+K_2^0$, K^++K^- as well as $\rho+\pi$ (but not $K_1^0+K_1^0$, $K_2^0+K_2^0$).

3. – Vector mesons and universality.

3·1. *Unstable particles and coupling constants.* – From the quantitative point of view the most important question in the vector theory of strong interactions is that of the universality of the vector couplings. We discuss this in detail for the ρ-meson.

In a somewhat old-fashioned approach one starts by considering the effective Lagrangian density

$$\mathscr{L}_{\text{eff.}} = if_{\rho\mathscr{N}\mathscr{N}}\rho_\mu\overline{\mathscr{N}}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\mathscr{N} + f_{\rho\pi\pi}\rho_\mu(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) + \dots . \tag{3.1}$$

Universality means that

$$f_{\rho\mathscr{N}\mathscr{N}} = f_{\rho\pi\pi} \text{ etc.}$$

First of all, how can we measure $f_{\rho\pi\pi}$? Let us consider the decay of the ρ-meson into two pions:

$$\rho\to\pi+\pi .$$

Applying the usual perturbation theory one finds that the decay rate is proportional to $f^2_{\rho\pi\pi}/4\pi$. But the ρ-meson is unstable, and the question arises whether we could justify such a simple reasoning. So we propose to study a theory of unstable particles in which the meaning of a (renormalized) coupling constant is uniquely defined. (For a detailed discussion of this problem see GELL-MANN and ZACHARIASEN [27]; the approach given here is a simple exposition of some of the ideas discussed in their paper.)

In order to avoid inessential complications due to the spin of the ρ-meson we first consider a hypothetical spin-zero, scalar meson, which we call σ, coupled to two distinguishable pions.

CASE I. Suppose σ were stable, *i.e.*, $m_\sigma < 2m_\pi$. Then, using the effective Lagrangian

$$\mathscr{L}_{\text{eff}} = g\sigma\pi\pi ,$$

one may compute, in the Born approximation, the differential cross-section for π-π scattering

$$\frac{d\sigma}{d\Omega} = \left|\frac{\exp[i\delta]\sin\delta}{p}\right|^2 = \frac{1}{4}\left(\frac{g^2}{4\pi}\right)^2\frac{1}{s}\frac{1}{(s-m_\sigma^2)^2},$$

where

$$s = (\text{total c.m. energy})^2,$$

$$p = \tfrac{1}{2}(s-4m_\pi^2)^{\frac{1}{2}},$$

provided that a term due to the exchange of a σ-meson is negligible. The Born approximation is expected to be bad, but dispersion theory (or any sensible theory of strong couplings) tells us that near the pole at $s=m_\sigma^2$, the Born approximation gives the right result

$$T \sim -\frac{g^2}{s-m_\sigma^2} \quad \text{near } s = m_\sigma^2 ,$$

where T is the s-wave invariant amplitude defined by

$$\frac{1}{8\pi\sqrt{s}}T = \frac{\exp[i\delta]\sin\delta}{p}.$$

This result suggests the following definition of the renormalized coupling constant

$$\frac{1}{g^2} = -\frac{d}{ds}\left(\frac{1}{T}\right)\bigg|_{s=m_\sigma^2}. \tag{3.2}$$

Note that $1/T$ vanishes at the pole $s=m_\sigma^2$.

CASE II. The σ is not stable against decay into two pions: $m_\sigma > 2m_\pi$. This time the σ-meson manifests itself as an s-wave resonance in π-π scattering. The quantity $p \operatorname{ctg} \delta$ vanishes as δ goes through $90°$ at $s = m_\sigma^2$. Because of

$$\frac{\exp[i\delta]\sin\delta}{p} = \frac{\sin\delta}{p\cos\delta - ip\sin\delta},$$

we see that

$$\operatorname{Re}\left(\frac{1}{T}\right) = \frac{p \operatorname{ctg}\delta}{8\pi\sqrt{s}},$$

becomes zero at resonance $s = m_\sigma^2$ just as $1/T$ does in the stable-particle case. This suggests the definition [27]

$$\frac{1}{g^2} = -\frac{\mathrm{d}}{\mathrm{d}s}\left[\operatorname{Re}\left(\frac{1}{T}\right)\right]\bigg|_{s=m_\sigma^2}, \tag{3.3}$$

in complete analogy with eq. (3.2).

We now use this definition to give a relation between the coupling constant g^2 and the parameters of the σ resonance. Let us assume a narrow resonance (small g^2). Then near the resonance we expect

$$\exp[i\delta]\sin\delta \approx \frac{m_\sigma \Gamma}{s - m_\sigma^2 + im_\sigma \Gamma},$$

where Γ is the full width at half-maximum. (Note $s - m_\sigma^2 \approx 2m_\sigma(W - m_\sigma)$, where $W = \sqrt{s}$.) From this relation and our definition (3.3) we obtain

$$\frac{1}{g^2} = \frac{p_{\text{res}}}{8\pi m_\sigma^2 \Gamma},$$

or

$$\Gamma = \left(\frac{g^2}{4\pi}\right)\frac{1}{4}\frac{(m_\sigma^2 - 4m_\pi^2)^{\frac{1}{2}}}{m_\sigma^2}. \tag{3.4}$$

We can easily verify that exactly the same expression can be obtained by computing the life-time or the decay width using the effective Lagrangian $g\sigma\pi\pi$. As a result, we see that the well-known formal identity between the pole terms in the sense of dispersion theory and the renormalized Born terms in the sense of perturbation theory can be extended to the case of unstable particles.

3'2. ρ-*decay*. – Let us apply our previous results to the more realistic case of a spin-1 meson decaying into two pions, and calculate the width of the

ρ-meson using the ordinary perturbation theory. The matrix element describing the $T=1$, $J=1^-$ ρ-meson decaying into two pions, say $\rho^0 \to \pi^+ + \pi^-$ is given by

$$M = \frac{-i}{\sqrt{8m_\rho \omega_1 \omega_2}} f_{\rho\pi\pi} \varepsilon_\mu^{(\rho)} (k_1 - k_2)_\mu \,,$$

where $\varepsilon_\mu^{(\rho)}$ is the polarization vector of the ρ-meson; k_{1_μ}, k_{2_μ} are the energy-momentum 4-vectors of the outgoing pions, ω_1 and ω_2, their energies. We have used that the relevant term in the Lagrangian (3.1) is of the form

$$if_{\rho\pi\pi}\varrho_\mu^0(\partial_\mu \pi^- \pi^+ - \pi^- \partial_\mu \pi^+) \,.$$

In the rest system of the decaying ρ, only the space components of $\varepsilon_\mu^{(\rho)}$ contribute; otherwise, we would be describing 3+1 degrees of freedom rather than the 3 degrees of freedom appropriate for a spin-1 particle. Therefore we have only to replace g in the σ-meson case by $f_{\rho\pi\pi}\boldsymbol{\varepsilon}^{(\rho)}\cdot(\boldsymbol{p}_1 - \boldsymbol{p}_2)$. The decay rate can readily be computed to be [27-29]

$$\Gamma(\rho \to 2\pi) = \frac{f^2_{\rho\pi\pi}}{4\pi}\frac{2}{3}\frac{p^3_{\text{res}}}{m^2_\rho} = \frac{1}{12}\frac{f^2_{\rho\pi\pi}}{4\pi} m_\rho \left(1 - \frac{4m^2_\pi}{m^2_\rho}\right)^{\frac{3}{2}} .$$

The measured value of

$$\Gamma_\rho \approx (100 \div 125)\ \text{MeV},$$

gives

$$\frac{f^2_{\rho\pi\pi}}{4\pi} \approx 2.0 \div 2.5 \,.$$

3'3. *S-wave pion-nucleon scattering.* – To test the universality hypothesis we must obtain $f_{\rho\mathcal{N}\mathcal{N}}$. Nucleon-nucleon forces are not useful for the determination of this constant for several reasons. First of all, the nucleon-nucleon interaction is best known for the p-p system; but in p-p scattering it is impossible to disentangle the ρ contribution from the ω contribution. A more serious difficulty is that no realiable methods exist for computing the contributions due to uncorrelated pions. Also an anomalous moment-type coupling of the ρ to the nucleon results in further complications. Finally, we could not obtain the relative sign of $f_{\rho\pi\pi}$ and $f_{\rho\mathcal{N}\mathcal{N}}$ which is of crucial importance from the point of view of universality.

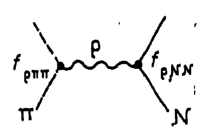

Fig. 3.

Let us look at low-energy pion-nucleon scattering. If we could somehow isolate the ρ exchange contribution shown in Fig. 3, we would be able to

measure the product $f_{\rho\pi\pi} f_{\rho\mathcal{N}\mathcal{N}}$. First of all note that the «potential» due to any isospin-1 object must be proportional to

$$\boldsymbol{T}_\pi \cdot \boldsymbol{T}_{\mathcal{N}} = \boldsymbol{T}_\pi \cdot \frac{\boldsymbol{\tau}_{\mathcal{N}}}{2} = \begin{cases} -1 & \text{for } T = \frac{1}{2}, \\ \frac{1}{2} & \text{for } T = \frac{3}{2}. \end{cases}$$

If the proportionality factor turns out to be positive (and it actually does), we get an attractive $T=\frac{1}{2}$ «potential» and a repulsive $T=\frac{3}{2}$ «potential». In this way one may hope to obtain a simple explanation for the experimentally observed attractive $T=\frac{1}{2}$ and the repulsive $T=\frac{3}{2}$ s-state interactions.

Calculating the contribution of our graph to the s-wave amplitude, we find

$$\frac{\sin 2\delta_1}{2p} - \frac{\sin 2\delta_3}{2p} = \frac{3}{4} \frac{f_{\rho\pi\pi} f_{\rho\mathcal{N}\mathcal{N}}}{4\pi} \frac{\omega m_{\mathcal{N}}}{W} \frac{1}{p^2} \ln\left(1 + \frac{4p^2}{m_\rho^2}\right), \tag{3.6}$$

where δ_1 and δ_3 are the s-wave phase shifts for the $T=\frac{1}{2}$, $T=\frac{3}{2}$ states; p the momentum in the c.m. system; ω the pion energy, and W the total c.m. energy. The logarithm term comes from the s-wave projection of the ρ-meson propagator

$$\frac{1}{-t + m_\rho^2} = \frac{1}{2p^2(1 - \cos\theta) + m_\rho^2},$$

since

$$\frac{1}{2}\int_{-1}^{1} \mathrm{d}(\cos\theta) \frac{1}{2p^2(1-\cos\theta) + m_\rho^2} = \frac{1}{4p^2} \ln\left(1 + \frac{4p^2}{m_\rho^2}\right).$$

Incidentally there exists a completely general rule for the character of forces due to the exchange of a $T=1$ ρ-meson if the universality principle holds. Take the $\pi^+ p$ system which is pure $T=\frac{3}{2}$. Here only a neutral ρ can be exchanged. This exchange will produce something like a Coulomb force except that the force is short-ranged. The universality principle says that $f_{\rho\pi\pi} = +f_{\rho\mathcal{N}\mathcal{N}}$, *i.e.*, the «charges» are equal when the third components of the isospins are both positive, which means that we get a repulsive force in the $T=\frac{3}{2}$ state. If the universality principle is right, and if only vector-type couplings are considered, the forces due to ρ exchange between two particles whose isospins are parallel are always repulsive.

Let us go back to the formula for $\pi\mathcal{N}$ scattering. The s-wave π-$\mathcal{N}$ scattering at low energies was considered to be mysterious in the sense that nobody could explain the isospin-dependence in a convincing manner. Assuming that the s-wave isospin-dependent amplitude is dominated by the ρ exchange, we

find from the measured value of $a_1 - a_3$

$$\frac{f_{\rho\pi\pi} f_{\rho\mathcal{N}\mathcal{N}}}{4\pi} \approx 2.5 .$$

A more sophisticated approach tries to take into account the energy-dependence of the s-wave phase shift [30]. The results of HAMILTON, SPEARMAN and WOOLOCK [31], who tried to fit this energy-dependence, with rescattering corrections taken into account, show

$$\frac{f_{\rho\pi\pi} f_{\rho\mathcal{N}\mathcal{N}}}{4\pi} \approx 2.1 \pm .3 .$$

((For the « experts » I may remark that the parameter C_1 of BOWCOCK, COTTINGHAM and LURIÉ [30] is related to ours by

$$C_1 = -\frac{1}{3}\left(\frac{f_{\rho\pi\pi} f_{\rho\mathcal{N}\mathcal{N}}}{4\pi}\right) .$$

So we have found a good agreement with the universality principle

$$f_{\rho\pi\pi} = f_{\rho\mathcal{N}\mathcal{N}} ,$$

since

$$\frac{f^2_{\rho\pi\pi}}{4\pi} \approx 2.0 \div 2.5 .$$

from the decay width of the ρ-meson, and

$$\frac{f_{\rho\mathcal{N}\mathcal{N}} f_{\rho\pi\pi}}{4\pi} \approx 2.1 \pm .3 ,$$

from π-$\mathcal{N}$ scattering.

3·4. *Other processes.* – It would be nice to test the universality principle in other reactions, *e.g.*, K$\mathcal{N}$ scattering. But it is very difficult to isolate the ρ contribution in such processes because we don't know how to calculate contributions other than the ρ-exchange contribution.

We can, however, consider the following interesting speculation. Whenever the OPE (one-pion-exchange) contribution is forbidden by symmetry considerations, the isospin-dependent amplitude for any low-energy scattering is dominated by the exchange of a single ρ-meson coupled universally to the isospin current. This hypothesis implies a simple rule about the resulting

forces. The ρ-exchange force is attractive when isospins are antiparallel, repulsive when they are parallel. A special case of this rule was already illustrated in the $\pi\mathcal{N}$ case:

$$T = \tfrac{3}{2} \uparrow\uparrow \text{ repulsive},$$

$$T = \tfrac{1}{2} \uparrow\downarrow \text{ attractive}.$$

Note also that the force must be of the form $\boldsymbol{T}_1 \cdot \boldsymbol{T}_2$.

The simple rule works remarkably well in the five cases examined so far.

i) $\pi\mathcal{N}$ scattering. Already discussed.

ii) $K\mathcal{N}$ scattering. K^+p scattering (pure $T=1$) is strongly repulsive [32]. $K\mathcal{N}$ scattering in $T=0$ may be repulsive but not as strongly as in $T=1$.

iii) $\overline{K}\mathcal{N}$ scattering. Y_0^* (1405) may well be a $\overline{K}\mathcal{N}$ s-wave bound state in the sense of DALITZ and TUAN [33] as discussed in Ticho's course. Y_1^* (1385) is most likely not a $\overline{K}\mathcal{N}$ s-wave bound state. $\therefore$ $T=0$ ($\uparrow\downarrow$) is more attractive.

iv) $\overline{K}K$ scattering. There is some evidence for a strong s-wave $K_1^0K_1^0$ interaction in the $T=0$, $G=+1$ state [34].

v) $\pi\pi$ scattering. The $T=0$ s-wave interaction is probably strongly attractive regardless of whether the so-called ABC effect [35] is interpretable in terms of a large scattering length [36]. The $T=2$ s-wave interaction seems to be weak [37].

Generally speaking, in this kind of theory, bound states and resonances are obviously more likely in states of lower isospins.

A similar and even simpler rule works for the ω-meson which we assume to be coupled to the hypercharge current. (If the ω-meson is the kind coupled to the baryon current rather than to the hypercharge current, then the following argument should be applied to a second $T=0$ vector meson yet to be discovered.) Note

$$Y(K) = Y(\mathcal{N}) = +1\,,$$

$$Y(\overline{K}) = -1 = -Y(\mathcal{N})\,,$$

where Y stands for hypercharge. Since like hypercharges repel while unlike hypercharges attract if the universality principle is right, the $K\mathcal{N}$ force is « on the average » repulsive while the $\overline{K}\mathcal{N}$ force is « on the average » attractive.

With both ρ and ω exchange we have the effective potentials [8]

$$\left\{\begin{array}{ll} K\mathcal{N}: & V_\omega + V_\rho(\tau_K \cdot \tau_{\mathcal{N}})\,, \\ \overline{K}\mathcal{N}: & -V_\omega + V_\rho(\tau_K \cdot \tau_{\mathcal{N}})\,, \end{array}\right. \tag{3.7}$$

where

$$\tau_K \cdot \tau_{\mathcal{N}} = \begin{cases} 1 & \text{for} \quad T = 1 \,, \\ -3 & \text{for} \quad T = 0 \,. \end{cases}$$

Note that the signs of V_ω and V_ρ are determined to be both positive (positive means repulsive) in the vector theory based on the universality principle, but are arbitrary in any other theory. There is some evidence that the simple description given here corresponds to reality [38, 39].

3'5. *Dispersion-theoretic approach to universality.* – To end this section we indicate how the universality principle might be formulated on the basis of dispersion theory. Let us go back to the ρ case and consider its effect on electromagnetic form factors. Suppose the ρ-meson dominates the charge form factor of every strongly interacting particle. Take the π^+ and p cases for the sake of definiteness as shown in Fig. 4. The constant $\gamma_{\gamma\rho}$ is the effective coupling constant of γ to ρ. We continually ignore the complications due to the instability of the ρ-meson. (For a more rigorous discussion see GELL-MANN and ZACHARIASEN [27].) Under the assumption that the ρ-meson completely dominates, we have, for the charge form factors,

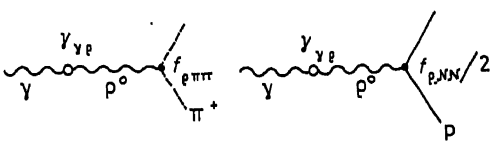

Fig. 4.

$$(3.8)\qquad \begin{cases} eF_\pi(t) = \dfrac{\gamma_{\gamma\rho} f_{\rho\pi\pi}}{-t + m_\rho^2}\,, \\[2ex] \dfrac{e}{2} F_1^{(V)}(t) = \dfrac{\gamma_{\gamma\rho} f_{\rho\mathcal{N}\mathcal{N}}/2}{-t + m_\rho^2}\,, \end{cases}$$

where $e/2$ comes from the fact that only $\frac{1}{2}$ of the electric charge of the proton is due to the isovector part of the form factor $F_1^{(V)}(t)$. At zero momentum transfer the form factors are normalized so that $F_\pi(0) = F_1^{(V)}(0) = 1$. Since $\gamma_{\gamma\rho}$ is common to both expressions, the universality of the *electric* charges provides the constraint $f_{\rho\pi\pi} = f_{\rho\mathcal{N}\mathcal{N}}$, just the universality result again, provided that the ρ-meson *completely* dominates both form factors.

Unfortunately the model of nucleon structure based on the idea that the ρ-meson *completely* dominates the isovector charge form factor runs into difficulty. In such a model, putting $\langle r^2\rangle_n^{\frac{1}{2}} = 0$ as observed, we would have $\langle r^2\rangle_p^{\frac{1}{2}} \approx 0.65$ fermi instead of the measured value (0.85 ± 0.05) fermi. This point will be discussed in detail in Drell's lessons.

One final point. If the ρ-meson completely dominated the charge form

factors, then the effective coupling between γ and ρ would be given by

$$\gamma_{\gamma\rho} = \frac{em_\rho^2}{f_{\rho\mathcal{N}\mathcal{N}}} = \frac{em_\rho^2}{f_{\rho\pi\pi}} = \frac{em_\rho^2}{f_\rho}, \tag{3.9}$$

which follows from (3.8) by setting $t=0$. Note that $\gamma_{\gamma\rho}$ is *inversely* proportional to the strong interaction constant f_ρ. This is just the opposite of the perturbation-theoretic result; in perturbation theory we expect $\gamma_{\gamma\rho}$ to be proportional to f_ρ since we consider

$$\gamma \xrightarrow{e} \left\{\begin{matrix} \pi^+ + \pi^- \\ p + \bar{p}\text{ etc.} \end{matrix}\right\} \xrightarrow{f_\rho} \varrho\,.$$

4. – Decay modes of the ω-meson.

4·1. $\omega \to e^+ + e^-$, $\mu^+ + \mu^-$. – The ω-meson with $T=0$, $J=1^-$, $G=C=-1$ was predicted by NAMBU [6] as a particle which may be helpful for our understanding of the isoscalar form factor of the nucleon. Its coupling to the photon, $\omega \rightleftarrows \gamma$, must be strong if the ω-meson is to contribute substantially to the isoscalar form factor (see Fig. 1). But because γ is coupled to any pair of charged particle-antiparticle, we expect that the ω is also coupled to any charged particle pair. So we must have [6]

$$\omega \to \gamma \to \begin{cases} e^+ + e^-\,, \\ \mu^+ + \mu^-\,, \\ \pi^+ + \pi^-\,. \end{cases}$$

If the ω-meson dominates the isoscalar form factor, then the coupling constant between ω and γ is given by

$$\gamma_{\gamma\omega} = \frac{em_\omega^2}{2f_\omega}, \tag{4.1}$$

in complete analogy with what we did for the ρ-meson case [see eq. (3.9)]. The constant f_ω that appears in (4.1) is defined by

$$\mathcal{L}_{\text{eff}} = f_\omega \omega_\mu (i\bar{\mathcal{N}}\gamma_\mu \mathcal{N} + \ldots) = f_\omega \omega_\mu (i\bar{p}\gamma_\mu p + i\bar{n}\gamma_\mu n + \ldots)\,. \tag{4.2}$$

(Our definition of the f_ρ and f_ω are related to those of GELL-MANN [5, 27] by $f_\rho = 2\gamma_\rho$, $f_\omega = \sqrt{3}\gamma_\omega$.) We can summarize our rule as follows:

$$\begin{cases} \omega \text{ dominates} \Rightarrow \text{insert } \gamma_{\gamma\omega} = \dfrac{em_\omega^2}{2f_\omega} \text{ at the } \gamma\text{-}\omega \text{ junction} \\[2ex] \rho \text{ dominates} \Rightarrow \text{insert } \gamma_{\gamma\rho} = \dfrac{em_\rho^2}{f_\rho} \text{ at the } \gamma\text{-}\rho \text{ junction}\,. \end{cases} \tag{4.3}$$

By straightforward calculations, the decay rate into a lepton pair is given by [41]

$$\Gamma(\omega \to e^+ + e^-, \mu^+ + \mu^-) = \left(\frac{1}{137}\right)^2 \frac{1}{12} \frac{m_\omega}{f_\omega^2/4\pi} \left(1 - \frac{4m_{e,\mu}^2}{m_\omega^2}\right)^{\frac{1}{2}} \tag{4.4}$$

provided that the ω-meson completely dominates the isoscalar form factor. If the ω does not dominate completely, the above result must be multiplied by the correction factor C_ω^2 where C_ω is the coefficient of the ω contribution in the phenomenological representation of the form factor [42]

$$F_1^{(s)}(t) = \frac{m_\omega^2 C_\omega}{-t + m_\omega^2} + 1 - C_\omega , \tag{4.5}$$

as discussed in Drell's course. Our final numerical result is given by

$$\Gamma(\omega \to e^+ + e^-) \approx \Gamma(\omega \to \mu^+ + \mu^-) \approx \frac{C_\omega^2}{(f_\omega^2/4\pi)} \times 3 \text{ keV} .$$

Note that the decay rate is inversely proportional to the strong interaction coupling constant squared, as advertized.

If the muon had an anomalous interaction, the $(e^+e^-)/(\mu^+\mu^-)$ branching ratio might be very different. This ratio may provide an interesting test of the commonly believed idea that the muon is a pure Dirac particle. The process $\omega \to \mu^+\mu^-$ could reveal a possible anomaly of the muon at momentum transfer squared equal to 0.6 $(\text{GeV}/c)^2$. So far the $\mu^+\mu^-$ and the e^+e^- decay modes of the ω-meson have noot been looked for.

4'2. $\omega \to \pi^+ + \pi^-$. – Let us consider the decay

$$\omega \to \pi^+ + \pi^- .$$

This process violates *G*-conjugation invariance; therefore it cannot be a strong one. But as the ω has the same quantum numbers as the photon (as far as the charge-conjugation parity and the spin-parity are concerned), the decay is possible via

$$\omega \to \gamma \to \pi^+ + \pi^- .$$

If we neglect the electromagnetic structure of the π-meson, calculations are again straightforward. We obtain for the partial decay width

$$\Gamma^{(0)}(\omega \to \pi^+ + \pi^-) = \frac{1}{48} \left(\frac{1}{137}\right)^2 \frac{m_\omega}{(f_\omega^2/4\pi)} \left(1 - \frac{4m_\pi^2}{m_\omega^2}\right)^{\frac{3}{2}} C_\omega^2 . \tag{4.6}$$

As the ω mass (780 MeV) is very close to the center of the ρ-resonance (750 MeV), one expects a strong final-state p-wave interaction between the two pions, which leads to an enormous enhancement of the decay rate. This amounts to saying that we cannot neglect the electromagnetic structure of the pion. So we insert the pion form factor $F_\pi(t)$ at the $\gamma\pi\pi$ vertex as shown in Fig. 5.

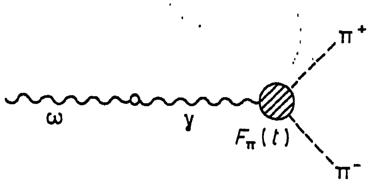

Fig. 5.

If we assume that the pion form factor $F_\pi(t)$ is dominated by the ρ-meson, we obtain by means of dispersion theory (cf. Drell's lessons)

$$F(t)=\frac{m_\rho^2}{-t+m_\rho^2-im_\rho\Gamma_\rho}\,,$$

which is supposed to be accurate in the neighborhood of $t\sim m_\rho^2$: Multiplying the width Γ^0 of the point-pion interaction by $|F_\pi(m_\omega^2)|^2$ we obtain [41]

$$(4.7)\quad \Gamma(\omega\to\pi^++\pi^-)=\Gamma^{(0)}(\omega\to\pi^++\pi^-)\,|F_\pi(m_\omega^2)|^2\approx\frac{0.7\ \text{keV}}{f_\omega^2/4\pi}\,C_\omega^2\,|F_\pi(m_\omega^2)|^2\,.$$

If we insert the mass and the width of the ρ-meson, we see that $|F_\pi(m_\omega^2)|^2$ may well be as large as 50. Thus we indeed have a large enhancement factor that multiplies $\Gamma^{(0)}$. It may be added that this estimate is not as realiable as our earlier estimates for the e^+e^- and $\mu^+\mu^-$ modes because we have neglected other possible graphs, *e.g.*, $\omega\to\gamma+\pi^0\to\pi^++\pi^-$.

The very existence of the decay mode $\omega\to\pi^++\pi^-$ implies electromagnetic mixing between the ρ and the ω, as first discussed by GLASHOW [43]. Under certain conditions one may observe interesting interference phenomena between the ρ resonance and the two-pion decay mode of the ω-meson as discussed in detail by BERNSTEIN and FEINBERG [44].

4.3. *The* $\omega\rho\pi$ *vertex and* $\omega\to\pi^0+\gamma$, $\pi^++\pi^-+\pi^0$. – We now consider the process

$$\omega\to\rho+\pi\,.$$

Although this decay mode is energetically forbidden, the $\omega\rho\pi$ vertex is very useful for computing other processes. The simplest invariant matrix element which phenomenologically describes this vertex is (apart from the usual kinematical factors)

$$(4.8)\qquad \frac{f_{\omega\rho\pi}}{m}\,\varepsilon_{\mu\nu\lambda\sigma}k_\mu^{(\omega)}\varepsilon_\nu^{(\omega)}k_\lambda^{(\rho)}\varepsilon_\sigma^{(\rho)}\,,$$

where $\varepsilon_\nu^{(\omega)}$, $\varepsilon_\sigma^{(\varrho)}$ are the 4-dimensional polarization vectors of the ω and the ρ-mesons. The constant m is a reference mass, which is introduced to make $f_{\omega\rho\pi}$ dimensionless. Note that the expression (4.8) satisfies the requirement that in the rest system of the $\omega(\rho)$-meson, the time component of $\varepsilon_\mu^{(\omega)}$ $(\varepsilon_\mu^{(\varrho)})$ does not contribute. Also note that this expression changes its sign under parity, which just compensates the odd parity of the pion.

By means of the $\omega\rho\pi$ vertex we now estimate the branching ratio of the two processes

$$\omega \to \pi^0 + \gamma\,,$$

$$\omega \to \pi^+ + \pi^- + \pi^0\,.$$

Let us assume that these processes proceed via the $\rho+\pi$ intermediate states as shown in Fig. 6. In Fig. 6*a* we insert for the $\gamma\rho$ coupling constant

$$\gamma_{\gamma\rho} = \frac{e m_\rho^2}{f_\rho}\,.$$

To the extent that the ρ-meson completely dominates the pion form factor, we can assume $f_\rho = f_{\rho\pi\pi}$, where $f_{\rho\pi\pi}$ has already been estimated from the decay width of the ρ-meson.

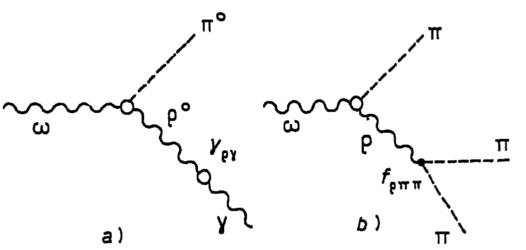

Fig. 6.

In both graphs in Fig. 6, we have the unknown constant $f_{\omega\rho\pi}/m$. But if we consider the ratio of the decay widths, this unknown constant cancels. In this way we obtain

$$\frac{\Gamma(\omega \to \pi^0+\gamma)}{\Gamma(\omega \to \pi^+ + \pi^- + \pi^0)} \approx 17\,\%\,, \tag{4.9}$$

as shown by GELL-MANN, SHARP and WAGNER [45] and by HORI *et al.* [46].

The branching ratio estimated above seems to be in good agreement with the experimental result of the CERN-Paris group [47] who investigated the process

$$\bar{\mathrm{p}} + \mathrm{p} \to \omega + \mathrm{K} + \bar{\mathrm{K}}\,,$$

where the ω's decay either into $\pi^+ + \pi^- + \pi^0$ or into neutral particles. This experiment shows

$$\frac{\omega \to \text{neutrals}}{\omega \to \pi^+ + \pi^- + \pi^0} = 21 \pm 7\,\%\,.$$

This experimental result agrees very well with the prediction (4.9) if we assume that the « neutral particles » of the CERN-Paris group are predominantly $\pi^0+\gamma$. This assumption seems fairly reasonable since the $2\pi^0+\gamma$ mode is likely to be negligible as shown by SINGER [48]. (Note in this connection that the $(\pi^+\pi^-\gamma)/(2\pi^0\gamma)$ ratio is required to be 2 since the dipion system with $C=+1$ must be in $T=0$.) A similar value for the experimental ratio has been obtained by the Berkeley hydrogen bubble chamber group [49] who studied

$$K^-+p \to \Lambda+\omega\ .$$

Their value is $(25\pm10)\%$ in good agreement with the value obtained from the $\bar{p}$ annihilation experiment.

One might expect that our ρ-dominance model for $\omega\to\pi^++\pi^-+\pi^0$ predicts a Dalitz plot which is substantially different from the Dalitz plot predicted by the « simplest-matrix-element model » ($\varepsilon_{\mu\nu\lambda\sigma}\varepsilon_\mu^{(\omega)}k_\nu^{(1)}k_\lambda^{(2)}k_\sigma^{(3)}$ in the covariant notation which reduces to $\boldsymbol{p}\times\boldsymbol{q}$) discussed in Rosenfeld's course. (It is easy to show that as $m_\rho\to\infty$, the « ρ-dominance model » reduces to the « simplest-matrix-element model »). Explicit calculations show, however, that as far as the density of points in the Dalitz plot is concerned, the two models differ at most by 10%.

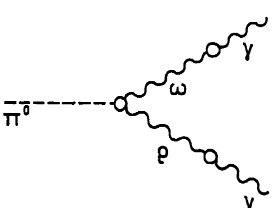

Fig. 7.

Finally, as pointed out by GELL-MANN and ZACHARIASEN [27, 50] we can estimate the strength of the $\omega\rho\pi$ vertex from the π^0-decay into 2 γ-rays if we assume that the π^0-decay amplitude is dominated by $\pi^0\to\rho^0+\omega$ followed by $\rho\to\gamma$, $\omega\to\gamma$ as shown in Fig. 7. Again we insert em_ρ^2/f_ρ at the $\rho\gamma$ junction and $em_\omega^2/2f_\omega$ at the $\omega\gamma$ junction. The decay width $\Gamma(\pi^0\to2\gamma)$ is known from the observed π^0 lifetime to be about 3 eV. So once $f_\omega/4\pi$ is known, we can calculate the constant $f_{\omega\rho\pi}/m$, and from this the width $\Gamma(\omega\to\pi^++\pi^-+\pi^0)$. The result is as follows [45]:

$$\Gamma(\omega\to\pi^++\pi^-+\pi^0)\approx 250\ \text{kev}\times\frac{f_\omega^2}{4\pi}\ . \tag{4.10}$$

The above estimates, however, might be unreliable because essentially the same method applied to

$$\pi^0\to e^++e^-+\gamma\ ,$$

disagrees very badly with experiments. The observed mass distribution of e^+e^- pairs [51, 52] does not fit at all into our model in which only the ρ and ω are assumed to be important. Even the sign of the parameter a that characterizes the structure of the π^0 is given incorrectly [53]. We may note

that the theoretical prediction on the $(\pi^0\gamma)/(\pi^+\pi^-\pi^0)$ ratio, which agrees with observation, rests only on the assumption that the ρ-meson dominates whereas in obtaining the theoretical distribution of the e^+e^- pair in π^0-decay (which disagrees with observation) we had to assume that both the ρ and ω dominate.

4'4. *Experimental methods for determining the ω width.* – In order to obtain the rate for any of the modes we have discussed, it is not sufficient to measure just the branching ratios; it is essential to measure absolutely the rate of at least one of the decay modes. So far, only an upper limit (~ 20 MeV) has been set for the total width of the ω-meson.

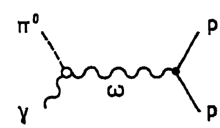

Fig. 8.

In order to determine the width $\Gamma(\omega\to\pi^0+\gamma)$ we may consider the following process [54]

$$\gamma+p\to\pi^0+p\,.$$

If we assume that this reaction is dominated at high energies by the exchange of an ω-meson as shown in Fig. 8, we can measure the product of the ω width and the $\omega\mathcal{N}\mathcal{N}$ coupling constant. This would lead to the differential cross-section

$$(4.11)\quad \frac{d\sigma}{d\Omega}=12\left(\frac{f_\omega^2}{4\pi}\right)\frac{\Gamma(\omega\to\pi^0+\gamma)}{(1-m_\pi^2/m_\omega^2)^3m_\omega^3}\frac{kq\omega^2}{(\Delta^2+m_\omega^2)^2}\left[\beta^2\sin^2\theta+\frac{\Delta^2}{2W^2}(1-\beta\cos\theta)^2\right],$$

$$\Delta^2=2k\omega(1-\beta\cos\theta)-m_\pi^2;$$

$q=\pi^0$ momentum, $\omega=\pi^0$ energy, $k=$ photon energy;

$W=$ total c.m. energy;

$\beta=q/\omega;$

where we have ignored possible contributions due to an anomalous moment type coupling of the ω to the nucleon (which is unimportant for photoproduction in forward directions). Recently a very precise angular distribution has been obtained for this process at $E_\gamma^{(\text{lab})}=1140$ MeV at Caltech by TALMAN *et al.* [55]. There is some evidence that the angular distribution cannot be explained in terms of a finite «reasonable» number of powers of $\cos\theta$. But if we assume that the ω-exchange diagram dominates, we can explain the major qualitative features of the angular distribution. From this it has been deduced that

$$\Gamma(\omega\to\pi^0+\gamma)\,\frac{f_\omega^2}{4\pi}==\approx 2\text{ MeV}\,.$$

A more direct approach for obtaining the strength of the $\omega\pi\gamma$ vertex itself makes use of the photoproduction of the ω-meson

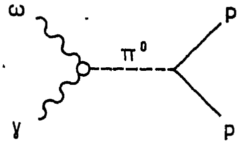

Fig. 9.

$$\gamma+p \to p+\omega ,$$

provided that the one-pion-exchange mechanism is operative (cf. Fig. 9). In this graph the $\pi^0\bar{p}p$ vertex is well known; therefore we could measure $\Gamma(\omega \to \pi^0+\gamma)$ itself if the angular distribution were known very accurately. The one-pion-exchange angular distribution is given by [56]

$$\frac{d\sigma}{d\Omega} = 6\frac{G^2_{\pi\mathcal{N}\mathcal{N}}}{4\pi}\frac{\Gamma(\omega \to \pi^0+\gamma)}{(1-m^2_\pi/m^2_\omega)^3 m^3_\omega}\frac{qk\omega^2}{W^2}\frac{\Delta^2(1-\beta\cos\theta)^2}{(\Delta^2+m^2_\pi)^4}, \tag{4.12}$$

$\Delta^2 = 2k\omega(1-\beta\cos\theta) - m^2_\omega$;

W = total c.m. energy;

q = ω-momentum, $\omega = \omega$-energy, k = photon-energy;

$\beta = q/\omega$;

$G_{\pi\mathcal{N}\mathcal{N}}/4\pi \approx 15$.

So far the photoproduction of the ω-meson has not been studied with precision.

At first sight the reactions

$$\pi^\pm+p \to p+\pi^\pm+\omega ,$$

seem to be very suitable for determing the ω-3π vertex because they have been shown experimentally to be very frequent at $p_\pi^{(\text{lab})} \approx (2 \div 3)$ GeV/c for both π^+ and π^- (Columbia-Rutgers [57], Berkeley [58]). The graph we have in mind is shown in Fig. 10 [59]. But unfortunately at $p_\pi^{(\text{lab})} \approx (2 \div 3)$ GeV/c, there is no evidence that this process goes via the one-pion-exchange mechanism. There seems to be a strong πp final-state interaction due to the 3-3 resonance so that we essentially have $\pi+p \to \mathcal{N}^*_{\frac{3}{2}}+\omega$. Nevertheless this method for determining the strength of the ω-3π vertex may work at higher energies.

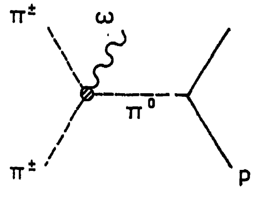

Fig. 10.

Finally we consider electron-positron annihilation processes in future colliding beam experiments being planned at Frascati and Stanford. The e^- and the e^+ create a *timelike* photon which in turn creates an ω-meson, « sitting all by itself » in the laboratory system. The ω-meson then decays into $\pi^++\pi^-+\pi^0$, $\pi^0+\gamma$, $\pi^++\pi^-$, e^++e^-, etc. Such contributions from the *real*

intermediate ω-meson can enhance the electromagnetic annihilation cross-sections by many orders of magnitude, as emphasized particularly by CABIBBO and GATTO [60]. For a rough estimate we may use the Breit-Wigner formula

$$(4.13)\quad \sigma(e^+ + e^- \to (\text{final state}) = 3\pi\lambda\!\!\!^{-2}\,\frac{\Gamma(\omega\to e^+ + e^-)\,\Gamma(\omega\to\text{final state})}{(E-m_\omega)^2+\Gamma_\omega^2/4}\,.$$

The «final state» is any possible state that may be enhanced in e^-e^+ annihilation processes because of the ω-meson, and Γ_ω stands for the total decay width of the ω-meson for all channels. In actual experiments it may be easier not to observe the peak itself but to measure the cross-section averaged over a suitable energy interval. We then have

$$(4.14)\qquad \bar\sigma = \frac{1}{2\,\Delta E}\int\limits_{m_\omega-\Delta E}^{m_\omega+\Delta E}\sigma(E)\,\mathrm{d}E = \frac{3}{2}\,\pi\lambda\!\!\!^{-2}\,\frac{\Gamma(\omega\to e^+ + e^-)\,\Gamma(\omega\to\text{final state})}{2\,\Delta E\,\Gamma_\omega}\,,$$

which would enable us to determine both the various branching ratios and the total decay rate Γ_ω.

5. – Unitary symmetry.

5·1. *Basic motivation.* – One of the most striking features of strong interactions is that there are many particles or resonant states with the same spin-parity and baryon number, but with different isospin and strangeness. In fact, some of the strongly interacting particles seem to form supermultiplets —for istance, the π, the η and the K are all 0^- particles. Just as the existence of ordinary multiplets (*e.g.*, π^+, π^-, π^0) allowed us to establish the law of charge-independence (long before charge-independence was directly tested for the S-matrix), the existence of the supermultiplets may imply a new symmetry higher than the symmetry implied by charge-independence.

Let us look for a moment at the CHEW-FRAUTSCHI plot [61] of strongly interacting particles discussed in Rosenfeld's course. As you recall, in such a plot angular momentum (which is regarded as a continuous variable) is plotted against $(\text{mass})^2$. The correlations among particles considered by CHEW and FRAUTSCHI are vertical in the sense that states with different spins but the same strangeness and isospin are connected via Regge trajectories. In contrast, the correlations introduced in a higher symmetry scheme such as the unitary symmetry model are horizontal in the sense that states with the same spin-parity but different strangeness and isospin are related. The two approaches are not necessarily in conflict with each other. Maybe they are even complementary (as emphasized by GELL-MANN). However, the «extreme Reggeists» (*e.g.*, CHEW and FRAUTSCHI [61]) believe that there are no hidden

symmetries and that the whole of strong-interaction physics including the conservation of baryons should follow from the twin principles of saturated unitarity and maximal analyticity. (Such extreme points of view are not universally shared by other theoretical physicists.)

5'2. *Unitary symmetry in two dimensions.* – I want to make it clear in the very beginning that you don't have to be esoteric group-theoreticians to be able to follow my lectures on unitary symmetry. Anybody who knows how to multiply matrices should be able to derive almost all of the physically interesting consequences of unitary symmetry. Indeed, you are already familiar with unitary symmetry in two dimensions even if you don't realize it.

Usually one discusses isospin rotations in complete analogy with rotations in a 3-dimensional real Euclidian space. Just as invariance under rotations in the ordinary Euclidian space implies angular-momentum conservation, invariance under rotations in isospin space implies isospin conservation. (In the group-theoretical classification this symmetry group is called O_3.) But the notion of charge-independence may be introduced in an entirely different way as we shall see below.

Consider two primitive objects, *e.g.*, the doublet, p and n. (This is the simplest example possible apart from the singlet which is too simple.) As is well-known, any isospin rotation can be completely characterized by its effect on the p-n doublet

$$\exp\left[i\frac{\boldsymbol{\tau}}{2}\cdot\hat{n}\theta\right]\begin{pmatrix}p\\ n\end{pmatrix}=\exp\left[i\sum_{j}^{3}\tau_j\theta_j/2\right]\begin{pmatrix}p\\ n\end{pmatrix}, \tag{5.1}$$

where the τ_j ($j=1, 2, 3$) are the traceless, Hermitian Pauli-matrices, and $\hat{n}$ is a unit vector parallel to the axis of the isospin rotation in question. Now (5.1) can be written in a different form:

$$\exp[i\boldsymbol{\tau}\cdot\hat{n}\theta/2)=\begin{pmatrix}\alpha & \beta\\ -\beta^* & \alpha^*\end{pmatrix}, \tag{5.2}$$

where α and β are *complex* numbers satisfying

$$|\alpha|^2+|\beta|^2=1\,. \tag{5.3}$$

Equation (5.2) follows immediately if we note

$$\exp\left[i\boldsymbol{\tau}\cdot\hat{n}\,\frac{\theta}{2}\right]=\cos\frac{\theta}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+i\boldsymbol{\tau}\cdot\hat{n}\sin\frac{\theta}{2}=$$

$$=\begin{pmatrix}\cos(\theta/2)+in_3\sin(\theta/2) & (in_1+n_2)\sin(\theta/2)\\ (in_1-n_2)\sin(\theta/2) & \cos(\theta/2)-in_3\sin(\theta/2)\end{pmatrix},$$

which is obviously of the form (5.2). Note also that (5.3) is satisfied.

Any isospin rotation can be characterized by 3 real numbers, *e.g.*, θ_1, θ_2 and θ_3 in eq. (5.1). In our new form (5.2) we still have 3 real independent parameters since we have two complex numbers (equivalent to 4 real numbers) and one relation between them. The matrix (5.2) together with the condition (5.3) is unitary. The constraint (5.3) amounts to det $=1$ (unimodular condition).

We have shown that instead of talking about rotations in some mysterious space we can simply consider unitary transformations on two primitive objects. Charge-independence can now be regarded as the consequence of invariance under unitary unimodular transformations of the form (5.2) with the constraint (5.3).

We may parenthetically remark that the matrix (5.2) with the condition (5.3) is not the most general 2×2 unitary matrix. The most general unitary matrix is no longer required to be unimodular, and can be written as

$$\exp[i\gamma]\begin{pmatrix}\alpha & \beta \\ -\beta^* & \alpha^*\end{pmatrix}, \tag{5.4}$$

where γ is real and α and β again satisfy the unimodular condition (5.3). Note that $\exp[i\gamma]$ affects the p and the n in the same way. In field theory the phase transformation $\exp[i\gamma]$ is nothing more than the gauge transformation that generates the conservation of nucleon number (or baryon number or hypercharge since we are talking about « nonstrange » objects). The group of transformations that are represented by the unitary unimodular matrix (5.2) with (5.3) is called SU(2) whereas the group characterized by the unitary matrix (5.4) is called U(2). The group U(2) factors into the product $\mathrm{U}(1)\otimes\mathrm{SU}(2)$ where U(1) is a one-parameter unitary group and SU(2) is a three-parameter unitary group.

More complicated objects such as the charge-triplet pion may be built up if we start with the doublet (pn) and the antidoublet ($\bar{\mathrm{p}}\bar{\mathrm{n}}$). This does not necessarily mean that we have to consider the pions as bound states of nucleons and antinucleons but rather that the pion trasforms as a bound state of a nucleon and an antinucleon as far as its isospin properties are concerned.

Consider now the outer product of the doublet (pn) and of the antidoublet ($\bar{\mathrm{p}}\bar{\mathrm{n}}$). In matrix form it reads

$$\mathscr{M}=\begin{pmatrix}\bar{p}p & \bar{n}p \\ \bar{p}n & \bar{n}n\end{pmatrix}. \tag{5.5}$$

This matrix has mixed properties under isospin rotations—there is a singlet

($T=0$) part and a triplet ($T=1$) part. The trace of the $\mathcal{M}$-matrix

$$(\bar{p}p+\bar{n}n)\,, \tag{5.6}$$

transforms like a singlet, as is well known to physicists. We can, of course, show the invariance of the trace of the $\mathcal{M}$-matrix under unitary transformations using formal mathematical techniques. (This is left as an exercise.) Since this trace transforms like a $T=0$ meson it may be identified with the η-meson. We can now form a traceless matrix Π by subtracting

$$\frac{1}{2}\,(\bar{p}p+\bar{n}n)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

from (5.5). We then obtain

$$\Pi=\begin{pmatrix}\dfrac{\bar{p}p-\bar{n}n}{2} & \bar{n}p\\ \bar{p}n & \dfrac{\bar{n}n-\bar{p}p}{2}\end{pmatrix}=\begin{pmatrix}\pi^0/\sqrt{2} & \pi^+\\ \pi^- & -\pi^0/\sqrt{2}\end{pmatrix}. \tag{5.6}$$

This traceless matrix does describe the charge-triplet pion.

To sum up, we have shown how to construct the $T=0$ η-meson and the $T=1$ π-meson as «bound states» of the $\mathcal{N}\bar{\mathcal{N}}$ system. We have presented this in detail since our approach can be immediately generalized to the case where we have three (rather than two) primitive objects.

5'3. *Unitary-symmetry model based on the Sakata triplet.* – The SU(2) group takes into account the known internal symmetries of strong-interaction physics as long as we confine ourselves only to nonstrange particles. We can construct more complicated objects—for istance, the 3-3 isobar which is a quartet—but the states we obtain always have the property

$$Q=T_3+\frac{B}{2}\,.$$

In other words, we will never be able to construct strange particles if we use nucleons and antinucleons as our only building blocks. If we are to construct strange-particle states, we must have (at least) one more primitive object with strangeness $\neq 0$. The most natural and economical thing to do is to add the Λ hyperon to the nucleon doublet, as proposed by SAKATA [62]. At first sight this seems quite satisfactory because Λ, apart from carrying strange-

ness, is a singlet. Thus we are led to consider the Sakata triplet (p, n, Λ) in analogy with the nucleon doublet (p, n).

Now that we have three primitive objects, we consider unitary unimodular transformations which are characterized by 3×3 matrices. This corresponds to the group SU(3). Note that a unitary unimodular transformation on the « Sakatons » mixes not only p and n, but also the Λ and the nucleon. The large mass difference between the Λ and the nucleon serves to emphasize the approximate nature of our new symmetry scheme since the $\mathcal{N}$—Λ mass difference term is not invariant under unitary unimodular transformations that mix the Λ and the nucleon.

It is well known in group theory that in order to characterize a continuous group such as SU(3) it is sufficient to examine only the behavior under infinitesimal transformations (*i.e.*, those which differ from the identity transformation only very slightly). Let us therefore investigate the generators of infinitesimal transformations of the SU(3) group. But first, recall that an infinitesimal transformation of the SU(2) group can be written as

$$1+\frac{i\tau_i\,\delta\theta_i}{2},$$

where the τ_i's are the usual 2×2 traceless, Hermitian, Pauli matrices, and the $\delta\theta_i$'s are real infinitesimal parameters that characterize the isospin rotation. An immediate generalization allows us to write down an infinitesimal transformation of the SU(3) group as

$$1+\frac{i\lambda_j\,\delta\theta_j}{2}, \tag{5.7}$$

where the λ_j's are now 3×3 traceless Hermitian matrices, and the $\delta\theta_j$'s are again real infinitesimal parameters. It is easy to convince oneself just by counting the number of components that there are $(3^2-1)=8$ independent traceless λ_j's just as there are $(2^2-1)=3$ independent τ-matrices. These λ_j's are assumed to act on the column matrix

$$\begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix},$$

just as the τ_i's act on

$$\begin{pmatrix} p \\ n \end{pmatrix}.$$

We write down an explicit representation for the λ_j's found in GELL-MANN's paper [20] although we shall never need it. λ_1, λ_2 and λ_3 are just like τ_1, τ_2 and τ_3; they do not mix nonstrange and strange states. Explicitly

$$\lambda_j = \left(\begin{array}{cc:c} \multicolumn{2}{c:}{\tau_j} & 0 \\ & & 0 \\ \hdashline 0 & 0 & 0 \end{array}\right) \qquad \text{for } j = 1, 2 \text{ and } 3\,.$$

On the other hand, λ_4, λ_5, λ_6 and λ_7 do mix the strange singlet (Λ) and the nonstrange doublet ($\mathcal{N}$).

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & 0 & 0 \end{pmatrix}.$$

The matrix λ_8 is a diagonal matrix:

$$\lambda_8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \end{pmatrix}.$$

It distinguishes the Λ hyperon from the nucleon. So the eigenvalues of the λ_8 are very much like the strangeness or the hypercharge quantum number. Note that we can diagonalize simultaneously only two of the eight λ-matrices —in our explicit representation, λ_3 and λ_8.

As far as baryon number, isospin and strangeness are concerned, the Sakata model can describe any strongly interacting state. For instance, we can construct a meson octet ($\pi^{\pm,0}$, η, $K^{+,0}$, K^-, $\overline{K}^0$) by taking the outer product of the Sakata triplet and the anti-Sakata triplet just as we have obtained the π and the η in the SU(2) case. For instance, $K^+=(\overline{\Lambda}p)$ etc. Note that the strangeness quantum number is just the number of $\overline{\Lambda}$'s minus the number of Λ's. From a certain point of view, this is the most economical model, and, for this reason, unitary symmetry based on the Sakata triplet has been investigated by a number of peopl—particularly by YAMAGUCHI [63], by WESS [64]

and by IKEDA, OGAWA and OHNUKI [65]. But there are some unpleasant features.

First, why should we choose p, n and Λ as the members of the « privileged class » when there are other equally acceptable candidates? In fact, since the relative $\Lambda\Sigma$ parity is now known to be most likely even [66-68], it is perhaps more natural to group the Λ and the Σ together. In addition, in the unitary symmetry model based on the Sakata triplet, it can be shown to be most natural to let the Ξ hyperon belong to yet another representation, the $J=\frac{3}{2}^+$ family together with the π-$\mathcal{N}$ 3-3 resonance. The assignment of spin $\frac{3}{2}$ to Ξ has not yet been ruled out experimentally, but preliminary data of the Brookhaven-Syracuse and the UCLA groups indicate that there is a large up-down asymmetry in the decay of Ξ-particles produced in the reaction $K^-+p \to \Xi+K$. If this tendency persists in future experiments, then the Ξ spin must be $\frac{1}{2}$ by the so-called LEE-YANG test [69]. (If the decaying particle has a high spin, the decay products tend to be emitted preferentially in the production plane rather than in directions normal to the plane.)

Finally let us see how symmetry based on the Sakata triplet has fared in the acid test of experiments. One of the predictions of this model is that the process

$$\bar{p}+p \to K_1^0+K_2^0 ,$$

is forbidden as shown by LEVINSON *et al.* [70]. (This can readily be seen if we recognize that invariance under unitary symmetry transformations imply in particular invariance under $n \rightleftarrows \Lambda$, $p \rightleftarrows \bar{p}$, $K^0 \rightleftarrows \bar{K}^0$, etc.) However, since unitary symmetry is only approximate, there may be some violation of the rule. For instance, we expect that the annihilation rate into $K_1^0+K_2^0$ might be about 1/10 as compared to other competing processes such as

$$\begin{aligned} p+\bar{p} &\to K^+ + K^- \\ &\to \pi^+ + \pi^- , \end{aligned}$$

which are fully allowed by the symmetry scheme. Experimentally the branching ratio has been found to be [71]

$$\pi^+\pi^- : K^+K^- : K_1^0K_2^0 \approx 4.0 : 1.3 : 0.6 ,$$

as discussed in Armenteros' seminar. This does not look too hopeful for the unitary symmetry model based on the Sakata triplet.

Let us sum up the defects of unitary symmetry à la Sakata.

i) Σ is « ugly ».

ii) A Ξ spin assignment of $\frac{3}{2}$ is most natural.

iii) $p+\bar{p} \to K_1^0+K_2^0$ is forbidden, contrary to observation.

5·3. *Baryons and mesons in the octet model.* – We now consider unitary symmetry based on an octet of baryons—$\mathcal{N}$, Λ, Σ and Ξ. This is the famous octet model or the « eightfold way », which has been worked out by GELL-MANN [5, 20] and NE'EMAN [21]. In this model the « primitive objects » are hidden in the sense that they do not correspond to any observable strongly interacting states. Just the same, in analogy with the Sakata case, we introduce as primitive objects of this theory a baryon supermultiplet

$$\begin{pmatrix} D^+ \\ D^0 \\ S^0 \end{pmatrix}. \tag{5.8}$$

and also a boson supermultiplet

$$(\varkappa^-, \bar{\varkappa}^0, \sigma^0)\,. \tag{5.9}$$

The quantum numbers of the components of the baryon supermultiplet (5.8) are the same as those of the Sakata triplet (p, n and Λ), *i.e.*, $B=1$, $T=\frac{1}{2}$ for D^+ and D^0, and $B=1$, $T=0$ for S^0. The quantum numbers of the components of the boson multiplet (5.9) are the same as those of K^-, $\overline{K}{}^0$ and η, *i.e.*, $B=0$, $T=\frac{1}{2}$ for $\varkappa^-$ and $\bar{\varkappa}^0$ and $B=0$, $T=0$ for σ^0. (Strictly speaking σ^0 is not quite like η^0; $\bar{\sigma}^0$ must be distinguishable from σ^0 if our fictitious model is to work.)

From the primitive objects (5.8) and (5.9) we construct the observed baryon octet ($\mathcal{N}$, Λ, Σ, Ξ) just as we constructed π and η out of p, n, $\overline{\text{p}}$ and $\overline{\text{n}}$ in the SU(2) case. We have the direct product of (5.8) and (5.9):

$$\begin{pmatrix} D^+\varkappa^- & D^+\bar{\varkappa}^0 & D^+\sigma^0 \\ D^0\,\varkappa^- & D^0\,\bar{\varkappa}^0 & D^0\,\sigma^0 \\ S^0\,\varkappa^- & S^0\,\bar{\varkappa}^0 & S^0\,\sigma^0 \end{pmatrix}. \tag{5.10}$$

From this matrix let us subtract its trace, which is a singlet under SU(3) transformations. The singlet we must subtract is

$$\frac{D^+\varkappa^- + D^0\bar{\varkappa}^0 + S^0\sigma^0}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

which is completely analogous to

$$\frac{\overline{p}p + \overline{n}n}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We then obtain the following octet ($3^2-1=8$).

$$
(5.11)\qquad \begin{pmatrix} \dfrac{2D^+\varkappa^- - D^0\bar{\varkappa}^0 - S^0\sigma^0}{3} & D^+\bar{\varkappa}^0 & D^+\sigma^0 \\ D^0\varkappa^- & \dfrac{-D^+\varkappa^- + 2D^0\bar{\varkappa}^0 - S^0\sigma^0}{3} & D^0\sigma^0 \\ S^0\varkappa^- & S^0\bar{\varkappa}^0 & \dfrac{-D^+\varkappa^- - D^0\bar{\varkappa}^0 + 2S^0\sigma^0}{3} \end{pmatrix}.
$$

We shall not « prove » in this course that the octet we have obtained is irreducible (*i.e.*, no further decomposition is possible) with respect to the symmetry group considered. This we may accept in analogy with the SU(2) case.

It is possible to identify the various elements of this matrix (5.11) with observed stable baryons from the point of view of the quantum numbers B and T. For instance, by recalling that (D^+, D^0) transforms like (p, n) and σ^0 like η^0, we see that $D^+\sigma^0$ and $D^0\sigma^0$ must form a doublet, which we identify with (p, n). Similarly $S^0\varkappa^-$ and $S^0\bar{\varkappa}^0$ are to be identified with Ξ^- and Ξ^0 since they form a doublet whose center of charge is $-\frac{1}{2}$. As for the $S=-1$ baryons, $D^+\bar{\varkappa}^0$ and $D^0\varkappa^-$ must be Σ^+ and Σ^- respectively. Then, by charge-independence, the linear combination

$$
\frac{D^+\varkappa^- - D^0\bar{\varkappa}^0}{\sqrt{2}},
$$

must correspond to Σ^0. What is left over must be an isosinglet, which we identify with the Λ hyperon

$$
\frac{D^+\varkappa^- + D^0\bar{\varkappa}^0 - 2S^0\sigma^0}{\sqrt{6}} \sim \Lambda ,
$$

where $\sqrt{6}$ has been iserted so that the Λ-state is correctly normalized.

With these identifications we can rewrite the traceless matrix (5.11) as follows

$$
(5.12)\qquad \mathscr{B} = \begin{pmatrix} \dfrac{\Sigma^0}{\sqrt{2}} + \dfrac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\dfrac{\Sigma^0}{\sqrt{2}} + \dfrac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\dfrac{2}{\sqrt{6}}\Lambda \end{pmatrix}.
$$

We have constructed a unitary-symmetry scheme in which the stable baryons emerge in a natural way as an octet, in contrast to the Sakata case

where the Σ belongs to a $J=\frac{1}{2}$ supermultiplet which is different from the $J=\frac{1}{2}$ supermultiplet to which p, n and Λ belong, and Ξ belongs to yet another supermultiplet, most likely with $J=\frac{3}{2}^+$. We recall that the primitive objects were introduced just as a mathematical device for constructing the actual particle states in order to keep track of the quantum numbers, B, S and T. Once we have found the baryon octet with the correct transformation properties, we can forget about the primitive objects.

We can now write down an octet corresponding to the observed pseudoscalar mesons. It must have the same matrix structure as the baryon octet. We have

$$\mathscr{P}=\begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\dfrac{2}{\sqrt{6}}\eta^0 \end{pmatrix}. \tag{5.13}$$

This octet fits beautifully into the octet of the observed pseudoscalar mesons. (The η must be even under $C=G$ if the π is odd under G. This follows because both the third component of the isovector-pseudoscalar density and the isoscalar-pseudoscalar density are even under C.)

In the same way we have the vector meson octet

$$\mathscr{V}=\begin{pmatrix} \dfrac{\varrho^0}{\sqrt{2}}+\dfrac{\omega^0}{\sqrt{6}} & \varrho^+ & M^+ \\ \varrho^- & -\dfrac{\varrho^0}{\sqrt{2}}+\dfrac{\omega^0}{\sqrt{6}} & M^0 \\ M^- & \bar{M}^0 & -\dfrac{2}{\sqrt{6}}\omega^0 \end{pmatrix}. \tag{5.14}$$

We have assumed that the ω-meson is a member of the unitary octet together with ρ and M. It is possible that the *observed* 780 MeV ω is a unitary singlet provided that there is another $T=0$ vector meson at ~ 1 GeV. We shall come back to this point when we discuss the mass formula. The M's have strangeness $=1$. They form a $Y=S=1$ doublet ($T=\frac{1}{2}$). They can well be identified with the 880 MeV K*'s if the K* spin turns out to be unity.

5·4. *Interactions in the octet model.* – Yukawa-type interactions are linear in a baryon field, an antibaryon field, and a pseudoscalar meson field. It is most convenient to write them down in matrix form. The appropriate matrix

representation for the antibaryons turns out to be the following.

(5.15)
$$\overline{\mathscr{B}} = \begin{pmatrix} \dfrac{\overline{\Sigma^0}}{\sqrt{2}} + \dfrac{\overline{\Lambda^0}}{\sqrt{6}} & \overline{\Sigma^-} & \overline{\Xi^-} \\ \overline{\Sigma^+} & -\dfrac{\overline{\Sigma^0}}{\sqrt{2}} + \dfrac{\overline{\Lambda^0}}{\sqrt{6}} & \overline{\Xi^0} \\ \bar{p} & \bar{n} & -\dfrac{2}{\sqrt{6}}\overline{\Lambda^0} \end{pmatrix}.$$

We are interested in constructing interactions that are invariant under unitary symmetry transformations. Noting that the trace is invariant, we have only two possible independent terms:

$$\mathrm{Tr}\,(\overline{\mathscr{B}}\mathscr{P}\mathscr{B}) \quad \text{and} \quad \mathrm{Tr}\,(\overline{\mathscr{B}}\mathscr{B}\mathscr{P})\,.$$

Instead, we may consider the following linear combinations:

(5.16)
$$\begin{cases} \text{Pure } D\text{-type:} & \mathrm{Tr}\,(\overline{\mathscr{B}}\mathscr{P}\mathscr{B} + \overline{\mathscr{B}}\mathscr{B}\mathscr{P})\,, \\ \text{Pure } F\text{-type:} & \mathrm{Tr}\,(\overline{\mathscr{B}}\mathscr{P}\mathscr{B} - \overline{\mathscr{B}}\mathscr{B}\mathscr{P})\,. \end{cases}$$

In is straightforward to find the appropriate coupling constants for the D and F-type couplings. In an Appendix the D and F-type interactions are written down explicitly. The fact that there are two independent Yukawa-type interactions invariant under unitary symmetry transformations can be shown to be a direct consequence of the fact that the representation 8 appears twice in the decomposition of $8 \otimes 8$ (see eq. (5.26) below).

The D-type interactions are also invariant under the substitutions

(5.17)
$$\begin{cases} \mathscr{B} \to \mathscr{B}^T \\ \overline{\mathscr{B}} \to \overline{\mathscr{B}}^T \\ \mathscr{P} \to \mathscr{P}^T\,, \end{cases}$$

where T stands for « transposed ». On the other hand, the F-type interactions are invariant under

(5.18)
$$\begin{cases} \mathscr{B} \to \mathscr{B}^T \\ \overline{\mathscr{B}} \to \overline{\mathscr{B}}^T \\ \mathscr{P} \to -\mathscr{P}^T\,. \end{cases}$$

These substitutions are known as the hypercharge reflection [5] R. More explicitly

$$(5.19)\qquad \left\{\begin{array}{lll} \begin{pmatrix} p \\ n \end{pmatrix} \rightleftarrows \begin{pmatrix} \Xi^- \\ \Xi^0 \end{pmatrix} & \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma_+ \end{pmatrix} \rightleftarrows \begin{pmatrix} \Sigma^- \\ \Sigma^0 \\ \Sigma_+ \end{pmatrix} & \Lambda \rightleftarrows \Lambda\,, \\[2ex] \begin{pmatrix} K^+ \\ K^- \end{pmatrix} \rightleftarrows \pm \begin{pmatrix} K^- \\ K^+ \end{pmatrix} & \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \rightleftarrows \pm \begin{pmatrix} \pi^- \\ \pi^0 \\ \pi^+ \end{pmatrix} & \eta \rightleftarrows + \eta\,, \end{array}\right.$$

where the upper (lower) signs are appropriate for the D (F) type. We emphasize that the most general Yukawa interactions invariant under unitary symmetry are an arbitrary linear combination of the D type and the F type. R-invariance, which enables us to select one form or the other is an additional symmetry operation which does not belong to the unitary-symmetry group.

As can be seen from Appendix, for the pure D-type interactions we obtain

$$g^2_{\pi\Lambda\Sigma} = \tfrac{4}{3} g^2_{\pi\mathcal{N}\mathcal{N}}\,,$$

$$g^2_{\pi\Sigma\Sigma} = 0\,, \qquad \text{etc.}$$

The vanishing of the $\pi\Sigma\Sigma$ coupling can readily be seen by noting that a typical term in the $-i\bar{\boldsymbol{\Sigma}}\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}$ interaction, say $\pi^0(\bar{\Sigma}^+\Sigma^+ - \bar{\Sigma}^-\Sigma^-)$, changes its sign under (5.17). In the F-type interactions the pion is coupled to a pseudoscalar density which transforms isotopically like the total isospin. No $\boldsymbol{\pi}\cdot\bar{\Lambda}\boldsymbol{\Sigma}$ coupling appears. From what we know about the $\Lambda\mathcal{N}$ forces deduced from hyperfragment binding energies, the $\pi\Lambda\Sigma$ coupling seems to be sizable [72]. So the pure F-type couplings seem to be ruled out. Also in the D-type interactions, $g_{\pi\mathcal{N}\mathcal{N}} = -g_{\pi\Xi\Xi}$ whereas in the F-type interactions $g_{\pi\mathcal{N}\mathcal{N}} = +g_{\pi\Xi\Xi}$. Different results are obtained by considering different linear combinations of the D and the F type; for example, a 50-50 mixture would make $g_{\pi\Xi\Xi} = 0$.

An important consequence of the model is that the K couplings cannot be made much weaker than the π couplings in any combination of the D and the F type. Phenomenologically it appears that the π couplings are about 10 times stronger than the K couplings if the pseudoscalar coupling constants are compared. (Compare, for instance, the $K\Lambda\mathcal{N}$, $K\Sigma\mathcal{N}$ couplings with the $\pi\mathcal{N}\mathcal{N}$ coupling in photoproduction experiments.) We must, however, point out that unitary symmetry is badly broken by the large value of the mass ratio $m_K/m_\pi \sim 3.5$. As it is difficult to test « broken symmetries » there is a considerable amount of arbitrariness. Note, in this connection, that unitary

symmetry can be maintained if the pseudovector coupling constants are compared since the largeness of $(m_K/m_\pi)^2$ would just compensate the smallness of the pseudoscalar K coupling constants.

Let us now consider the couplings of the vector mesons to the baryons. We again have two possible couplings:

$$\text{(5.20)} \qquad \begin{cases} \text{Pure } D: & \mathrm{Tr}\,(\overline{\mathscr{B}}\mathscr{V}\mathscr{B}+\overline{\mathscr{B}}\mathscr{B}\mathscr{V})\,, \\ \text{Pure } F: & \mathrm{Tr}\,(\overline{\mathscr{B}}\mathscr{V}\mathscr{B}-\overline{\mathscr{B}}\mathscr{B}\mathscr{V})\,, \end{cases}$$

where $\mathscr{B}$, $\overline{\mathscr{B}}$ and $\mathscr{V}$ are the matrix representations for the baryon octet, the antibaryon octet and the vector meson octet defined earlier. By explicit calculations, the results of which are given in Appendix, we see that the D type couplings bear no resemblance whatsoever to the universal couplings of the ρ and the ω to the conserved isospin current and the hypercharge current. On the other hand, the F-type couplings of the vector mesons to the baryons can be seen to be a very natural generalization of the vector theory of strong interactions in which the ρ is coupled to the isospin current and the ω is coupled to the hypercharge current. In addition, we have also the $S=1$ M-meson coupled to the strangeness-changing current

$$(-\sqrt{3}\,\overline{\mathscr{N}}\Lambda-\overline{\mathscr{N}}\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}+\ldots)\,\mathrm{M}\,.$$

It might be possible to detect this kind of interaction in associated production if reactions such as $\pi^-+p\to\Lambda+K^0$ are dominated by the exchange of the M-meson (which, as we have remarked, may well be the 880 MeV K*).

From a more formal point of view we may remark that in the F-type coupling scheme the vector mesons are coupled to the current generated by the gauge transformation of unitary symmetry

$$1+i\lambda_i\delta\theta_i/2\,.$$

The currents generated with $\lambda_i=\lambda_1$, λ_2 and λ_3 are the three components of the isospin current to which the ρ is coupled; with $\lambda_i=\lambda_8$ we generate the hypercharge current to which the ω is coupled. Finally with $\lambda_i=\lambda_4,\ldots,\lambda_7$ we generate strangeness-changing currents to which the M and the $\overline{\mathrm{M}}$ are coupled; these strangeness-changing currents are conserved only in the limits where the mass differences among the baryons go to zero.

In the octet model with the F-type interactions of the vector mesons there is a relation between the coupling constant f_ω and f_ρ:

$$\text{(5.21)} \qquad \frac{f_\omega^2}{4\pi}=\frac{3}{4}\frac{f_\rho^2}{4\pi}=3\frac{\gamma^2}{4\pi}\,,$$

where γ is the vector meson coupling constant defined in GELL-MANN's paper [5, 20]. From the width of the ρ-meson, we obtain

$$\frac{f_\rho^2}{4\pi} \approx 2.0\,,$$

which, in turn, implies

$$\frac{f_\omega^2}{4\pi} \approx 1.5\,.$$

Nuclear force calculations [73, 74] performed recently seem to give a larger value for $f_\omega^2/4\pi$. This discrepancy may be due to the possible existence of another $T=0$ vector meson.

The couplings of the vector mesons to the pseudoscalar mesons of the form $\mathscr{V}\mathscr{P}\mathscr{P}$ must be of the F type. This follows from charge conjugation invariance. For example, in the D-type coupling scheme we would have a term like $\rho^0\eta^0\pi^0$ which is noninvariant under charge conjugation. The F-type couplings of the vector mesons to the pseudoscalar mesons of the form $\mathscr{V}\mathscr{P}\mathscr{P}$ imply that the ρ is coupled to the isospin of the pion with the same strength as it is coupled to the isospin of the K-particle, in agreement with the universality principle.

There are two couplings of the vector mesons to the pseudoscalar mesons which may be « directly » observed: $\rho \to 2\pi$ and $M \to K+\pi$. Using Appendix and eq. (3.5) we readily obtain

$$\frac{\Gamma(M \to K+\pi)}{\Gamma(\rho \to 2\pi)} = \frac{3}{4}\frac{p_{K\pi}^3/m_M^2}{p_{\pi\pi}^3/m_\rho^2}\,. \tag{5.23}$$

If the M-meson is the K* resonance at 880 MeV, then from the observed ρ width of 100 MeV, we obtain $\Gamma(M) \approx 30$ MeV which is to be compared with the observed width of K*: $\Gamma(K^*) \approx 47$ MeV as quoted in Ticho's course.

In the unitary symmetry model there is an open possibility for a vector meson coupled to the baryon current. This will be a unitary singlet vector meson since the baryon current is of the form

$$(\bar{p}p+\bar{n}n+\bar{\Lambda}\Lambda+\ldots) = \mathrm{Tr}\,(\bar{\mathscr{B}}\mathscr{B})\,,$$

which is obviously a unitary singlet. (In fact, this is the unique bilinear form of $\bar{\mathscr{B}}$ and $\mathscr{B}$ wich transforms like a unitary singlet.) Note also that it is impossible to construct a vector current with pseudoscalar mesons that transforms like a unitary singlet (*e.g.*, one cannot construct a current bilinear in the η-meson). So if there exists a unitary singlet vector meson, it must necessarily be the kind coupled to the baryon current.

5'5. *Predictions of the octet model.* – Let us now summarize the predictions of the eightfold way based on the Gell-Mann-Ne'eman octet. First of all, all members of a given symmetry multiplet must have the same spin-parity. From this point of view the most crucial test is the spin of Ξ which, in the octet model, must be $\frac{1}{2}$. Also there must exist a $T=\frac{1}{2}$ $K\pi$ resonance with spin 1 which may well be the 880 MeV K^*. With respect to parity, once we define the Λ parity to be even by convention, the $\Lambda\Sigma$ parity and the $\Xi\mathcal{N}$ parity must be even, and the K must be pseudoscalar. When the octet model was proposed the $\Lambda\Sigma$ parity was not known to be even, nor was there any evidence for the $T=0$ pseudoscalar η-meson.

The second prediction of unitary symmetry is that all members of a given unitary symmetry multiplet must have the same mass. This we know is not fulfilled; otherwise unitary symmetry would have been discovered many years ago. However, if unitary symmetry is broken only weakly, it is possible to derive a formula for the mass spectrum.

For pedagogical purposes it is instructive to go back to our fictitious model of baryons in terms of D^+, D^0, S^0, $\varkappa^-$, $\bar{\varkappa}^0$ and σ^0. The very breakdown of unitary symmetry means that the D-doublet mass need not be the same as the S^0-singlet mass; similarly the $\varkappa\sigma$ mass difference need not be zero. In this fictitious model we assume that the forces that bind the (D^+, D^0, S^0) set and the ($\varkappa^-$, $\bar{\varkappa}^0$, σ^0) set are assumed to be independent of isospin and strangeness; otherwise we would introduce higher-order violations of unitary symmetry. Using

$$D^+\sigma^0 \sim p\,, \qquad S^0\varkappa^- \sim \Xi^-, \qquad D^+\bar{\varkappa}^0 \sim \Sigma^+,$$
$$\frac{D^+\varkappa^- + D^0\bar{\varkappa}^0 - 2S^0\sigma^0}{\sqrt{6}} \sim \Lambda^0\,,$$

we obtain

$$\begin{aligned}
m_{\mathcal{N}} &= m_D + m_\sigma - E_B\,,\\
m_\Lambda &= \tfrac{2}{6}(m_D + m_\varkappa) + \tfrac{4}{6}(m_S + m_\sigma) - E_B\,,\\
m_\Sigma &= m_D + m_\varkappa - E_B\,,\\
m_\Xi &= m_S + m_\varkappa - E_B\,,
\end{aligned}$$

where E_B stands for the binding energy. From this we obtain

$$\frac{m_{\mathcal{N}} + m_\Xi}{2} = \frac{3m_\Lambda + m_\Sigma}{4}\,, \tag{5.24}$$

as first derived by GELL-MANN [5, 20]. Experimentally the left-hand side is about 1127 MeV while for the right-hand side we have 1134 MeV. We might

remark that eq. (5.24) can also be written as

$$-\mathscr{L}_m = m_1 \operatorname{Tr}(\overline{\mathscr{B}}\mathscr{B}) + m_2 \operatorname{Tr}(\overline{\mathscr{B}}\lambda_8\mathscr{B}) + m_3 \operatorname{Tr}(\overline{\mathscr{B}}\mathscr{B}\lambda_8)\,, \tag{5.25}$$

where $\mathscr{L}_m$ stands for the sum of the mass terms in the effective Lagrangian.

A similar relation holds for the pseudoscalar mesons, but, as suggested by Feynman, it is better to work with (mass)^2. Then for the pseudoscalar octet we have

$$m_K^2 = \frac{3m_\eta^2 + m_\pi^2}{4}\,. \tag{5.26}$$

Experimentally, for the left-hand side we have (496 MeV)^2 whereas for the right-hand side we have (480 MeV)^2.

For the vector mesons the mass relation is not so good. The analogous formula with the observed ρ and K^* mass predicts the $T=0$ member of the octet at 930 MeV (rather than 780 MeV). Perhaps the observed ω-meson is a unitary singlet and a second $T=0$ vector meson yet to be discovered is the $T=0$ member of the octet. Or else, it may well be that the two $T=0$ vector mesons get mixed up in a complicated way; perhaps such mixing is responsible for the breakdown of unitary symmetry which would otherwise be exact. But all this is extremely speculative.

Attempts have been made to extend the unitary symmetry model based on the Gell-Mann-Ne'eman octet to the low-lying baryon isobars. Since the isospin of $\mathcal{N}^*_{\frac{3}{2}}$ (3-3 resonance) is $\frac{3}{2}$, the representation 8 discussed previously is obviously inadequate. We must construct representations with higher dimensions. Just as we have decomposed the outer product of the representation 3 and the representation $\overline{3}$ (which is inequivalent to the 3) into a singlet and an octet

$$\overline{3}\otimes 3 = 1 \oplus 8\,,$$

we can construct the product $8\otimes 8$ and decompose it as follows:

$$8\otimes 8 = 1\oplus 8\oplus 8\oplus 10\oplus \overline{10}\oplus 27\,. \tag{5.27}$$

Each representation can further be broken down into conventional charge-independence multiplets as shown in Table II. This table can be obtained by standard group theory, as shown in BEHRENDS *et al.* [75]; it can also be obtained in a more pedestrian way, as outlined, for instance, in a paper by GLASHOW and SAKURAI [76].

Back to physics! Any baryon isobar that can decay into a pseudoscalar meson and a baryon must belong to one or another of the representations in Table II if unitary symmetry is to be a useful symmetry for baryon isobars.

TABLE II.

1	$Y=\ \ 0$	$T=0$
8	$Y=\ \ 1$ $Y=\ \ 0$ $Y=-1$	$T=\frac{1}{2}$ $T=0, 1$ $T=\frac{1}{2}$
10	$Y=\ \ 1$ $Y=\ \ 0$ $Y=-1$ $Y=-2$	$T=\frac{3}{2}$ $T=1$ $T=\frac{1}{2}$ $T=0$
$\overline{10}$	$Y=\ \ 2$ $Y=\ \ 1$ $Y=\ \ 0$ $Y=-1$	$T=0$ $T=\frac{1}{2}$ $T=1$ $T=\frac{3}{2}$
27	$Y=\ \ 2$ $Y=\ \ 1$ $Y=\ \ 0$ $Y=-1$ $Y=-2$	$T=1$ $T=\frac{1}{2}, \frac{3}{2}$ $T=0, 1, 2$ $T=\frac{1}{2}, \frac{3}{2}$ $T=1$

The existence of one member of the supermultiplet must imply the existence of all other members of the supermultiplet; otherwise unitary symmetry would be rather violently violated.

In an attempt to fit some of the low-lying isobars to our unitary symmetry scheme, let us recall the following experimental pieces of information.

i) The spin of Y_1^* (1385) is most likely $\frac{3}{2}$ [77].

ii) Y_0^* (1405) is most likely an s-wave ($\overline{K}\mathcal{N}$) bound state resonance; Y_0^* (1520) is a $J=\frac{3}{2}^-$ (d-wave) resonance.

iii) There is no evidence whatsoever for a resonance in K^+p scattering. It is then natural to let $\mathcal{N}^*_{\frac{3}{2}}$ and Y_1^* belong to the representation 10. (The representation 27 could accomodate both $\mathcal{N}^*_{\frac{3}{2}}$ and Y_1^*, but it would also predict a $J=\frac{3}{2}^+$ Y_0^*, and a $p_{\frac{3}{2}}$ K^+p resonance, etc.) This assignment has an important consequence: There must exist a $T=\frac{1}{2}$ $\pi\Xi$ resonance. The «prediction» has been fulfilled [24, 78] as discussed in Ticho's course, although so far there is no evidence for the $J=\frac{3}{2}^+$ spin-parity assignment.

This is not the whole story. OKUBO [79] was able to generalize the Gell-Mann mass formula (5.24) to any unitary symmetry multiplet. It reads

$$m = m_0\left\{1 + aY + b\left[T(T+1) - \frac{Y^2}{4}\right]\right\}, \tag{5.28}$$

from which follows Gell-Mann's relation for the octet discussed previously. For the representation 10, we have the linear relation $T=1+Y/2$ so that the quadratic Y^2 and T^2 terms cancel. Then we must get equal spacing in the mass spectrum, as first emphasized by GELL-MANN [80, 81].

$$m = m_0'(1 + a'Y) .$$

This has indeed been observed to a fantastic degree of accuracy

$\mathcal{N}^*_{\frac{3}{2}}$	1235 MeV,
Y^*_1	1385 MeV,
$\Xi^*_{\frac{1}{2}}$	1535 MeV.

It is also worth-while to observe that if the dimensionless parameters a and b in eq. (5.27) are determined from the baryon octet, then the spacing parameter for the 10 is also correctly predicted [76].

Finally this representation predicts an $S=-3$, $T=0$, $Q=-1$ spin $\frac{3}{2}^+$ hyperon, say Z^-. If we take the mass formula seriously, it must be stable against decay via strong and electromagnetic interactions since the predicted mass is ~ 1685 MeV [76, 80, 81]. (The $\overline{\mathrm{K}}\Xi$ threshold is at 1820 MeV.) The conjectured hyperon could be created in

$$\mathrm{K}^- + \mathrm{p} \to Z^- + \mathrm{K}^+ + \mathrm{K}^0 , \qquad \bar{\mathrm{p}} + \mathrm{p} \to Z^- + \overline{Z^-}, \text{ etc.},$$

and would decay into $\pi+\Xi$, $\overline{\mathrm{K}}+\Lambda$ and $\overline{\mathrm{K}}+\Sigma$ via weak interactions [82]. Should such an object be found experimentally, our confidence in unitary symmetry would grow by an order of magnitude.

* * *

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APPENDIX

A) The *D* type couplings of pseudo scalar mesons with baryons (γ_5's omitted)

$$
\begin{aligned}
\sqrt{2}\,\mathrm{Tr}(\overline{\mathscr{B}}\mathscr{P}\mathscr{B}+\overline{\mathscr{B}}\mathscr{B}\mathscr{P}) = {}& \pi^0\left(\bar{p}p-\bar{n}n+\frac{2}{\sqrt{3}}\overline{\Sigma^0}\Lambda+\frac{2}{\sqrt{3}}\bar{\Lambda}\Sigma^0-\overline{\Xi^0}\Xi^0+\overline{\Xi^-}\Xi^-\right)+\\
&+\pi^+\left(\sqrt{2}\,\bar{p}n+\frac{2}{\sqrt{3}}\overline{\Sigma^+}\Lambda+\frac{2}{\sqrt{3}}\bar{\Lambda}\Sigma^-+\sqrt{2}\,\overline{\Xi^0}\Xi^-\right)+\\
&+\pi^-\left(\sqrt{2}\,\bar{n}p+\frac{2}{\sqrt{3}}\overline{\Lambda\Sigma^+}+\frac{2}{\sqrt{3}}\overline{\Sigma^-}\Lambda+\sqrt{2}\,\overline{\Xi^-}\Xi^0\right)+\\
&+K^+\left(-\frac{1}{\sqrt{3}}\bar{p}\Lambda+\bar{p}\Sigma^0+\sqrt{2}\,\bar{n}\Sigma^--\frac{1}{\sqrt{3}}\bar{\Lambda}\Xi^-+\Sigma^0\Xi^-+\sqrt{2}\,\overline{\Sigma^+}\Xi^0\right)+\\
&+K^0\left(-\frac{1}{\sqrt{3}}\bar{n}\Lambda-\bar{n}\Sigma^0+\sqrt{2}\,\bar{p}\Sigma^+-\frac{1}{\sqrt{3}}\bar{\Lambda}\Xi^0-\overline{\Sigma^0}\Xi^0+\sqrt{2}\,\overline{\Sigma^-}\Xi^-\right)+\\
&+K^-\left(-\frac{1}{\sqrt{3}}\bar{\Lambda}p+\overline{\Sigma^0}p+\sqrt{2}\,\overline{\Sigma^-}n-\frac{1}{\sqrt{3}}\overline{\Xi^-}\Lambda+\overline{\Xi^-}\Sigma^0+\sqrt{2}\,\overline{\Xi^0}\Sigma^+\right)+\\
&+\bar{K}^0\left(-\frac{1}{\sqrt{3}}\bar{\Lambda}n-\overline{\Sigma^0}n+\sqrt{2}\,\overline{\Sigma^+}p-\frac{1}{\sqrt{3}}\overline{\Xi^0}\Lambda-\overline{\Xi^0}\Sigma^0+\sqrt{2}\,\overline{\Xi^-}\Sigma^-\right)+\\
&+\eta\left(-\frac{1}{\sqrt{3}}\bar{p}p-\frac{1}{\sqrt{3}}\bar{n}n-\frac{2}{\sqrt{3}}\bar{\Lambda}\Lambda+\frac{2}{\sqrt{3}}\overline{\Sigma^+}\Sigma^++\frac{2}{\sqrt{3}}\overline{\Sigma^0}\Sigma^0+\right.\\
&\left.+\frac{2}{\sqrt{3}}\overline{\Sigma^-}\Sigma^--\frac{1}{\sqrt{3}}\overline{\Xi^0}\Xi^0-\frac{1}{3}\overline{\Xi^-}\Xi^-\right);
\end{aligned}
$$

To obtain the *D* type couplings of vector mesons with baryons, just let

$$\pi^0\,\bar{p}i\gamma_5 p\to\varrho^0\,\bar{p}i\gamma_\mu p\,,\qquad \eta\bar{\Lambda}i\gamma_5\Lambda\to\omega\bar{\Lambda}i\gamma_\mu\Lambda\,,\ \text{etc.}\,.$$

B) The *F* type couplings of vector mesons with baryons (γ_μ''s omitted)

$$
\begin{aligned}
\sqrt{2}\,\mathrm{Tr}(\overline{\mathscr{B}}\mathscr{V}\mathscr{B}-\overline{\mathscr{B}}\mathscr{B}\mathscr{V}) = {}& \varrho^0(\bar{p}p-\bar{n}n+2\overline{\Sigma^+}\Sigma^+-2\overline{\Sigma^-}\Sigma^-+\overline{\Xi^0}\Xi^0-\overline{\Xi^-}\Xi^-)+\\
&+\varrho^+(\sqrt{2}\,\bar{p}n-2\overline{\Sigma^+}\Sigma^0+2\overline{\Sigma^0}\Sigma^--\sqrt{2}\,\overline{\Xi^0}\Xi^-)+\\
&+\varrho^-(\sqrt{2}\,\bar{n}p-2\overline{\Sigma^0}\Sigma^++2\overline{\Sigma^-}\Sigma^0-\sqrt{2}\,\overline{\Xi^-}\Xi^0)+\\
&+M^+(-\sqrt{3}\,\bar{p}\Lambda-\bar{p}\Sigma^0-\sqrt{2}\,\bar{n}\Sigma^-+\sqrt{3}\bar{\Lambda}\Xi^-+\overline{\Sigma^0}\Xi^-+\sqrt{2}\,\overline{\Sigma^+}\Xi^0)+
\end{aligned}
$$

$$+ M^0(-\sqrt{3}\bar{n}\Lambda + \bar{n}\Sigma^0 - \sqrt{2}\bar{p}\Sigma^+ + \sqrt{3}\,\bar{\Lambda}\Xi^0 - \bar{\Sigma}^0\Xi^0 + \sqrt{2}\bar{\Sigma}^-\Xi^-) +$$
$$+ M^-(-\sqrt{3}\,\bar{\Lambda}p - \bar{\Sigma}^0 p - \sqrt{2}\,\bar{\Sigma}^- n + \sqrt{3}\,\bar{\Xi}^-\Lambda + \bar{\Xi}^-\Sigma^0 + \sqrt{2}\,\bar{\Xi}^0\,\Sigma^+) +$$
$$+ \Lambda^0(-\sqrt{3}\,\bar{\Lambda}n + \bar{\Sigma}^0 n - \sqrt{2}\,\bar{\Sigma}^+ p + \sqrt{3}\,\bar{\Xi}^0\Lambda - \bar{\Xi}^0\Sigma^0 + \sqrt{2}\,\bar{\Xi}^-\Sigma^-) +$$
$$+ \omega(\sqrt{3}\,\bar{p}p + \sqrt{3}\,\bar{n}n - \sqrt{3}\,\bar{\Xi}^0\Xi^0 - \sqrt{3}\,\bar{\Xi}^-\Xi^-)\,.$$

To obtain the F type couplings of pseudoscalar mesons with baryons, just let

$$\varrho^0(\bar{p}i\gamma_\mu p) \to \pi^0(\bar{p}i\gamma_5 p)\,, \qquad M^-\bar{\Lambda}i\gamma_\mu p \to K^-\bar{\Lambda}i\gamma_5 p\,, \text{ etc.}.$$

To obtain the couplings of vector mesons with pseudoscalar mesons, just let

$$i(\bar{p}\gamma_\mu\Lambda - \bar{\Lambda}\gamma_\mu\Xi^-) \to i(K^-\partial_\mu\eta - \eta\partial_\mu K^-)\,,$$
$$i(\bar{\Sigma}^+\gamma_\mu\Sigma^0 - \bar{\Sigma}^0\gamma_\mu\Sigma^-) \to i(\pi^-\partial_\mu\pi^0 - \pi^0\partial_\mu\pi^-)\,, \text{ etc.}$$

C) Our coupling constants differ from those of Gell-Mann's [5,20] in the sense that his couplings read

$$\gamma_\varrho[\varrho^0(\bar{p}p - \bar{n}n + 2\bar{\Sigma}^+\Sigma^+ - \dots) + \varrho^+(\sqrt{2}\,\bar{p}n - \dots) + \varrho^-(\sqrt{2}\,\bar{n}p - \dots)] +$$
$$+ \gamma_\omega\,[\omega(\sqrt{3}\,\bar{p}p + \sqrt{3}\,\bar{n}n - \dots)]\,,$$

where our couplings read

$$f_\rho\left[\varrho^0\left(\frac{\bar{p}p}{2} - \frac{\bar{n}n}{2} + \bar{\Sigma}^-\Sigma^- - \dots\right) + \varrho^+\left(\frac{\bar{p}n}{\sqrt{2}} - \dots\right) + \varrho^-\left(\frac{\bar{n}p}{\sqrt{2}} - \dots\right)\right] +$$
$$+ f_\omega[\omega(\bar{p}p + \bar{n}n - \dots)]\,.$$

In other words

$$f_\rho = 2\gamma_\rho\,, \qquad f_\omega = \sqrt{3}\,\gamma_\omega\,.$$

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$$\frac{\Gamma(\omega \to \pi^0 + \gamma)}{\Gamma(\pi^0 \to 2\gamma)} \approx 2 \left(\frac{m_\omega^2 - m_\pi^2}{m_\omega m_\pi}\right)^3 \left(\frac{\gamma_\omega}{e}\right)^2 .$$

Note that their γ_ω is related to our f_ω by $f_\omega = \sqrt{3}\gamma_\omega$.
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Selected Topics on Strange Particles

Strange-Particle Decays.

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Introduction.

In these lectures we discuss the weak decays of strange particles. In particular, we discuss the present evidence concerning the $\Delta I = \frac{1}{2}$ rule in the nonleptonic decays, and the $\Delta I = \frac{1}{2}$ and $\Delta S = \Delta Q$ rules in the leptonic decays of strange particles.

We consider the hyperon decays

$$\Lambda \to \mathcal{N} + \pi ,$$

$$\Sigma \to \mathcal{N} + \pi ,$$

and

$$\Xi \to \Lambda + \pi ;$$

and the K-meson decays

$$K \to 2\pi ,$$

$$K \to 3\pi ,$$

and

$$K \to \pi + L + \nu ,$$

where L (lepton) stands for e or μ.

I will assume that the students are partly familiar with the material in GELL-MANN and ROSENFELD [1]. I will furthermore try to avoid repeating material given here at Varenna by Professor ROSENFELD.

The five lectures are as follows:

I. Simple introductory examples illustrating invariance (or lack of invariance) with respect to S, I, I_3, P, T, and C in weak decays.

II. Review of the definition and measurement of the decay parameters α, β, and γ in hyperon decay.

III. $\Delta I = \frac{1}{2}$ rule for the nonleptonic decays $K \to 2\pi$, $\Lambda \to \mathcal{N} + \pi$, $\Xi \to \Lambda + \pi$, and $\Sigma \to \mathcal{N} + \pi$.

IV. $K \to 3\pi$ and the $\Delta I = \frac{1}{2}$ rule.

V. The $\Delta I = \frac{1}{2}$ rule for leptonic K-decays.

There will be no attempt to give complete references, especially to « well-known » results.

1. – Introductory examples.

We begin by considering the quantities S, I, I_3, P, T, and C. All of these (except T) are conserved in the strong interactions but not in the weak interactions. T (time-reversal) invariance is usually assumed to hold in both the strong and weak reactions. (There is no experimental evidence to the contrary).

To illustrate nonconservation of (S, I, I_3) in weak (decay) interactions, consider $\Lambda \to p + \pi^-$. We have

$$(S = -1,\ I = 0,\ I_3 = 0)_\Lambda \to (0,\ \tfrac{1}{2} \text{ or } \tfrac{3}{2},\ -\tfrac{1}{2})_{p\pi^-}\,.$$

Thus none of S, I, or I_3 is conserved. Notice that $|\Delta I_3| = \frac{1}{2}$ but that $\Delta I = \frac{1}{2}$ or $\frac{3}{2}$.

We now turn our attention briefly to P, T, and C, using a minimum of formalism.

In considering the meaning of P (parity) conservation (or nonconservation) we will use mirrors. The space inversion $x, y, z \to -x, -y, -z$ is (for example) equivalent to the reflection $x, y, z \to -x, y, z$, followed by a rotation R of 180° about the x axis, $x, y, z \to x, -y, -z$. Since R is assumed to have no observable consequences (*i.e.*, the orientation of the system with respect to Andromeda, for instance, is assumed to be irrelevant), it is sufficient to consider only reflections in a mirror. The behavior of an axial vector (spin) or of a polar vector (linear momentum) upon reflection in a mirror is shown in Fig. 1.

To designate a spin we usually use $\Uparrow$ instead of . Sometimes we use if the spin is perpendicular to the paper.

We now consider, as an example of P conservation, the strong process $\pi^- + p \to \Lambda + K^0$. Suppose the target proton is unpolarized. Let the plane

of the paper be the production plane. Consider the three production configurations of Fig. 2, which differ only as to the orientation of the spin of the Λ.

In case (i) the Λ spin is perpendicular to the production plane. In (ii) the Λ spin lies in the production plane. In (iii) the Λ spin is opposite to that in case (ii). If we view process (i) in a mirror held parallel to the pro-

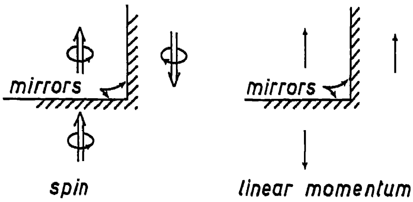

Fig. 1.

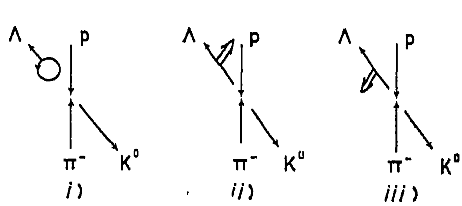

Fig. 2.

duction plane (plane of the paper) we « see » a process which we call (i′). A real process (*i.e.*, with no mirror) that looks like (i′) is also called (i′). Notice that, in our example, (i′) happens to be indistinguishable from (i). Similarly the process (ii′) looks like (iii), and (iii′) looks like (ii). The following statements are all equivalent:

(*a*) « The process is invariant under reflection ».

(*b*) « Parity is conserved in the process ».

(*c*) « The process p and its reflected process p' occur with equal probability ».

Thus if parity is conserved, processes (ii) and (ii′)—that is (ii) and (iii)—occur with equal probability; therefore the Λ polarization components in the production plane must average to zero. Similarly (i) and (i′) occur with equal probability. But these are the same process. Therefore a net polarization perpendicular to the production plane [as in (i)] is allowed (but not required).

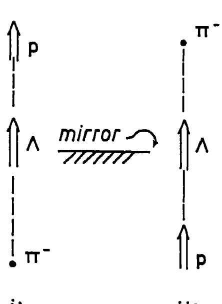

Fig. 3.

As a matter of fact, one finds experimentally that, in $\pi^-+p \rightarrow \Lambda+K^0$, the Λ's often have polarization of nearly 100% derpendicular to the production plane, but are never polarized in the production plane [2].

Next consider the weak process $\Lambda \rightarrow p+\pi^-$. Consider the decay configurations (i) and (ii) of Fig. 3. Here we have suppressed the arrows corresponding to the vectors representing linear momentum. We represent a spin-zero pion by a dot, and a spin-$\frac{1}{2}$ particle by $\Uparrow$, and think of the picture as a diagram in momentum space; the position x, y, z of the particle on the diagram gives

its momentum p_x, p_y, p_z. (We will use this convention several times more in this lecture.)

The decay (ii) is the reflection of the decay (i), for a mirror oriented as indicated. (Of course for *any* orientation of the mirror, (ii) is obtained by reflection of (i), followed by some rotation of the entire process. We have chosen the orientation of the mirror so as to preserve the Λ-spin direction, and thus avoid an additional irrelevant rotation.) If P were conserved in the decay, then process (i) and its reflection (ii) would occur with equal probability. Thus there would be no decay asymmetry for a polarized source of Λ's—as many protons would be emitted parallel and antiparallel to the Λ polarization. The large « up-down » decay asymmetries (with respect to the production plane) that are observed experimentally show that P is not conserved in $\Lambda \to p+\pi^-$, and also in most of the other hyperon decays. The large asymmetries often observed correspond to nearly maximum parity nonconservation in the decay, and to Λ's strongly polarized in the production process. The decay asymmetry determines $\alpha_\Lambda p_\Lambda$, where α_Λ is the decay parameter, and p_Λ is the Λ polarization. That is, $p_\Lambda=$(number of $+$ spins) minus (number of $-$ spins) divided by the total number of Λ's. These quantities will be discussed in more detail in the second lecture.

Next we consider the consequences of T (time-reversal) invariance for hyperon decay. We will use the same type of pictures as before: diagrams in three-dimensional momentum space, with double-shafted arrows to represent spins. The application of T to a physical state leads to a new state related to the original state through reversal of all linear momenta and spins. Furthermore an outgoing wave becomes an incoming wave. (Think of a playback of a movie film in reverse.) An incoming wave does not correspond to an observable « final » state of free particles—the incoming particles must interact before one obtains an outgoing wave that can correspond to final free particles. Furthermore, consider a process in which an initial state i, say a Λ, evolves into a final state f, say $p+\pi^-$. Then in the time-reversed picture the sense of evolution is reversed, and $p+\pi^-$ evolves into Λ. This process is of course unobservable by presently conceivable technique. However, in quantum mechanics, interchange of i and f in $\langle\psi_f|H|\psi_i\rangle\equiv m$ merely corresponds to complex conjugation, and thus does not affect $|m|^2$. Our pictures of course correspond to $|m|^2$. We therefore draw pictures in which the initial and final states are both present, with labels i and f, and include a step called « complex conjugation » (c.c.) which does not change the picture but interchanges i and f.

Consider an initial state that consists of a Λ at rest (and therefore at the origin in p_x, p_y, p_z space) with spin along $+z$. It evolves into a final state that is an outgoing proton with momentum along $+x$ and spin along $+y$. This is picture (i), Fig. 4. (We have not chosen this configuration by accident,

of course.) Now apply time reversal, T, to (i), to get (ii). Under T the Λ spin reverses, the decay-proton spin and linear momentum reverse, and the outgoing proton wave becomes an incoming wave. The sense of evolution is reversed so Λ is final, f, instead of initial, i. Next apply c.c., to inter-

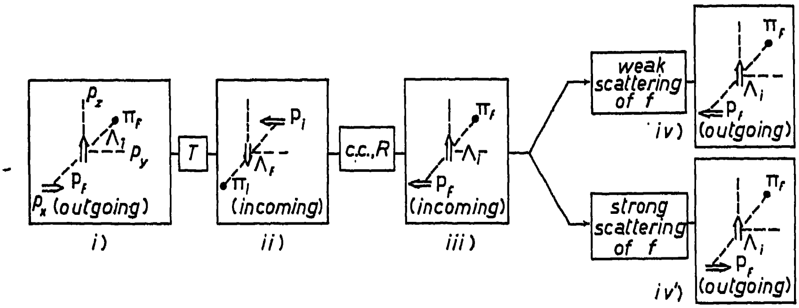

Fig. 4.

change i and f. Also perform a rotation R, of the entire process by 180° about the y axis, so that the Λ spin is again along $+z$. R and c.c. give (iii), and are assumed to have no observable consequences. Finally, let the incoming p-π^- wave scatter and become an outgoing wave, corresponding to an observable final state. Here, if we were using the formalism, we would obtain an s-matrix element factor. Instead we will merely give two extreme illustrations. One extreme is a « weak scattering » in the final (f) state, so weak in fact that « nothing happens », and the incoming wave becomes an outgoing wave with the same linear momenta and spins. This is picture (iv). In the other extreme example there is a strong spin-flip scattering and the proton spin is reversed without deflection of the linear momentum, to give picture (iv′).

The following statements are equivalent:

(a) « Time-reversal invariance holds in Λ-decay ».

(b) « The decay corresponding to (iv) (for weak final-state scattering) or (iv′) (for strong scattering) occurs with the same probability as that corresponding to (i) ».

From the pictures we see that if the π^--p scattering is weak and if T invariance holds, then the Λ polarization corresponding to (i) is exactly cancelled by the equally probable decay (iv), so that there is zero net polarization of the type (i). On the other hand, if the π-p scattering is strong, as in (iv′), a net polarization *can* be obtained. However, if the π-p scattering phase shifts are known (at the decay momentum) the effect of the scattering can be exactly taken into account, and one can still test T invariance. We need not write down the formulas, which are well known [1].

The decay parameter corresponding to the Λ polarization shown in (i) is called β, with $-1 \leqslant \beta \leqslant 1$. We have $\beta = 0$ if T invariance holds and the f scattering is weak. This parameter will be discussed in the second lecture. It is clear from the discussion of Fig. 4 that one needs polarized Λ's in order to measure β_Λ.

There are two measurements of β for hyperon decays so far. CRONIN and OVERSETH [2] find for $\Lambda \to p + \pi^-$ a value $\beta_\Lambda = 0.19 \pm 0.19$. This value is consistent with T invariance and the known π-p phase shifts. Another result is that of the U.C.-Berkeley-U.C.L.A. experiment. The experimenters find [2] for $\Xi^- \to \Lambda + \pi^-$ the preliminary result $\beta_{\Xi^-} = -0.68 \pm 0.27$. The experimental uncertainty is of course large, but the large value of β, if substantiated, probably indicates a strong Λ-π interaction. This should not be surprising, since the Ξ mass is near that of the Y_1^* resonance [2].

Lastly we consider C invariance. Again we use the example of Λ-decay. Charge conjugation C applied to the process $\Lambda \to p + \pi^-$ gives the process $\overline{\Lambda} \to \overline{p} + \pi^+$. If C invariance holds then these two decays should occur with equal amplitudes for the same configuration of momentum and spins.

Insufficient experimental information is available for $\overline{\Lambda}$-decay. However, CPT invariance allows us to substitute PT for C. We can then consider the effect of PT invariance on Λ-decay, since PT does not change Λ into $\overline{\Lambda}$. Our pictures will be similar to those used previously. We will prove that PT invariance would, in the absence of final-state interactions, give zero for the « up-down » decay parameter α_Λ. We start with configuration (i), of Fig. 5,

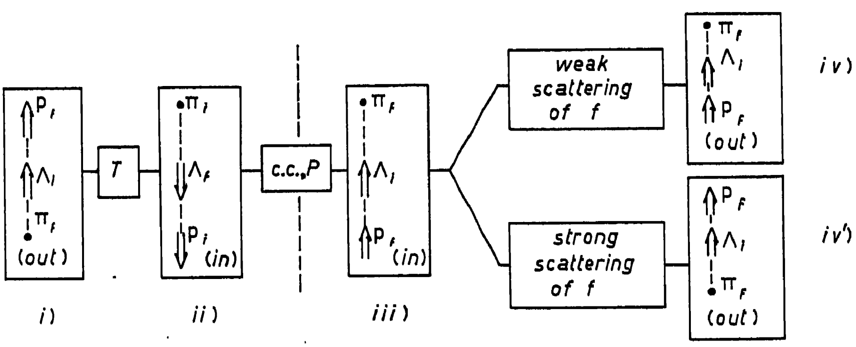

Fig. 5.

which implies a source of polarized Λ's. Application of T gives (ii), with reversed linear momenta and spins, with incoming p-π^-, and with i (initial) and f (final) reversed. Complex conjugation (c.c.) and reflection P in a *vertical* mirror (chosen to eliminate the need for a further rotation to orient the Λ spin) give (iii). The incoming f (final) wave scatters and becomes an outgoing f state.

A weak f scattering (« nothing happens ») is shown in (iv). A strong scattering, in which the π^- and p reverse their linear momenta (180° scattering) is shown in (iv′). The following statements are equivalent:

(*a*) « PT-invariance is satisfied in $\Lambda \to p + \pi^-$ ».

(*b*) « Decay configuration (iv), for weak final state scattering [or (iv′) for a particular strong scattering] has the same probability as configuration (i) ».

We see that any up-down decay asymmetry implied by (i) is completely cancelled by (iv), for weak scattering. Thus PT invariance (*i.e.*, C invariance) guarantees $\alpha_\Lambda = 0$, in the absence of π-p final-state interactions. At the momentum of Λ-decay (100 MeV/c) the π-p scattering phase shifts are very small [1], so that « weak scattering » holds. Experimentally the decay parameter α_Λ is large. We conclude that $\Lambda \to p + \pi^-$ does *not* satisfy PT invariance. This was first pointed out by R. Gatto [3].

2. – Decay parameters.

In this lecture we consider in detail the hyperon decay parameters α, β, and γ, and how they are measured. In every case we have a parent particle of spin $\frac{1}{2}$ decaying into a daughter of spin $\frac{1}{2}$ plus a pion (spin zero). The decays of interest are $\Lambda \to \mathcal{N} + \pi$, $\Sigma \to \mathcal{N} + \pi$, and $\Xi \to \Lambda + \pi$. Instead of speaking of « parent » and « daughter », we will for convenience take $\Lambda \to p + \pi^-$ as a model, most of the time.

Since the Λ has $J = \frac{1}{2}$, the p-π^- system can only be in the state $S_{\frac{1}{2}}$ or $P_{\frac{1}{2}}$. Call S and P the corresponding amplitudes. Let ψ_+ describe the $\pi^- + p$ spin and space configuration for $(J, J_z) = (\frac{1}{2}, +\frac{1}{2})$, and ψ_- that for $(J, J_z) = (\frac{1}{2}, -\frac{1}{2})$. We can use the Clebsch-Gordan coefficients of Table I to construct $\psi_\pm$. We will represent the proton's spin state by

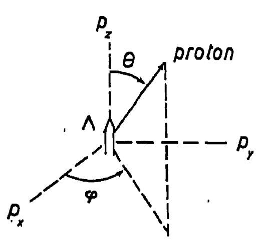

Fig. 6.

$$\Uparrow \equiv \begin{pmatrix}1\\0\end{pmatrix} \equiv \left(\frac{1}{2}, +\frac{1}{2}\right) \qquad \text{and} \qquad \Downarrow \equiv \begin{pmatrix}0\\1\end{pmatrix} \equiv \left(\frac{1}{2}, -\frac{1}{2}\right).$$

The orbital angular momentum state of $\pi^- + p$ is given by $Y_l^m(\theta, \varphi)$, where θ and φ are the polar and azimuthal angles of emission of the *proton* with respect to the z axis (see Fig. 6).

The appropriate spherical harmonics are Y_0^0 for the $S_{\frac{1}{2}}$ state of π-p, and Y_1^1, Y_1^0, and Y_1^{-1} for $P_{\frac{1}{2}}$. We use

$$Y_0^0 = 1\,, \tag{1a}$$

$$Y_1^{+1} = -\sqrt{\tfrac{3}{2}}\,\sin\theta \exp[i\varphi]\,, \tag{1b}$$

(1c) $$Y_1^0 = \sqrt{3}\cos\theta\,,$$

(1d) $$Y_1^{-1} = \sqrt{\tfrac{3}{2}}\sin\theta\,\exp[-i\varphi]\,.$$

That part of ψ_+ that corresponds to $S_{\frac{1}{2}}$ can be written down without using the table. It is just $SY_0^0\Uparrow = S\cdot 1\cdot\Uparrow$. To obtain the $P_{\frac{1}{2}}$ part, we use Table I, which gives the composition of 1 (P-wave) $\times\frac{1}{2}$ (spin).

Looking in the column $(\frac{1}{2}, +\frac{1}{2})$ [because we want ψ_+] we find the decomposition

$$(\tfrac{1}{2}, +\tfrac{1}{2}) = \sqrt{\tfrac{2}{3}}\,(1, +1)(\tfrac{1}{2}, -\tfrac{1}{2}) - \sqrt{\tfrac{1}{3}}\,(1, 0)(\tfrac{1}{2}, +\tfrac{1}{2})\,.$$

Putting the $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ parts together, we have

$$\psi_+ = SY_0^0\Uparrow + P[\sqrt{\tfrac{2}{3}}\,Y_1^1\Downarrow - \sqrt{\tfrac{1}{3}}\,Y_1^0\Uparrow]\,.$$

Similarly, using the table, we get

$$\psi_- = SY_0^0\Downarrow + P[\sqrt{\tfrac{1}{3}}\,Y_1^0\Downarrow - \sqrt{\tfrac{2}{3}}\,Y_1^{-1}\Uparrow]\,.$$

Using the spherical harmonics of eq. (1), we have

(2a) $$\begin{aligned}\psi_+ &= S\Uparrow + P\,[-\sin\theta\,\exp[i\varphi]\Downarrow - \cos\theta\Uparrow] = \\ &= (S - P\cos\theta)\Uparrow - P\sin\theta\exp[i\varphi]\Downarrow\,,\end{aligned}$$

(2b) $$\begin{aligned}\psi_- &= S\Downarrow + P[\cos\theta\Downarrow - \sin\theta\exp[-i\varphi]\Uparrow] = \\ &= (S + P\cos\theta)\Downarrow - P\sin\theta\exp[-i\varphi]\Uparrow\,.\end{aligned}$$

The decay angular distribution for ψ_+ is given by $|\psi_+|^2 = \psi_+^*\psi_+$. We use the orthogonality of the spin functions, namely

$$\Uparrow^*\Uparrow \equiv (1, 0)\begin{pmatrix}1\\0\end{pmatrix} = 1\,, \qquad \Downarrow^*\Downarrow \equiv (0, 1)\begin{pmatrix}0\\1\end{pmatrix} = 1\,, \qquad \Uparrow^*\Downarrow = \Downarrow^*\Uparrow = 0;$$

so that

$$|\psi_+|^2 = |S - P\cos\theta|^2 + |-P\sin\theta|^2 = |S|^2 + |P|^2 - 2\,\mathrm{Re}\,S^*P\cos\theta\,.$$

$$|\psi_-|^2 = |S + P\cos\theta|^2 + |-P\sin\theta|^2 = |S|^2 + |P|^2 + 2\,\mathrm{Re}\,S^*P\cos\theta\,.$$

It is customary to define

(3) $$\alpha \equiv \frac{2\,\mathrm{Re}\,S^*P}{|S|^2 + |P|^2}\,,$$

(4) $$\beta \equiv \frac{2\,\mathrm{Im}\,S^*P}{|S|^2 + |P|^2}\,,$$

(5) $$\gamma \equiv \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2}\,.$$

(note that $\alpha^2+\beta^2+\gamma^2=1$.) Then, from the above,

$$|\psi_+|^2=[|S|^2+|P|^2][1-\alpha\cos\theta]\,, \tag{6}$$

$$|\psi_-|^2=[|S|^2+|P|^2][1+\alpha\cos\theta]\,. \tag{7}$$

Now suppose a collection of Λ's is partially polarized, with a fraction f_+ in the state ψ_+, and a fraction f_- in the state ψ_-, with $f_++f_-=1$. Then the weighted decay angular distribution is given by

$$|\psi|^2=f_+|\psi_+|^2+f_-|\psi_-|^2=[|S|^2+|P|^2]\{(f_++f_-)-\alpha(f_+-f_-)\cos\theta\}\,.$$

The polarization p of the collection of Λ's is defined to be

$$p_\Lambda=(f_+-f_-)/(f_++f_-)\,, \tag{8}$$

with $-1<p_\Lambda<+1$, so that

$$|\psi|^2=[|S|^2+|P|^2]\{1-\alpha p\cos\theta\}\,. \tag{9}$$

The decay distribution for N-decays is thus given by

$$\mathrm{d}N=N[1-\alpha p\cos\theta]\,\frac{\mathrm{d}\,(\cos\theta)}{2}\,. \tag{10}$$

Notice that

$$\int_{-1}^{1}\mathrm{d}N=N\,,\qquad\text{and}\qquad\int_{-1}^{1}\cos\theta\cdot\mathrm{d}N=\frac{-N\alpha p}{3}\,,$$

so that

$$-\alpha p=\frac{3}{N}\int_{-1}^{1}\cos\theta\,\mathrm{d}N\Rightarrow\frac{3\sum\limits_i\cos\theta_i}{N}\,, \tag{11}$$

where the sum is over all the decays, and where the arrow means « corresponds, for large numbers, to ». Equation (11) is often used by experimenters. An equivalent formula is $-\alpha p=2\,(\text{up}-\text{down})/(\text{up}+\text{down})$.

We see that measurement of « up — down asymmetry » does not give α, but gives αp. Since the sign and magnitude of p are generally unknown, a measurement of αp gives a lower limit to $|\alpha|$. That is, $|\alpha|=|\alpha p|/|p|\geqslant|\alpha p|$.

In order to measure α directly one can measure the longitudinal polarization of the decay protons from an *unpolarized* collection of Λ's. This is

easily seen as follows. First, consider only proton emission along the $\pm z$ axis. From eqs. (6) and (7), with $\cos\theta = \pm 1$ we obtain the relative probabilities shown in Fig. 7. Notice that because of angular momentum conservation the proton spin direction must be the same as that of the Λ, for emission along the z axis (quantization axis), because the π-p orbital angular momentum can have no component along the proton's linear momentum, and therefore cannot flip the baryon spin. The definition of the longitudinal polarization of the proton, along its velocity $\hat{v}$ with respect to the Λ rest frame, is given by an expression analogous to eq. (8). Using Fig. 7, we get, for the longitudinal polarization,

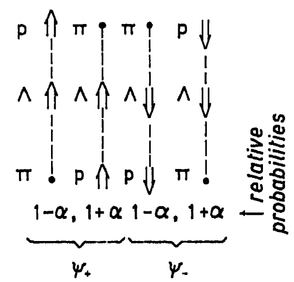

Fig. 7.

$$p(\text{long.}) = \frac{N_+ - N_-}{N_+ + N_+} = \frac{(1-\alpha) + (1-\alpha) - (1+\alpha) - (1+\alpha)}{(1-\alpha) + (1-\alpha) + (1+\alpha) + (1+\alpha)} = -\alpha_\Lambda\,,$$

where $N_\pm$ refer to $\pm\hat{v}$, and where we have used equal weights for ψ_+ and ψ_-. Since the Λ collection is unpolarized, all quantization directions are equivalent, so that we can always choose the z axis to be along the direction of emission of the proton and be assured that ψ_+ and ψ_- have equal populations. The above result therefore is general.

One still has the problem of *measuring* this longitudinal polarization of the daughter. In the case of $\Xi \to \Lambda + \pi$ one can measure the decay asymmetry of the daughter Λ with respect to the direction of $\boldsymbol{v}_\Lambda - \boldsymbol{v}_\Xi$, and thus determine $\alpha_\Lambda p_\Lambda$ (longitudinal), using eq. (11). But p_Λ (long.) $= -\alpha_\Xi$. Thus one measures $\alpha_\Lambda \alpha_\Xi$ [2].

In the case of $\Lambda \to p + \pi^-$ one can scatter the decay proton for instance from carbon to look for a scattering asymmetry, using a spark chamber [2]. Notice that if the Λ (unpolarized collection) decays at rest in the laboratory system, then the proton has a purely longitudinal polarization in the laboratory system (where the carbon scatterer is at rest). When this proton scatters from carbon (spin zero) there cannot be any «left-right» scattering asymmetry, merely from the symmetry of the initial p-carbon configuration. There also cannot by any front-back (0° *vs.* 180°) scattering asymmetry that depends on the proton's longitudinal polarization. This follows from parity conservation in the strong p-carbon reaction. We can see this with our mirror. Suppose an incoming «spin-head-on» (as opposed to «tail-on») proton likes to scatter «strongly» (*i.e.*, through 180°) from carbon. If the tail-on collision does *not* like to occur, we have a means of determining the polarization. However, the image of a head-on collision in a mirror held parallel to the proton velocity is a tail-on collision. By P conservation the two processes have the same

probability. Thus head-on and tail-on protons *both* scatter strongly (or weakly) and we cannot distinguish the two polarizations (since parity *is* conserved in the strong reactions).

One gets around this by using fast Λ's that decay in flight. Then the decay protons, which have a polarization along $\boldsymbol{v}_p - \boldsymbol{v}_\Lambda$, can have a component $\perp$ to $\boldsymbol{v}_p - \boldsymbol{v}_C$. It is then possible to get azimuthal asymmetry in the scattering. This is illustrated in Fig. 8, which is our usual diagram in velocity space. We choose the carbon at rest. The Λ is shown without an arrow, since it is unpolarized. If $\boldsymbol{v}_p - \boldsymbol{v}_\Lambda$ is along $\pm\hat{x}$ or $\pm\hat{z}$ we see that the proton has a transverse polarization of approximately $-\alpha_\Lambda \cos\theta$, where θ is as shown. If $\boldsymbol{v}_p - \boldsymbol{v}_\Lambda$ is along $\boldsymbol{v}_\Lambda - \boldsymbol{v}_C$, *i.e.*, along $\pm\hat{y}$, there is no transverse polarization. Thus about $\frac{4}{6}$ of the decays are useful.

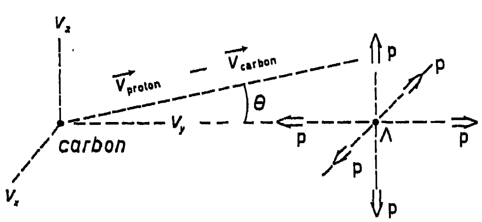

Fig. 8.

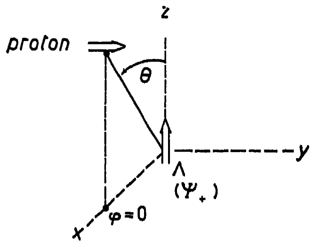

Fig. 9.

We turn now to the problem of measuring the decay parameter β. It was already mentioned in the first lecture that β is a measure of T invariance, and also it was shown that the proton polarization shown in Fig. 4 (i) (for a polarized Λ) must average to zero if T invariance holds and the scattering is weak (as it is in Λ-decay). We will calculate the slightly more general proton polarization component shown in Fig. 9. We choose the Λ state ψ_+, *i.e.*, 100% polarized Λ's along $+z$. (Our final answer can then be multiplied by p_Λ if $p_\Lambda \neq +1$). We choose the decay configuration with $\varphi=0$, as shown in Fig. 9. This simplifies the formulas and corresponds to an unessential rotation of the axes. We wish to calculate $\langle\sigma_y\rangle$. We have, for the state ψ_+,

$$\langle\sigma_y\rangle = \frac{\psi_+^* \sigma_y \psi_+}{\psi_+^* \psi_+}\,. \tag{12}$$

The denominator is given by eq. (6). To calculate the numerator we use

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \Uparrow \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \Downarrow \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad \sigma_y \Uparrow = i \Downarrow\,, \quad \sigma_y \Downarrow = -i \Uparrow;$$

$$\psi_+ = (S - P\cos\theta)\Uparrow - P\sin\theta \Downarrow\,, \quad \sigma_y \psi_+ = i[(S - P\cos\theta)\Downarrow + P\sin\theta \Uparrow];$$

$$\Uparrow^*\Uparrow = \Downarrow^*\Uparrow = 1\,, \quad \Uparrow^*\Downarrow = \Downarrow^*\Uparrow = 0;$$

$$\begin{aligned}\psi_+^*\,\sigma_v\,\psi_+ &= [(S-P\cos\theta)^*\Uparrow^* - P^*\sin\theta\Downarrow^*]i[(S-P\cos\theta)\Downarrow + P\sin\theta\Uparrow] = \\ &= i\,\{(S-P\cos\theta)^*P\sin\theta - P^*\sin\theta(S-P\cos\theta)\} = \\ &= i\,\{2i\,\mathrm{Im}\,S^*P\sin\theta\} \equiv -\,(|S|^2+|P|^2)\beta\sin\theta\,.\end{aligned}$$

Finally then, for $\varphi=0$, and $p_\Lambda=1$,

$$\langle\sigma_v\rangle = \frac{-\beta\sin\theta}{1-\alpha\cos\theta}\,. \tag{13}$$

Clearly, for $p_\Lambda\neq1$ we have, for $\varphi=0$,

$$\langle\sigma_v\rangle = \frac{-\beta p_\Lambda\sin\theta}{1-\alpha p_\Lambda\cos\theta}\,. \tag{14}$$

It is clear that our choice of $\varphi=0$ was unessential, and eq. (44) gives, more generally, the azimuthal or $\hat{\varphi}$ component of polarization.

The case shown in Fig. 4 (i) has $\varphi=0$, $\theta=90°$, $p_\Lambda=1$, so that $\langle\sigma_v\rangle=-\beta$. Since we had previously concluded that this polarization must vanish if T-invariance holds, we see that β is a measure of lack of T-invariance (for weak final-state interactions). If T-invariance holds, S and P are « relatively real », *i.e.*, S/P is real (for weak final-state interactions).

The problem of measurement of $\langle\sigma_v\rangle$ (of the proton in Fig. 9) is illustrated in Fig. 10. We see that as far as transverse proton polarization is concerned, we could use Λ's at rest in the laboratory system and have four out of six « useful directions ». However, we need polarized Λ's, and polarized Λ's are not produced at rest; furthermore, the proton would then have only a few MeV, and would not penetrate a scatterer of reasonable thickness. For fast Λ's we see that only $\frac{2}{6}$ of the decays are useful—those with $\boldsymbol{v}_p-\boldsymbol{v}_\Lambda$ along $\pm\hat{y}$ in Fig. 10. Of course, in the decay $\Xi^-\to\Lambda+\pi^-$, « all four » directions of Λ emission in the production plane are useful.

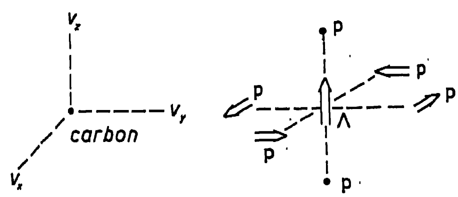

Fig. 10.

One may ask: How can one in a single experiment measure α, using an unpolarized sample of parent hyperons, and β, using polarized parents? The answer is that, since the parent polarization must be perpendicular to the production plane, one obtains « effectively » unpolarized parents if one throws away information as to the orientation of the production plane. Crucial to

this argument is the fact that, for a spin-$\frac{1}{2}$ parent, the decay distribution, eq. (9), is *linear* in $p_\Lambda \cos\theta$, and so the term containing p_Λ averages to zero when we average over the distribution.

We turn now to the measurement of γ. From eq. (3) we see that α is unchanged by the interchange of S and P. Thus if we know α we know the relative amounts of S and P, but don't know «which is which». That is, $|S|/|P|=\frac{10}{1}$ and $|S|/|P|=\frac{1}{10}$ are indistinguishable. From eq. (5) we see that it is the *sign* of γ that tells us the correct ratio. (The magnitude of γ is already known once α and β are known, since $\gamma^2=1-\alpha^2-\beta^2$).

To determine which is which (S or P) we consider first the limiting case of pure S-wave decay for a 100% polarized Λ, ψ_+. For pure S-wave there is no orbital angular momentum to flip the spin, and the proton polarization is the same as that of the Λ for all directions of emission. This also follows from eq. (1) if we set $P=0$ to get $\psi_+=S\Uparrow$.

We next calculate $\langle\sigma_x\rangle$ and $\langle\sigma_z\rangle$ for the general case. We still set $\varphi=0$ for convenience. (Since we have already calculated $\langle\sigma_y\rangle$ in eq. (13), we are at present interested only in $\langle\sigma_x\rangle$ and $\langle\sigma_z\rangle$.) We use

$$\sigma_x=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \sigma_x\Uparrow=\Downarrow,\quad \sigma_x\Downarrow=\Uparrow;\quad \sigma_x\psi_+=\sigma_x[(S-P\cos\theta)\Uparrow-P\sin\theta\Downarrow]=$$
$$=[(S-P\cos\theta)\Downarrow-P\sin\theta\Uparrow].$$

$$(15)\qquad \begin{aligned}\psi_+^*\sigma_x\psi_+&=[(S-P\cos\theta)^*\Uparrow^*-P^*\sin\theta\Downarrow^*][(S-P\cos\theta)\Downarrow-P\sin\theta\Uparrow]=\\ &=-(S-P\cos\theta)^*P\sin\theta-P^*\sin\theta(S-P\cos\theta)=\\ &=2|P|^2\sin\theta\cos\theta-2\operatorname{Re}S^*P\sin\theta=\\ &=|P|^2\sin2\theta-(|S|^2+|P|^2)\alpha\sin\theta.\end{aligned}$$

Similarly,

$$\sigma_z=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\qquad \sigma_z\Uparrow=\Uparrow,\qquad \sigma_z\Downarrow=-\Downarrow;$$

$$\sigma_z\psi_+=\sigma_z[(S-P\cos\theta)\Uparrow-P\sin\theta\Downarrow]=[(S-P\cos\theta)\Uparrow+P\sin\theta\Downarrow],$$

$$(16)\qquad \begin{aligned}\psi_+^*\sigma_z\psi_+&=[(S-P\cos\theta)^*\Uparrow^*-P^*\sin\theta\Downarrow^*][(S-P\cos\theta)\Uparrow+P\sin\theta\Downarrow]=\\ &=|S-P\cos\theta|^2-|P|^2\sin^2\theta=\\ &=|S|^2+|P|^2(\cos^2\theta-\sin^2\theta)-2\operatorname{Re}S^*P\cos\theta=\\ &=|S|^2+|P|^2\cos2\theta-(|S|^2+|P|^2)\alpha\cos\theta.\end{aligned}$$

We can combine (16) and (15) into a vector $\boldsymbol{\sigma}$ in the xz plane. (We are not concerned with σ_y at the moment; we are *not* assuming $\langle\sigma_y\rangle=0$.) We find

$$(17)\qquad \begin{aligned}\psi_+^*\boldsymbol{\sigma}\psi_+&=\psi_+^*[\sigma_z\hat{z}+\sigma_x\hat{x}]\psi_+=\\ &=|S|^2\hat{z}+|P|^2[(\cos2\theta)\hat{z}+(\sin2\theta)\hat{x}]-(|S|^2+|P|^2)\alpha[\cos\theta\hat{z}+\sin\theta\hat{x}].\end{aligned}$$

But $\cos\theta\hat{z}+\sin\theta\hat{x}\equiv\hat{q}(\theta)$, where $\hat{q}$ is the unit vector along the proton momentum (in the Λ rest frame). And $(\cos 2\theta)\hat{z}+(\sin 2\theta)\hat{x}\equiv\hat{n}(2\theta)$, where $\hat{n}(2\theta)$ is a unit vector in the $\hat{z}\hat{q}$ plane, making an angle 2θ with $\hat{z}$.

Finally we obtain, from these definitions and eqs. (16) and (6),

$$\langle\boldsymbol{\sigma}\rangle=\frac{|S|^2\hat{z}+|P|^2\hat{n}(2\theta)-(|S|^2+|P|^2)\alpha\hat{q}(\theta)}{(|S|^2+|P|^2)[1-\alpha\cos\theta]}\,. \tag{18}$$

In addition there is a y component given by eq. (13) or by (14). If we do not have $p=+1$ (pure ψ_+ state) we obtain, by a weighted average over ψ_+ and ψ_-, the final general result

$$\langle\boldsymbol{\sigma}\rangle=\frac{p[|S|^2\hat{z}+|P|^2\hat{n}(2\theta)-2\,\mathrm{Im}\,S^*P\sin\theta\hat{\varphi}]-2\,\mathrm{Re}\,S^*P\hat{q}(\theta)}{(|S|^2+|P|^2)[1-\alpha p\cos\theta]}\,. \tag{19}$$

For a pure P-wave, $|S|=0$, $\alpha=0$, $\beta=0$ and we obtain, for the proton polarization

$$\langle\boldsymbol{\sigma}\rangle_{P\text{-wave}}=p\hat{n}(2\theta)\,. \tag{20}$$

Then the proton spin lies in the plane of the emission of the proton (and the Λ polarization z axis). Proton emission at angle θ gives proton spin at 2θ.

For pure S-wave we have

$$\langle\boldsymbol{\sigma}\rangle_{S\text{-wave}}=p\hat{z}\,. \tag{21}$$

The two extremes are illustrated in Fig. 11.

We can now put the proton polarization [eqs. (19) and (14)] in a simpler form if we go to the unit vectors $\hat{q}$, $\hat{\theta}$, and $\hat{\varphi}$ corresponding to spherical coordinates. See Fig. 12. Here $\hat{q}(\theta,\varphi)$ is a unit vector in the direction of emis-

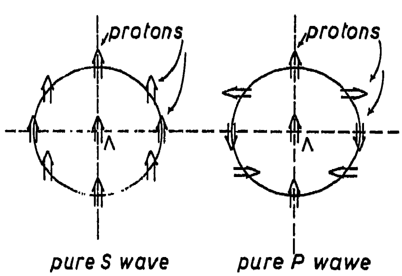

Fig. 11.

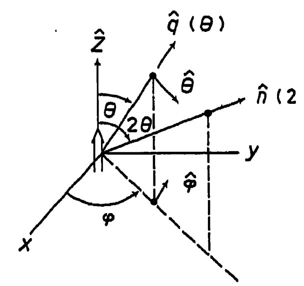

Fig. 12.

sion of the proton, relative to Λ rest frame, and $\hat{\theta}$ and $\hat{\varphi}$ are unit vectors corresponding to increase in θ and φ. By inspection of Fig. 12, we see

$$\hat{z}=\hat{q}\cos\theta-\hat{\theta}\sin\theta\,,$$

$$\hat{n}(2\theta)=\hat{q}\cos\theta+\hat{\theta}\sin\theta\,.$$

Therefore for one part of the numerator of (19), we have

$$|S|^2\hat{z}+|P|^2\hat{n}(2\theta)=(|S|^2+|P|^2)\,\hat{q}\cos\theta-(|S|^2-|P|^2)\,\hat{\theta}\sin\theta=$$
$$=(|S|^2+|P|^2)[\hat{q}\cos\theta-\gamma\hat{\theta}\sin\theta]\,.$$

Accordingly from eq. (19) we find the general expression for the daughter polarization $\langle\boldsymbol{\sigma}\rangle$ in terms of the daughter emission direction $\hat{q}$, the parent polarization $p\hat{z}$, and the decay parameters α, β, and γ:

(22) $$\langle\boldsymbol{\sigma}\rangle=\frac{\hat{q}(p\cos\theta-\alpha)-p\sin\theta(\gamma\hat{\theta}+\beta\hat{\varphi})}{(1-\alpha p\cos\theta)}\,.$$

As checks, we see that for pure S-wave we have $\alpha=\beta=0$, $\gamma=+1$, and thus

$$\langle\boldsymbol{\sigma}\rangle=p[\hat{q}\cos\theta-\hat{\theta}\sin\theta]=p\hat{z}\,.$$

For pure P-wave we have $\alpha=\beta=0$, $y=-1$, and find

$$\langle\boldsymbol{\sigma}\rangle=p[\hat{q}\cos\theta+\hat{\theta}\sin\theta]=p\hat{n}(2\theta)\,.$$

The longitudinal polarization of the daughter along its direction of emission is given by

(23) $$\langle\boldsymbol{\sigma}\rangle\cdot\hat{q}=\frac{p\cos\theta-\alpha}{1-\alpha p\cos\theta}\,,$$

which reduces to $-\alpha$ if the Λ polarization, p is zero.

The expression (22) has the advantage that the usually used parameters α, β, and γ appear explicitly, and the unit vectors are orthogonal. Expression (19) has perhaps the advantage that it is easier to see the separate effects of S and P-wave.

From eq. (22), by squaring and adding the three orthogonal components (and by using $\alpha^2+\beta^2+\gamma^2=1$), we find the square of the magnitude of the proton's polarization vector,

(24) $$(\langle\boldsymbol{\sigma}\rangle)^2=1-\frac{(1-p^2)(1-\alpha^2)}{(1-\alpha p\cos\theta)^2}\,.$$

This means that if the Λ is 100% polarized ($p=\pm1$), then for any given direction of emission of the proton, the proton polarization is 100%, in *some* direction [given by (22)]. On the other hand, if the Λ polarization is $p\neq1$, the proton's polarization is not p but is given by eq. (24).

3. – $\Delta T = \frac{1}{2}$ rule for nonleptonic decay.

In this lecture we review the well-known [1] evidence that has led to the hypothesis of the $\Delta I = \frac{1}{2}$ rule, for nonleptonic decays. We consider the decays $K \to 2\pi$, $\Lambda \to \mathcal{N} + \pi$, $\Xi \to \Lambda + \pi$, and $\Sigma \to \mathcal{N} + \pi$, and will make calculations illustrating spurion technique [1].

3·1. *The decay* $K \to 2\pi$.

We consider the decays

$$K^+ \to \pi^+ + \pi^0 \,, \tag{25}$$

$$K^0 \to \pi^+ + \pi^- \,, \tag{26}$$

and

$$K^0 \to \pi^0 + \pi^0 \,, \tag{27}$$

Two pions can have total $I = 0$, 1, or 2. Let us use our Clebsch-Gordan Table II to construct the charge states for 2π with $I = 0$, 1, or 2. Reading from the table we get, using the notation $\psi(I, I_3)$,

$$\psi(0,0) = \sqrt{\tfrac{1}{3}}(1, +1)(1, -1) - \sqrt{\tfrac{1}{3}}(1, 0)(1, 0) + \sqrt{\tfrac{1}{3}}(1, -1)(1, 1) \,,$$

or

$$\psi(0, 0) = \sqrt{\tfrac{1}{3}}\{\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+\} \,. \tag{28}$$

Similarly we read, by inspection of the table,

$$\psi(1, 0) = \sqrt{\tfrac{1}{2}}\{\pi^+\pi^- - \pi^-\pi^+\} \,, \tag{29}$$

$$\psi(1, +1) = \sqrt{\tfrac{1}{2}}\{\pi^+\pi^0 - \pi^0\pi^+\} \,, \tag{30}$$

$$\psi(2, 0) = \sqrt{\tfrac{1}{6}}\{\pi^+\pi^- + 2\pi^0\pi^0 + \pi^-\pi^+\} \,, \tag{31}$$

$$\psi(2, +1) = \sqrt{\tfrac{1}{2}}\{\pi^+\pi^0 + \pi^0\pi^+\} \,. \tag{32}$$

We do not need any other components in considering reactions (25), (26), and (27).

In this notation $\pi^+\pi^-$ means that pion 1 is π^+, 2 is π^-. The expressions $\pi^+\pi^-$ and $\pi^-\pi^+$ do not mean the same thing, since 1 and 2 may be distinguishable by position or by energy.

By inspection of eqs. (28) through (32) we see that, upon interchanging the *charge* of 1 and 2, we have $\psi \to (-1)^I\psi$, for $I = 0$, 1, or 2.

Since the two pions are identical bosons, the total exchange of space (x),

spin (σ), and charge (Q) must leave ψ unchanged. The pions have no spin, so we have

$$(x)(Q) = +1\,,$$

i.e.,

$$(-1)^l(-1)^I = +1\,, \tag{33}$$

where l is the relative angular momentum of the two pions. For K-decay, the total spin $J=0$, so $l=0$, so $(-1)^I=+1$, so $I=0$ or 2 only. $I=0$ is excluded for K^+ by charge conservation, so K^+ can go to 2π only in the state $\psi(2, +1)$. However, K^0 can go either to $\psi(2, 0)$ or $\psi(0, 0)$.

In discussing K^0 we must distinguish between K_1^0 and K_2^0, which are even and odd, respectively under CP. For $\pi^0\pi^0$ or $\pi^+\pi^-$, CP has the same effect as interchanging the charge (when present) and space co-ordinates of the two particles. Therefore $CP=+1$ (identical bosons). Therefore K_1^0 can and K_2^0 cannot decay into $\pi^0\pi^0$ and $\pi^+\pi^-$.

Since K^+ and K^0 have $I=\frac{1}{2}$, the change of I in the decay of $K^+\to 2\pi$ is $|\Delta \boldsymbol{I}|\equiv\Delta I=2\pm\frac{1}{2}=\frac{5}{2}$ or $\frac{3}{2}$. In $K_1^0\to 2\pi$ we have $\Delta I=2\pm\frac{1}{2}=\frac{5}{2}$ or $\frac{3}{2}$; or $0+\frac{1}{2}=\frac{1}{2}$. Thus $\Delta I=\frac{1}{2}$ is available for $K_1^0\to 2\pi$, but not for $K^+\to 2\pi$. The rate $R(K^+\to\pi^++\pi^0)$ is only about 1/600 of $R(K_1^0\to 2\pi)$ [1]. The most natural explanation is that there is a selection rule that nearly forbids decays with $\Delta I=\frac{3}{2}$ or $\frac{5}{2}$ but allows those with $\Delta I=\frac{1}{2}$. This proposed selection rule is called the $\Delta I=\frac{1}{2}$ rule. If the $\Delta I=\frac{1}{2}$ rule were strictly obeyed, $K^+\to\pi^++\pi^0$ would be forbidden. Furthermore $K^0\to 2\pi$ would go only to $\psi(0, 0)$. By eq. (28) we see that in that case the branching ratio

$$B_1\equiv\frac{R(K_1^0\to\pi^0\pi^0)}{R(K_1^0\to\pi^0\pi^0)+R(K_1^0\to\pi^++\pi^-)}\equiv\frac{R(0\,0)}{R(0\,0)+R(+\,-)}\,, \tag{34}$$

would have the value

$$B_1=\frac{|-1|^2}{|1|^2+|-1|^2+|1|^2}=\frac{1}{3}\,.$$

This is close to what is in fact observed [1, 2].

Since $K^+\to\pi^++\pi^0$ does after all exist, we next calculate the effect on the prediction of $B_1=\frac{1}{3}$, using spurion technique [1].

The spurion $s(\Delta I, \Delta I_3)$ is introduced in order to keep track of the change of I in the decay. One can think of the spurion as carrying off $\Delta \boldsymbol{I}$, so that one now has $\boldsymbol{I}$ conservation in the decay. We illustrate this by considering the K-decay into 2π with $I=2$. We assume that $\Delta I=\frac{3}{2}$ occurs, but that there is no $\Delta I=\frac{5}{2}$. We use eqs. (31) and (32) to describe the 2π state. We

have, using the notation (I, I_3),

(35) $$K^+(\tfrac{1}{2}, +\tfrac{1}{2}) \to \psi(2, +1) + s(\tfrac{3}{2}, -\tfrac{1}{2})\,,$$

(36) $$K^0\,(\tfrac{1}{2}, -\tfrac{1}{2}) \to \psi(2, 0) \quad + s(\tfrac{3}{2}, -\tfrac{1}{2})\,.$$

Here we have chosen the $\Delta I = \frac{3}{2}$ spurion, and have chosen $\Delta I_3 = -\frac{1}{2}$ for the spurion in order to « conserve » I_3. Notice also that because of the famous formula

(37) $$\frac{Q}{e} = I_3 + \frac{S+B}{2}\,,$$

« conservation » of I_3 implies strangeness (S) conservation.

For convenience we now transpose the K and the spurion to opposite sides of the equation. To maintain conservation of I_3 we see, from eq. (35), that he must reverse the sign of I_3 when we transpose. From (37) this means that S also must reverse its sign. Equation (35) and (36) can be combined as

(38) $$K(\tfrac{1}{2}) \to \psi(2) + s(\tfrac{3}{2}, -\tfrac{1}{2})\,.$$

After transposing we get

(39) $$s(\tfrac{3}{2}, +\tfrac{1}{2}) \to \psi(2) + \bar{K}(\tfrac{1}{2})\,,$$

which means

(40) $$s(\tfrac{3}{2}, +\tfrac{1}{2}) \to \psi(2, +1) + K^-(\tfrac{1}{2}, -\tfrac{1}{2})\,,$$

and

(41) $$s(\tfrac{3}{2}, +\tfrac{1}{2}) \to \psi(2, 0) \quad + \bar{K}^0\,(\tfrac{1}{2}, +\tfrac{1}{2})\,.$$

We now go to the Clebsch-Gordan Table IV. According to eq. (39) we want to compose $2 \times \frac{1}{2}$ so as to get $(\frac{3}{2}, +\frac{1}{2})$. We therefore look under the column $(\frac{3}{2}, +\frac{1}{2})$ to find

(42) $$\begin{aligned}(\tfrac{3}{2}, +\tfrac{1}{2}) &= \sqrt{\tfrac{3}{5}}(2, +1)(\tfrac{1}{2}, -\tfrac{1}{2}) - \sqrt{\tfrac{2}{5}}\,(2, 0)(\tfrac{1}{2}, +\tfrac{1}{2}) = \\ &= \sqrt{\tfrac{3}{5}}(2, +1)K^- - \sqrt{\tfrac{2}{5}}(2, 0)\bar{K}^0 = \\ &= \sqrt{\tfrac{3}{5}}K^-\{\sqrt{\tfrac{1}{2}}(\pi^+\pi^0 + \pi^0\pi^+)\} - \sqrt{\tfrac{2}{5}}\bar{K}^0\,\{\sqrt{\tfrac{1}{6}}(\pi^+\pi^- + 2\pi^0\pi^0 + \pi^-\pi^+)\}.\end{aligned}$$

Similarly we consider the $\Delta I = \frac{1}{2}$ spurion. This cannot be obtained by composition of $2 \times \frac{1}{2}$, but only by $0 \times \frac{1}{2}$, so we have, analogous to (39),

(43) $$s(\tfrac{1}{2}, +\tfrac{1}{2}) \to \bar{K}(\tfrac{1}{2}) + \psi(0)\,,$$

or

$$(44)\qquad (\tfrac{1}{2}, +\tfrac{1}{2}) = (\tfrac{1}{2}, +\tfrac{1}{2})(0,0) = \bar{K}^0\sqrt{\tfrac{1}{3}}\{\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+\}\,.$$

Suppose now that both $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ occur, with amplitudes a_1 and a_3, respectively. Then from (44) and (42), the total amplitude is

$$\psi = a_1(\tfrac{1}{2}, +\tfrac{1}{2}) + a_3(\tfrac{3}{2}, \tfrac{1}{2})\,,$$

or

$$(45)\quad \psi = \bar{K}^0\left|(\pi^+\pi^- + \pi^-\pi^+)\left(\sqrt{\frac{1}{3}}\,a_1 - \sqrt{\frac{2}{30}}\,a_3\right) + \right.$$
$$\left. + \pi^0\pi^0\left(-\sqrt{\frac{1}{3}}\,a_1 - 2\sqrt{\frac{2}{30}}\,a_3\right)\right| + K^-\left[(\pi^+\pi^0 + \pi^0\pi^+)\sqrt{\frac{3}{10}}\,a_3\right].$$

We now remark that the overall relative phase between the $\bar{K}^0$ and K^- parts of ψ has no physical meaning. This is because charge conservation prevents « interference » between $\bar{K}^0$ and K^-. We can only compare intensities.

We next recall that it is K_1^0-decay we are interested in, not K^0 or $\bar{K}^0$. In eq. (45), that part of ψ proportional to $\bar{K}^0(\pi^+\pi^- + \pi^-\pi^+)$, for example, represents (after transposing), the amplitude for $K^0 \to (\pi^+\pi^- + \pi^-\pi^+)$. Now,

$$(46)\qquad K_1^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}}\,.$$

We have *chosen* the final state of 2π so that it corresponds to K_1^0-decay. Thus as far as decay into a state with $CP = +1$ (K_1^0-decay) is concerned, the K^0 and $\bar{K}^0$ behave in exactly the same way.

That is, they interfere constructively in just such a way as to give $CP = +1$. Therefore the amplitude for $\bar{K}^0 \to (\pi^+\pi^- + \pi^-\pi^+)$ is exactly the same as that for $K^0 \to (\pi^+\pi^- + \pi^-\pi^+)$. From eq. (46), the amplitude for $K_1^0 \to (\pi^+\pi^- + \pi^-\pi^+)$ is just $(1+1)/\sqrt{2} = \sqrt{2}$ times that for $K^0 \to (\pi^+\pi^- + \pi^-\pi^+)$. Therefore when we write rates, we have

$$(47)\qquad R(K_1^0 \to 2\pi) = (\sqrt{2})^2 R(K^0 \to 2\pi)\,.$$

[Note: This argument is rephrased following eq. (109).]

From eq. (45) we now write the decay rates

$$R(K^+ \to \pi^+\pi^0) \equiv R(+0) = \left[\left(\sqrt{\frac{3}{10}}\right)^2 + \left(\sqrt{\frac{3}{10}}\right)^2\right]|a_3|^2\,, \quad \textit{i.e.}\,,$$

$$(48)\qquad R(+0) = \frac{3}{5}|a_3|^2\,.$$

$$(49)\qquad R(K_1^0 \to \pi^+\pi^-) \equiv R(+-)2(1^2+1^2)\left|\sqrt{\frac{1}{3}}\,a_1 - \sqrt{\frac{1}{15}}\,a_3\right|^2,$$

$$(50)\qquad R(K_1^0 \to \pi^0\pi^0) \equiv R(00) = 2\left|-\sqrt{\frac{1}{3}}\,a_1 - 2\sqrt{\frac{1}{15}}\,a_3\right|^2.$$

Now choose units such that $a_1 \equiv 1$. Set $a_3 = |a_3|\exp[i\delta]$. Expand (49) and (50), neglecting quadratic terms in a_3, to get

$$(51)\qquad R(+-) = \frac{4}{3} - \frac{8}{3}\frac{1}{\sqrt{5}}|a_3|\cos\delta\,,$$

$$(52)\qquad R(0\,0) = \frac{2}{3} + \frac{8}{3}\frac{1}{\sqrt{5}}|a_3|\cos\delta\,,$$

so that in these units

$$(53)\qquad \begin{cases} R_1 \equiv R(+-) + R(0\,0) = 2\,. \\ R(+0) = \frac{3}{5}|a_3|^2 = \frac{3}{10}|a_3|^2 R_1\,, \end{cases}$$

so that

$$(54)\qquad |a_3| = \sqrt{\frac{10}{3}}\sqrt{\frac{R(+0)}{R_1}}\,.$$

Putting in numbers [1, 2], $R_1 = 550R(+0)$, so

$$(55)\qquad |a_3| = 0.078\,|a_1|\,.$$

Now

$$B_1 \equiv \frac{R(0\,0)}{R_1} = \frac{1}{3} + \frac{4}{3}\frac{1}{\sqrt{5}}\cos\delta\,\frac{|a_3|}{|a_1|}\,,$$

or finally,

$$B_1 = 0.333 + 0.047\cos\delta\,.$$

For $-1 < \cos\delta < 1$ we get

$$(56)\qquad 0.29 \leqslant B_1 \leqslant 0.38\,,$$

as the prediction of the $\Delta I = \frac{1}{2}$ rule.

If there were no π-π interaction at 200 MeV/c in the S state, a_3 and a_1 would be relatively real, by T-invariance. Then $\cos\delta = \pm 1$.

Recent values of B_1 are [2]

0.260 ± 0.024, ANDERSON *et al.*,

0.294 ± 0.021, CHRÉTIEN *et al.*,

0.329 ± 0.013, BROWN *et al.*

All are consistent with eq. (56), although not completely with one another.

3·2. *The decay* $\Lambda \to \mathcal{N} + \pi$. – Since $I = 0$ for the Λ, we can have $\Delta I = \frac{1}{2}$ or $\frac{3}{2}$. We write the spurion reactions

$$s(\tfrac{1}{2}, -\tfrac{1}{2}) \to \bar{\Lambda}(0, 0) N(\tfrac{1}{2}) \pi(1) . \tag{56}$$

The $\overline{\Lambda}(0, 0)$ contributes only a factor of unity, so, from Table I we find

$$(\tfrac{1}{2}, -\tfrac{1}{2}) = \bar{\Lambda}[\sqrt{\tfrac{1}{3}} n\pi^0 - \sqrt{\tfrac{2}{3}} p\pi^-] . \tag{57}$$

Similarly for $\Delta I = \frac{3}{2}$ we have, by inspection of Table I,

$$(\tfrac{3}{2}, -\tfrac{1}{2}) = \bar{\Lambda}[\sqrt{\tfrac{2}{3}} n\pi^0 + \sqrt{\tfrac{1}{3}} p\pi^-] . \tag{58}$$

The total amplitude is

$$\psi \equiv a_1(\tfrac{1}{2}, -\tfrac{1}{2}) + a_3(\tfrac{3}{2}, -\tfrac{1}{2}) ,$$

or

$$\psi = (\sqrt{\tfrac{1}{3}} a_1 + \sqrt{\tfrac{2}{3}} a_3) \bar{\Lambda} n\pi^0 + (-\sqrt{\tfrac{2}{3}} a_1 + \sqrt{\tfrac{1}{3}} a_3) \bar{\Lambda} p\pi^- . \tag{59}$$

The decay rates are therefore

$$R(\mathrm{n}\pi^0) = \tfrac{1}{3} |a_1 + \sqrt{2} a_3|^2 , \tag{60}$$

$$R(\mathrm{p}\pi^-) = \tfrac{1}{3} |\sqrt{2} a_1 - a_3|^2 . \tag{61}$$

We define the branching ratio

$$B_\Lambda \equiv \frac{R(\mathrm{p}\pi^-)}{R_\Lambda \equiv R(\mathrm{p}\pi^-) + R(\mathrm{n}\pi^0)} , \tag{62}$$

and notice that if the $\Delta I = \frac{1}{2}$ rule holds, *i.e.*, if $a_3 = 0$, then $B_\Lambda = \frac{2}{3}$.

If we expand (60) and (61), set $a_1 \equiv 1$, call $a_3 \equiv |a_3| \exp[i\delta]$, and neglect

$|a_3|^2$, we find

$$(63)\quad \begin{cases} R(\mathrm{n}\pi^0) = \dfrac{1}{3} + \dfrac{2\sqrt{2}}{3}|a_3|\cos\delta\,, \\ R(\mathrm{p}\pi^-) = \dfrac{2}{3} - \dfrac{2\sqrt{2}}{3}|a_3|\cos\delta\,, \\ R_\Lambda = R(\mathrm{n}\pi^0) + R(\mathrm{p}\pi^-) = 1\,, \\ B_\Lambda = \dfrac{R(\mathrm{p}\pi^-)}{R_\Lambda} = \dfrac{2}{3} - \dfrac{2\sqrt{2}}{3}|a_3|\cos\delta\,, \\ B_\Lambda = 0.660 - 0.95\,|a_3|\cos\delta\,. \end{cases}$$

Here we know that the $\mathcal{N}\pi$ scattering is weak, so we expect $\cos\delta = \pm 1$. Also, we have corrected $\frac{2}{3} \to 0.660$, for phase-space ($\mathrm{n}\pi^0$ is lighter than $\mathrm{p}\pi^-$).

A recent accurate value of B_Λ by ANDERSON *et al.* [2] gives $B_\Lambda = .685 \pm 0.0170$. If we take $\cos\delta = \pm 1$ then we see that we must choose $\cos\delta = -1$ and

$$(64)\qquad \frac{|a_3|}{|a_1|} = 0.026 \pm 0.018\,.$$

This result is of the same order as eq. (55), for $\mathrm{K}^+ \to 2\pi$. Of course the value of $|a_3|/|a_1|$ for Λ-decay need have no relation to that for $\mathrm{K} \to 2\pi$.

It is interesting to observe [4] that $B_\Lambda = \frac{2}{3}$ is obtained not only for $a_3 = 0$ but also for $a_3 = -2\sqrt{2}\,a_1$. This can be seen by inspection of eqs. (60) and (61). Let us examine this possibility more closely. So far we have discussed only branching ratios, as predicted by the $\Delta I = \frac{1}{2}$ rule. That is, in terms of the decay amplitudes S and P we have been considering only $|S|^2 + |P|^2 = R$. The $\Delta I = \frac{1}{2}$ rule predicts much more. For instance, eq. (57) holds for *every* decay configuration, for the $\Delta I = \frac{1}{2}$ decay, and thus holds for the S-wave and P-wave parts separately. That is, from (57), and using the subscript 1 to stand for the $\Delta I = \frac{1}{2}$ amplitude,

$$(65)\qquad A_1(\Lambda \to \mathrm{p}\pi^-) = -\sqrt{2}\,A_1(\Lambda \to \mathrm{n}\pi^0)\,,$$

which means, for $\Delta I = \frac{1}{2}$,

$$(66)\qquad S_1(\Lambda \to \mathrm{p}\pi^-) = -\sqrt{2}\,S_1(\Lambda \to \mathrm{n}\pi^0)\,,$$

and

$$(67)\qquad P_1(\Lambda \to \mathrm{p}\pi^-) = -\sqrt{2}\,P_1(\Lambda \to \mathrm{n}\pi^0)\,.$$

Then for the decay parameters α, β, and γ, defined in eqs. (3), (4), (5), we see

$$(68)\qquad \alpha_1\,(\Lambda \to \mathrm{p}\pi^-) = \alpha_1\ \ (\Lambda \to \mathrm{n}\pi^0)\,,$$

$$(69)\qquad \beta_1\,(\Lambda \to \mathrm{p}\pi^-) = \beta_1\ \ (\Lambda \to \mathrm{n}\pi^0)\,,$$

(70) $$\gamma_1(\Lambda \to p\pi^-) = \gamma_1 \ (\Lambda \to n\pi^0)\,,$$

(71) $$R_1(\Lambda \to p\pi^-) = 2R_1(\Lambda \to n\pi^0)\,.$$

Similarly if we had a *pure* $\Delta I = \frac{3}{2}$ decay, then, from eq. (58),

$$A_3(\Lambda \to p\pi^-) = (1/\sqrt{2})\, A_3(\Lambda \to n\pi^0)\,.$$

This again leads to *equality* of α_3, β_3, and γ_3, for $\Lambda \to p+\pi^-$ and $\Lambda \to n+\pi^0$, and a branching ratio $R_3(\Lambda \to p\pi^-) = \frac{1}{2}R_3(\Lambda \to n\pi^0)$.

Suppose now that one has a mixture of $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$. In general, the two decays ($\Delta I = \frac{3}{2}$ and $\frac{1}{2}$) should have different S/P ratios. In that case, the S/P ratios for $\Lambda \to p\pi^-$ and $n\pi^0$ will *not* be the same, in general. We can see this in detail as follows. For $\Delta I = \frac{1}{2}$ (designated by subscript 1), if we use π^- and π^0 to denote $\Lambda \to p+\pi^-$ and $n+\pi^0$, we have, from eq. (65), for the S- P-wave parts separately,

(72) $$S_1(\pi^-) = -\sqrt{2}\, S_1(\pi^0)\,,$$

and

(73) $$P_1(\pi^-) = -\sqrt{2}\, P_1(\pi^0)\,.$$

For $\Delta I = \frac{3}{2}$ (subscript 3) we hawe

(74) $$S_3(\pi^-) = (1/\sqrt{2})\, S_3(\pi^0)\,,$$

and

(75) $$P_3(\pi^-) = (1/\sqrt{2})\, P_3(\pi^0)\,.$$

Now suppose $\Delta I = \frac{1}{2}$ occurs with amplitude a_1, and $\Delta I = \frac{3}{2}$ with amplitude a_3, then

$$\begin{aligned}
S(\pi^-) &= a_1 S_1(\pi^-) + a_3 S_3(\pi^-) = -\sqrt{2}\, a_1 S_1(\pi^0) + (a_3/\sqrt{2})\, S_3(\pi^0)\,,\\
P(\pi^-) &= a_1 P_1(\pi^-) + a_3 P_3(\pi^-) = -\sqrt{2}\, a_1 P_1(\pi^0) + (a_3/\sqrt{2})\, P_3(\pi^0)\,,\\
S(\pi^0) &= a_1 S_1(\pi^0) + a_3 S_3(\pi^0)\,,\\
P(\pi^0) &= a_1 P_1(^0) + a_3 P_3(\pi^0)\,.
\end{aligned}$$

We see by inspection of these equations that if $a_3 = 0$ or if $a_1 = 0$, then $S(\pi^-)/P(\pi^-) = S(\pi^0)/P(\pi^0)$. The same is true if $S_1/P_1 = S_3/P_3$. In both cases α, β, and γ are the same for $p\pi^-$ and $n\pi^0$. The choice $a_3 = -2\sqrt{2}\, a_1$ gives $B_\Lambda = \frac{2}{3}$. In general, if $S_1/P_1 \neq S_3/P_3$, then a value for a_3/a_1 that gives $B = \frac{2}{3}$ leads to different values of α, β, and γ for $p\pi^-$ and $n\pi^0$.

It is thus important to check α, β, and γ for $\Lambda \to n\pi^0$. CORK *et al.* [5] have measured the up-down asymmetries for $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$ « simultaneously », *i.e.*, from Λ's produced in the same way. Therefore there is a single Λ polarization p_Λ. The decay asymmetries yield $\alpha(\pi^-)p_\Lambda$ and $\alpha(\pi^0)p_\Lambda$, and the ratio gives

$$\frac{\alpha(n\pi^0)}{\alpha(p\pi^-)} = +1.10 \pm 0.27 ,$$

in agreement with the $\Delta I = \frac{1}{2}$ rule.

BLOCK *et al.* [2] have measured $\gamma(n\pi^0)$ by an indirect method. The branching ratio for

$$\frac{{}^4\mathrm{He}_\Lambda \to (\pi^0 \text{ modes})}{{}^4\mathrm{He}_\Lambda \to (\pi^- \text{ modes})} ,$$

depends strongly on $S(\pi^0)/P(\pi^0)$. They find that S-wave predominates. They find

$$\gamma(n\pi^0) = +0.78^{+0.22}_{-0.43} .$$

This is to be compared with, for instance, the value obtained by CRONIN and OVERSETH [2],

$$\gamma(p\pi^-) = +0.78 \pm 0.04 .$$

Thus an « accidental » solution with $a_3 = -2\sqrt{2}\,a_1$, must (within the errors) also have the same S/P ratio for $\Delta I = \frac{3}{2}$ and $\frac{1}{2}$, to agree with experiment. Such a double accident seems unlikely.

3'3. *The decay* $\Xi \to \Lambda + \pi$. – To find the prediction for $\Delta I = \frac{1}{2}$ we write

$$s(\tfrac{1}{2}, +\tfrac{1}{2}) \to \overline{\Lambda}(0)\Xi(\tfrac{1}{2})\bar{\pi}(1) ,$$

$$(\tfrac{1}{2}, +\tfrac{1}{2}) = \overline{\Lambda}[\sqrt{\tfrac{2}{3}}(1, +1)(\tfrac{1}{2}, -\tfrac{1}{2}) - \sqrt{\tfrac{1}{3}}(1, 0)(\tfrac{1}{2}, \tfrac{1}{2})] = \overline{\Lambda}[\sqrt{\tfrac{2}{3}}\pi^+\Xi^- - \sqrt{\tfrac{1}{3}}\pi^0\Xi^0] ,$$

which gives (transposing)

$$R(\Xi^- \to \Lambda + \pi^-) = 2R(\Xi^0 \to \Lambda + \pi^0) . \tag{76}$$

The Ξ^- lifetime is about 1.2×10^{-10} s [2]. The Ξ^0 lifetime is not yet known well enough to test eq. (76).

3'4. *The decay* $\Sigma \to \mathcal{N} + \pi$. – The final state $\mathcal{N} + \pi$ can have $I = \frac{1}{2}$ or $\frac{3}{2}$. The Σ has $I = 1$. Therefore we can have $\Delta I = 1 \times \frac{1}{2} = \frac{3}{2}$ or $\frac{1}{2}$; or $1 \times \frac{3}{2} = \frac{5}{2}$, $\frac{3}{2}$, or $\frac{1}{2}$. We assume, for simplicity, that $\Delta I = \frac{5}{2}$ is absent, but include $\Delta I = \frac{3}{2}$ as well as $\frac{1}{2}$.

We write $\Sigma \to \mathcal{N} + \pi + s$. Transposing, we have $s \to \Sigma(\overline{\mathcal{N}\pi})$. From an example, say $\Sigma^- \to n + \pi^- + s$, we see that we have $\Delta I_3 = +\frac{1}{2}$ for the spurion. There are four possible transition amplitudes, corresponding to $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$, and $I(\overline{\mathcal{N}\pi}) = \frac{1}{2}$ or $\frac{3}{2}$. We write down the four charge states, using the notation

$$\psi(1,1) \equiv \psi(\Delta I = \tfrac{1}{2},\ I(\overline{\mathcal{N}\pi}) = \tfrac{1}{2}) ,$$

$$\psi(1,3) \equiv \psi(\Delta I = \tfrac{1}{2},\ I(\overline{\mathcal{N}\pi}) = \tfrac{3}{2}) ,$$

$$\psi(3,1) \equiv \psi(\Delta I = \tfrac{3}{2},\ I(\overline{\mathcal{N}\pi}) = \tfrac{1}{2}) ,$$

and

$$\psi(3,3) \equiv \psi(\Delta I = \tfrac{3}{2},\ I(\overline{\mathcal{N}\pi}) = \tfrac{3}{2}) .$$

Correspondingly we define the four decay amplitudes $A(1,1)$, $A(1,3)$, $A(3,1)$ and $A(3,3)$, and have the superposition

$$\psi(\Sigma\overline{\mathcal{N}\pi}) = A(1,1)\psi(1,1) + A(1,3)\psi(1,3) + A(3,1)\psi(3,1) + A(3,3)\psi(3,3). \tag{77}$$

We now write down $\psi(1,1)$, etc., using Tables I and III, and recalling that $\Delta I = +\frac{1}{2}$ in each case. To aid in reading the table we write an intermediate step, in a notation that is self-explanatory:

$$\begin{aligned}\psi(1,1) &= (\tfrac{1}{2}[1(\Sigma)\times\tfrac{1}{2}(\overline{\mathcal{N}\pi})];\ +\tfrac{1}{2}) = \sqrt{\tfrac{2}{3}}\,\Sigma^+(\tfrac{1}{2}, -\tfrac{1}{2}) - \sqrt{\tfrac{1}{3}}\,\Sigma^0(\tfrac{1}{2}, +\tfrac{1}{2}) = \\ &= \sqrt{\tfrac{2}{3}}\,\Sigma^+(\sqrt{\tfrac{1}{3}}\pi^0\overline{p} - \sqrt{\tfrac{2}{3}}\pi^-\overline{n}) - \sqrt{\tfrac{1}{3}}\,\Sigma^0(\sqrt{\tfrac{2}{3}}\pi^+\overline{p} - \sqrt{\tfrac{1}{3}}\pi^0\overline{n}) .\end{aligned} \tag{78}$$

$$\begin{aligned}\psi(1,3) &= (\tfrac{1}{2}[1(\Sigma)\times\tfrac{3}{2}(\overline{\mathcal{N}\pi})];\ +\tfrac{1}{2}) = \\ &= \sqrt{\tfrac{1}{2}}\Sigma^-(\tfrac{3}{2}, +\tfrac{3}{2}) - \sqrt{\tfrac{1}{3}}\Sigma^0(\tfrac{3}{2}, +\tfrac{1}{2}) + \sqrt{\tfrac{1}{6}}\Sigma^+(\tfrac{3}{2}, -\tfrac{1}{2}) = \\ &= \sqrt{\tfrac{1}{2}}\Sigma^-\pi^+\overline{n} - \sqrt{\tfrac{1}{3}}\Sigma^0(\sqrt{\tfrac{1}{3}}\pi^+\overline{p} + \sqrt{\tfrac{2}{3}}\pi^0\overline{n}) + \sqrt{\tfrac{1}{6}}\Sigma^+(\sqrt{\tfrac{2}{3}}\pi^0\overline{p} + \sqrt{\tfrac{1}{3}}\pi^-\overline{n}) .\end{aligned} \tag{79}$$

$$\begin{aligned}\psi(3,1) &= (\tfrac{3}{2}[1(\Sigma)\times\tfrac{1}{2}(\overline{\mathcal{N}\pi});\ +\tfrac{1}{2}) = \sqrt{\tfrac{1}{3}}\Sigma^+(\tfrac{1}{2}, -\tfrac{1}{2}) + \sqrt{\tfrac{2}{3}}\Sigma^0(\tfrac{1}{2}, +\tfrac{1}{2}) = \\ &= \sqrt{\tfrac{1}{3}}\Sigma^+(\sqrt{\tfrac{1}{3}}\pi^0\overline{p} - \sqrt{\tfrac{2}{3}}\pi^-\overline{n}) + \sqrt{\tfrac{2}{3}}\Sigma^0(\sqrt{\tfrac{2}{3}}\pi^+\overline{p} - \sqrt{\tfrac{1}{3}}\pi^0\overline{n})\end{aligned} \tag{80}$$

$$\psi(3,3) = \left(\frac{3}{2}\,1(\Sigma)\times\frac{3}{2}\,(\overline{\mathcal{N}\pi});\ +\frac{1}{2}\right) = \tag{81}$$

$$= \sqrt{\frac{2}{5}}\,\Sigma^-\left(\frac{3}{2}, +\frac{3}{2}\right) + \sqrt{\frac{1}{15}}\,\Sigma^0\left(\frac{3}{2}, +\frac{1}{2}\right) - \sqrt{\frac{8}{15}}\,\Sigma^+\left(\frac{3}{2}, -\frac{1}{2}\right) =$$

$$= \sqrt{\frac{2}{5}}\,\Sigma^-\pi^+\overline{n} + \sqrt{\frac{1}{15}}\,\Sigma^0\left(\sqrt{\frac{1}{3}}\,\pi^+\overline{p} + \sqrt{\frac{2}{3}}\,\pi^0\overline{n}\right) - \sqrt{\frac{8}{15}}\,\Sigma^+\left(\sqrt{\frac{2}{3}}\,\pi^0\overline{p} + \sqrt{\frac{1}{3}}\,\pi^-\overline{n}\right).$$

We could now write down the general superposition $\psi(\Sigma\mathcal{N}\pi)$ given by

eq. (77). However, since we are interested in the charge states rather than the I-spin states, we rewrite eq. (77) as

$$\begin{aligned}(82)\qquad \psi(\Sigma\overline{\mathcal{N}\pi}) \equiv\; & A(\Sigma^+\pi^0\overline{\mathrm{p}})\,\psi(\Sigma^+\pi^0\overline{\mathrm{p}}) + A(\Sigma^+\pi^-\overline{\mathrm{n}})\,\psi(\Sigma^+\pi^-\overline{\mathrm{n}}) + \\ & + A(\Sigma^0\pi^+\overline{\mathrm{p}})\,\psi(\Sigma^0\pi^+\overline{\mathrm{p}}) + A(\Sigma^0\pi^0\overline{\mathrm{n}})\,\psi(\Sigma^0\pi^0\overline{\mathrm{n}}) + A(\Sigma^-\pi^+\overline{\mathrm{n}})\,\psi(\Sigma^-\pi^+\overline{\mathrm{n}})\,.\end{aligned}$$

From eqs. (77) through (82) we obtain the amplitudes

$$(83)\qquad A(\Sigma^+\pi^0\,\overline{\mathrm{p}}) = \frac{\sqrt{2}}{3}A(1,1) + \frac{1}{3}A(1,3) + \frac{1}{3}A(3,1) - \frac{4}{3}\sqrt{\frac{1}{5}}\,A(3,3)\,,$$

$$(84)\qquad A(\Sigma^+\pi^-\overline{\mathrm{n}}) = -\frac{2}{3}A(1,1) + \frac{\sqrt{2}}{6}A(1,3) - \frac{\sqrt{2}}{3}A(3,1) - \frac{2}{3}\sqrt{\frac{2}{5}}\,A(3,3)\,,$$

$$(85)\qquad A(\Sigma^-\pi^+\overline{\mathrm{n}}) = \frac{\sqrt{2}}{2}A(1,3) + \sqrt{\frac{2}{5}}\,A(3,3)\,,$$

$$(86)\qquad A(\Sigma^0\,\pi^+\,\overline{\mathrm{p}}) = -\frac{\sqrt{2}}{3}A(1,1) - \frac{1}{3}A(1,3) + \frac{2}{3}A(3,1) + \frac{1}{3}\sqrt{\frac{1}{5}}\,A(3,3)\,,$$

$$(87)\qquad A(\Sigma^0\,\pi^0\,\overline{\mathrm{n}}) = \frac{1}{3}A(1,1) - \frac{\sqrt{2}}{3}A(1,3) - \frac{\sqrt{2}}{3}A(3,1) + \frac{1}{3}\sqrt{\frac{2}{5}}\,A(3,3)\,.$$

Unfortunately we cannot make use of eqs. (86) and (87), since $\Sigma^0 \to \mathcal{N}+\pi$ is unobservable because of the rapid death of the Σ^0 via the electromagnetic decay $\Sigma^0 \to \Lambda+\gamma$.

We are left with eqs. (83), (84), and (85). These equations hold for either the S-wave or the P-wave parts of the decay amplitude. If we wished we could write the equations twice, once with new subscripts for S and once for P. In general the separate terms are complex numbers. However, if the final $\mathcal{N}$-π interaction is small, then T invariance demands that the separate terms all be real, except for an unimportant phase factor common to all terms. The $\mathcal{N}$-π interaction is indeed negligible at the decay momentum [1]. We therefore take all the terms to be real. We now imagine eq. (83) (for instance) written twice (once with subscripts for S-wave, and once for P-wave). We can imagine a two-dimensional S-P space, and think of the equations (*i.e.*, S and P) as equations involving the S and P components of vectors. We combine the components and write, for instance,

$$(88)\qquad \boldsymbol{A}(1,1) \equiv A_s(1,1)\,\hat{\boldsymbol{S}} + A_p(1,1)\,\hat{\boldsymbol{P}}\,,$$

with $\hat{\boldsymbol{S}}$ and $\hat{\boldsymbol{P}}$ as unit vectors, and with similar expressions for $\boldsymbol{A}(1,3)$, $\boldsymbol{A}(3,1)$, and $\boldsymbol{A}(3,3)$. Since eqs. (83), (84), and (85) hold for both the S and P com-

ponents, they hold for the vectors. We can therefore imagine these equations rewritten, with the substitution of $\boldsymbol{A}(1,1)$ for $A(1,1)$, etc.

At first glance the right-hand sides of eqs. (83), (84), and (85) seem to involve the four independent vectors $\boldsymbol{A}(1,1)$, $\boldsymbol{A}(1,3)$, $\boldsymbol{A}(3,1)$, and $\boldsymbol{A}(3,3)$. However, we observe that $\boldsymbol{A}(1,1)$ and $\boldsymbol{A}(3,1)$ occur only in the combination

$$\boldsymbol{A}(1,1;3,1) \equiv \boldsymbol{A}(1,1) + \sqrt{\tfrac{1}{2}}\,\boldsymbol{A}(3,1)\,, \tag{89}$$

so that we may rewrite eqs. (83), (84), and (85) as follows:

$$A^0 \equiv \boldsymbol{A}(\Sigma^+\pi^0\,\bar{\mathrm{p}}) = \sqrt{\frac{2}{9}}\,\boldsymbol{A}(1,1;3,1) + \sqrt{\frac{1}{9}}\,\boldsymbol{A}(1,3) - \sqrt{\frac{16}{45}}\,\boldsymbol{A}(3,3)\,, \tag{90}$$

$$A^+ \equiv \boldsymbol{A}(\Sigma^+\pi^-\,\bar{\mathrm{n}}) = -\frac{2}{3}\,\boldsymbol{A}(1,1;3,1) + \sqrt{\frac{1}{18}}\,\boldsymbol{A}(1,3) - \sqrt{\frac{8}{45}}\,\boldsymbol{A}(3,3)\,, \tag{91}$$

$$A^- \equiv \boldsymbol{A}(\Sigma^-\pi^+\,\bar{\mathrm{n}}) = \sqrt{\frac{1}{2}}\,\boldsymbol{A}(1,3) + \sqrt{\frac{2}{5}}\,\boldsymbol{A}(3,3)\,. \tag{92}$$

From eqs. (90), (91), and (92) we form the linear combination

$$\sqrt{2}\,\boldsymbol{A}(\Sigma^+\pi^0\bar{\mathrm{p}}) + \boldsymbol{A}(\Sigma^+\pi^-\,\bar{\mathrm{n}}) - \boldsymbol{A}(\Sigma^-\pi^+\,\bar{\mathrm{n}}) = -\sqrt{\frac{18}{5}}\,\boldsymbol{A}(3,3)\,. \tag{93}$$

Using eq. (93), we can make the following observations. (*a*) Suppose the $\Delta I = \frac{1}{2}$ rule holds. Then $\boldsymbol{A}(3,1) = 0$ and $\boldsymbol{A}(3,3) = 0$. Since $\boldsymbol{A}(3,3) = 0$, eq. (93) corresponds to a closed triangle in the S-P plane. This is the well-known triangle of GELL-MANN and ROSENFELD [1]. (*b*) Suppose we have $\boldsymbol{A}(3,1) \neq 0$, but $\boldsymbol{A}(3,3) = 0$. Since $\boldsymbol{A}(3,1) \neq 0$, the $\Delta I = \frac{1}{2}$ rule does not hold. Nevertheless, according to eq. (93) we obtain a closed triangle in the S-P plane. Thus if we find a closed triangle (experimentally) we cannot rule out $\Delta I = \frac{3}{2}$. The linear combination of $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ given by eq. (89) cannot be resolved. (*c*) Suppose we have $\boldsymbol{A}(3,3) \neq 0$. Then eq. (93) corresponds to a closed quadrangle instead of a triangle. Equation (93) can be used to determine $\boldsymbol{A}(3,3)$. Of course then the $\Delta I = \frac{1}{2}$ rule does not hold exactly. (We already know this, from the decay $K^+ \to \pi^+\pi^0$.)

We turn now to our experimental knowledge of $\boldsymbol{A}(\Sigma^+\pi^0\bar{\mathrm{p}})$, $\boldsymbol{A}(\Sigma^+\pi^-\bar{\mathrm{n}})$, and $\boldsymbol{A}(\Sigma^-\pi^+\bar{\mathrm{n}})$. From the partial decay rates we know [1, 6] that

$$|\boldsymbol{A}(\Sigma^+\pi^0\bar{\mathrm{p}})| \approx |\boldsymbol{A}(\Sigma^+\pi^-\bar{\mathrm{n}})| \approx |\boldsymbol{A}(\Sigma^-\pi^+\bar{\mathrm{n}})|\,. \tag{94}$$

Therefore, *if* $\boldsymbol{A}(3,3) = 0$, we see from eq. (93) that the resulting triangle will be an isosceles right triangle with equal legs $|\boldsymbol{A}(\Sigma^+\pi^-\bar{\mathrm{n}})|$ and $|\boldsymbol{A}(\Sigma^-\pi^+\bar{\mathrm{n}})|$,

and hypotenuse $\sqrt{2}\,|A(\Sigma^+\pi^0\bar{p})|$. The decay parameter α_0, corresponding to $A(\Sigma^+\pi^0\bar{p})$—*i.e.*, to $\Sigma^+ \to p+\pi^0$—has been determined by BEALL *et al.* [7] by measuring the scattering asymmetry of the decay proton. Their result is $\alpha_0 = +0.78^{+0.08}_{-0.09}$. The other decay parameters, β_0 and γ_0, are not known. We assume $\beta_0 = 0$ (T invariance and small $\mathcal{N}$-π interaction). The decay parameter α^+, corresponding to $A(\Sigma^+\pi^-\bar{n})$,—*i.e.*, to $\Sigma^+ \to \to n+\pi^+$—has been measured by CORK *et al.* [5]; they measured in a single experiment the up-down asymmetry for $\Sigma^+ \to \to n+\pi^+$, to obtain $\alpha^+ p_{\Sigma^+}$, and for $\Sigma^+ \to p+\pi^0$, to obtain $\alpha_0 p_{\Sigma^+}$. The ratio gives α_+/α_0, and the known value of α_0 gives α_+. They find $\alpha_+ = +0.03 \pm 0.08$. In our present notation, $\alpha = = 2A_S A_P/(A_S^2 + A_P^2)$, so that $\alpha_+ \cong 0$ means that $A(\Sigma^+\pi^-\bar{n})$ is oriented along either the $\hat{S}$ axis or the $\hat{P}$ axis. Until γ_+ is measured we cannot choose between these alternative possibilities. The decay parameter α_-, corresponding to $A(\Sigma^-\pi^+\bar{n})$—*i.e.*, to $\Sigma^- \to n+\pi^-$—has been measured by TRIPP, WATSON, and FERRO-LUZZI [8], who obtain $\alpha_- = +0.16 \pm 0.21$, and by NUSSBAUM *et al.* [2], who obtain $\alpha_- = = +0.04 \pm 0.23$. Therefore $A(\Sigma^-\pi^+\bar{n})$ is oriented (approximately) either along $\hat{S}$ or along $\hat{P}$. There is as yet no knowledge of γ_-, so that either alternative is possible. If $A(3,3) = 0$, then according to eqs. (93) and (94) and the results $\alpha_+ \approx 0$ and $\alpha_- \approx 0$, we have the two possibilities

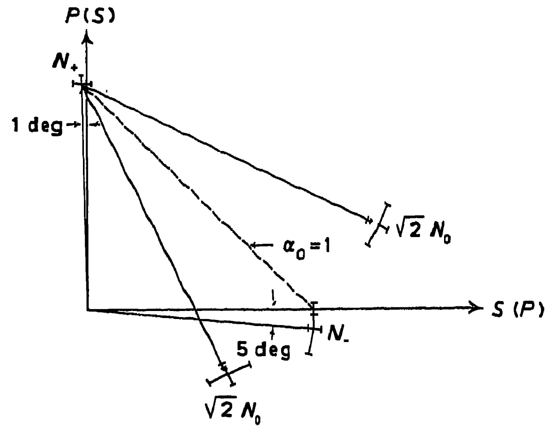

Fig. 13.

(95a) $$A(\Sigma^+\pi^-\bar{n}) \approx -\hat{S}\,,$$

(95b) $$A(\Sigma^-\pi^+\bar{n}) \approx \hat{P}\,,$$

(95c) $$A(\Sigma^+\pi^0\,\bar{p}) \approx \frac{\hat{S}+\hat{P}}{\sqrt{2}}\,,$$

or, instead,

(96a) $$A(\Sigma^+\pi^-\bar{n}) \approx -\hat{P}\,,$$

(96b) $$A(\Sigma^-\pi^+\bar{n}) \approx \hat{S}\,,$$

(96c) $$A(\Sigma^+\pi^0\,\bar{p}) \approx \frac{\hat{P}+\hat{S}}{\sqrt{2}}\,.$$

We have chosen units such that $|A(\Sigma^-\pi^+\bar{n})|\equiv 1$. Either solution could of course be multiplied by -1, or by $\exp[i\alpha]$, with no physical consequences.

Solutions (95) and (96) demand $\alpha_0\approx +1$. This is in disagreement with the measured value [7], $\alpha_0 = +.78^{+.08}_{-.09}$. In Fig. 13 we reproduce the diagram from TRIPP *et al* [8]. (Their notation N^+, N^-, and N_0 corresponds to our $A(\Sigma^+\pi^-\bar{n})$ $A(\Sigma^-\pi^+\bar{n})$, and $A(\Sigma^+\pi^0\bar{p})$. Their sign convention for α is opposite to ours.) The two possibilities for N_0 correspond to the two possibilities $\gamma_0>0$, and $\gamma_0<0$, *i.e.*, $|S|/|P|>1$ and <1. From the diagram and eq. (93) we find

$$\text{(97)} \qquad \frac{|A(3,3)|}{|A(\Sigma^-\pi^+\bar{n})|}\approx 0.23\pm 0.09 \qquad \text{or} \qquad 0.30\pm 0.10\,,$$

where the errors are only estimated from the diagram, and where the first possibility corresponds to $\gamma_0/\gamma_+>0$ and the second to $\gamma_0/\gamma_+<0$. In the diagram it is implicitly assumed that $\gamma_+/\gamma_-<0$; *i.e.*, that if $\Sigma^+\to n+\pi^+$ goes by S-wave, then $\Sigma^-\to n+\pi^-$ goes by P-wave, and vice versa. In other words it is assumed that the violation of the $\Delta I=\frac{1}{2}$ rule is small. Of course, if the $\Delta I=\frac{1}{2}$ rule does not hold, one can have $\gamma_+/\gamma_->0$; that is, both decays can go by S-wave or both by P-wave. We must have some reservations until γ_+, γ_-, and γ_0 are measured.

It is perhaps worth remarking that even if experiments finally tell us that, for example, $\Sigma^+\to n+\pi^+$ is pure P-wave, $\Sigma^-\to n+\pi^-$ is pure S-wave, and $\alpha_0=+1.0$ (instead of 0.78), then we still will not be able to rule out a large violation of the $\Delta I=\frac{1}{2}$ rule. For instance if in the example of eq. (95*a*) we replaced $-\hat{S}$ by $\hat{S}$, but left (95*b*) and (95*c*) as they are, we would obtain $\sqrt{\frac{15}{8}}A(3,3)=2\hat{S}$, instead of zero, as is seen from eq. (93). This type of ambiguity, and also the ambiguity corresponding to eq. (89), is not « inherent » but, as we see from eqs. (86) and (87), could be resolved if it were possible to measure the rates for $\Sigma^0\to p+\pi^-$ and $\Sigma^0\to n+\pi^0$.

4. – $K\to 3\pi$ and the $\Delta I=\frac{1}{2}$ rule.

In this lecture we consider the decays

$$K^+\to\pi^++\pi^++\pi^-\equiv(+\,+\,-)\,,$$

$$K^+\to\pi^++\pi^0+\pi^0\equiv(+\;0\;0)\,,$$

$$K_2^0\to\pi^++\pi^-+\pi^0\equiv(+\,-\,0)\,,$$

and

$$K_2^0\to\pi^0+\pi^0+\pi^0\equiv(0\;0\;0)\,.$$

The final (3π) state can have $I=0$, 1, 2, or 3. For K^+-decay we have $Q=+1$, so $I_3=+1$; therefore $I=0$ is excluded. Thus for $K^+\to 3\pi$ one has the pos-

sibilities $\Delta I = 1 \pm \frac{1}{2} = \frac{3}{2}$ or $\frac{1}{2}$, $2 \pm \frac{1}{2} = \frac{5}{2}$ or $\frac{3}{2}$, and $3 \pm \frac{1}{2} = \frac{7}{2}$ or $\frac{5}{2}$. For $K^0 \to 3\pi$ we shall see that K_1^0 goes to $I = 0$ or 2, and K_2^0 goes to $I = 1$ or 3. We consider only K_2^0-decay. Thus for $K_2^0 \to 3\pi$ one has the possibilities $\Delta I = 1 \pm \frac{1}{2} = \frac{3}{2}$ or $\frac{1}{2}$, and $3 \pm \frac{1}{2} = \frac{7}{2}$ or $\frac{5}{2}$.

Consider now the states $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$. Let $\boldsymbol{L}$ be the angular momentum of pion 1 relative to the c.m. of 2 and 3, and let $\boldsymbol{l}$ be the angular mentum of 2 and 3 in their c.m. Then $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{l}$. But $\boldsymbol{J} = 0$, since the spin of the K is zero. Therefore $|\boldsymbol{L}| = |\boldsymbol{l}|$. Therefore $P = (-1)^3(-1)^L(-1)^l = -1$ Thus for $K_1^0 \to 3\pi$, for which $CP = +1$, we have $C = -1$, and for $K_2^0 \to 3\pi$, we have $CP = -1$, and $C = +1$. (Here we are assuming CP invariance in the decay.) Since $3\pi^0$ obviously has $C = +1$, we see that $K_2^0 \to 3\pi^0$ is allowed, and $K_1^0 \to 3\pi^0$ is forbidden.

We next wish to show that for 3π, $I = 0$ and 2 have $C = -1$, and $I = 1$ and 3 have $C = +1$. There are several ways to show this. The easiest is to assume the theorem proven by Professor ROSENFELD in his accompanying lectures, namely

$$C = G(-1)^I .$$

For 3π we have $G = -1$, so we have $C = (-1)^{I+1}$.

Another way is to use the Clebsch-Gordan tables by brute force, as follows. The notation is (I, I_3).

$I = 0$.

We have from Table II,

$$\begin{aligned}(98)\qquad (0, 0)_{3\pi} = 1(\pi)\times 1(2\pi) &= \sqrt{\tfrac{1}{3}}\,\pi^+(1, -1)_{2\pi} - \sqrt{\tfrac{1}{3}}\,\pi^0(1, 0)_{\pi 2} + \\ &+ \sqrt{\tfrac{1}{3}}\,\pi^-(1, +1)_{3\pi} = \sqrt{\tfrac{1}{3}}\,\pi^+(\sqrt{\tfrac{1}{2}}\,\pi^0\pi^- - \sqrt{\tfrac{1}{2}}\,\pi^-\pi^0) - \\ &- \sqrt{\tfrac{1}{3}}\,\pi^0(\sqrt{\tfrac{1}{2}}\,\pi^+\pi^- - \sqrt{\tfrac{1}{2}}\,\pi^-\pi^+) + \sqrt{\tfrac{1}{3}}\,\pi^-(\sqrt{\tfrac{1}{2}}\,\pi^+\pi^0 - \sqrt{\tfrac{1}{2}}\,\pi^0\pi^+) .\end{aligned}$$

Under C, we have $\pi^0 \to \pi^0$, $\pi^+ \to \pi^-$, and $\pi^- \to \pi^+$, and by inspection of eq. (98) we see that $(0, 0)_{3\pi} \to -(0, 0)_{3\pi}$. That is, $C = -1$.

$I = 1$.

Here there are three possibilities. We have, first,

$$(99a)\qquad (1, 0)_{3\pi} = 1(\pi)\times 0(2\pi) = \pi^0(0, 0)_{2\pi} = \pi^0(\sqrt{\tfrac{1}{3}}\,\pi^+\pi^- - \sqrt{\tfrac{1}{3}}\,\pi^0\pi^0 + \sqrt{\tfrac{1}{3}}\,\pi^-\pi^+) ,$$

second,

$$\begin{aligned}(99b)\qquad (1, 0)_{3\pi} = 1(\pi)\times 1(2\pi) &= \sqrt{\tfrac{1}{2}}\,\pi^+(1, -1)_{2\pi} - \sqrt{\tfrac{1}{2}}\,\pi^-(1, +1)_{2\pi} = \\ &= \sqrt{\tfrac{1}{2}}\,\pi^+(\sqrt{\tfrac{1}{2}}\,\pi^0\pi^- - \sqrt{\tfrac{1}{2}}\,\pi^-\pi^0) - \sqrt{\tfrac{1}{2}}\,\pi_-(\sqrt{\tfrac{1}{2}}\,\pi^+\pi^0 - \sqrt{\tfrac{1}{2}}\,\pi^0\pi^+) ,\end{aligned}$$

and lastly,

(99c) $$(1,0)_{3\pi} = 1(\pi)\times 2(2\pi) =$$
$$= \sqrt{\frac{3}{10}}\,\pi^{+}(2,-1)_{2\pi} - \sqrt{\frac{2}{5}}\,\pi^{0}(2,0)_{2\pi} + \sqrt{\frac{3}{10}}\,\pi^{-}(2,+1)_{2\pi} =$$
$$= \sqrt{\frac{3}{10}}\,\pi^{+}\left(\sqrt{\frac{1}{2}}\,\pi^{0}\pi^{-} + \sqrt{\frac{1}{2}}\,\pi^{-}\pi^{0}\right) -$$
$$- \sqrt{\frac{2}{5}}\,\pi^{0}\left(\sqrt{\frac{1}{6}}\,\pi^{+}\pi^{-} + \sqrt{\frac{2}{3}}\,\pi^{0}\pi^{0} + \sqrt{\frac{1}{6}}\,\pi^{-}\pi^{+}\right) +$$
$$+ \sqrt{\frac{3}{10}}\,\pi^{-}\left(\sqrt{\frac{1}{2}}\,\pi^{+}\pi^{0} + \sqrt{\frac{1}{2}}\,\pi^{0}\pi^{+}\right).$$

We see that in all three cases, $C=+1$.

$I=2$.

There are two possibilities, first,

(100a) $$(2,0)_{3\pi} = 1(\pi)\times 1(2\pi) = \sqrt{\tfrac{1}{6}}\pi^{+}(1,-1)_{2\pi} + \sqrt{\tfrac{2}{3}}\pi^{0}(1,0)_{2\pi} +$$
$$+ \sqrt{\tfrac{1}{6}}\pi^{-}(1,+1) = \sqrt{\tfrac{1}{6}}\pi^{+}(\sqrt{\tfrac{1}{2}}\pi^{0}\pi^{-} - \sqrt{\tfrac{1}{2}}\pi^{-}\pi^{0}) +$$
$$+ \sqrt{\tfrac{2}{3}}\pi^{0}(\sqrt{\tfrac{1}{2}}\pi^{+}\pi^{-} - \sqrt{\tfrac{1}{2}}\pi^{-}\pi^{+}) + \sqrt{\tfrac{1}{6}}\pi^{-}(\sqrt{\tfrac{1}{2}}\pi^{+}\pi^{0} - \sqrt{\tfrac{1}{2}}\pi^{0}\pi^{+}),$$

and second,

(100b) $$(2,0)_{3\pi} = 1(\pi)\times 2(2\pi) = \sqrt{\tfrac{1}{2}}\pi^{-}(2,+1)_{2\pi} - \sqrt{\tfrac{1}{2}}\pi^{+}(2,-1)_{2\pi} =$$
$$= \sqrt{\tfrac{1}{2}}\pi^{-}(\sqrt{\tfrac{1}{2}}\pi^{+}\pi^{0} + \sqrt{\tfrac{1}{2}}\pi^{0}\pi^{+}) - \sqrt{\tfrac{1}{2}}\pi^{+}(\sqrt{\tfrac{1}{2}}\pi^{0}\pi^{-} + \sqrt{\tfrac{1}{2}}\pi^{-}\pi^{0}).$$

In both (100a) and (100b) we have $C=-1$.

$I=3$.

We have

(101) $$(3,0)_{3\pi} = 1(\pi)\times 2(2\pi) = \sqrt{\tfrac{1}{5}}\pi^{+}(2,-1)_{2\pi} + \sqrt{\tfrac{3}{5}}\pi^{0}(2,0)_{2\pi} +$$
$$+ \sqrt{\tfrac{1}{5}}\pi^{-}(2,+1)_{2\pi} = \sqrt{\tfrac{1}{5}}\pi^{+}(\sqrt{\tfrac{1}{2}}\pi^{0}\pi^{-} + \sqrt{\tfrac{1}{2}}\pi^{-}\pi^{0}) +$$
$$+ \sqrt{\tfrac{3}{5}}\pi^{0}(\sqrt{\tfrac{1}{6}}\pi^{+}\pi^{-} + \sqrt{\tfrac{2}{3}}\pi^{0}\pi^{0} + \sqrt{\tfrac{1}{6}}\pi^{-}\pi^{+}) + \sqrt{\tfrac{1}{5}}\pi^{-}(\sqrt{\tfrac{1}{2}}\pi^{+}\pi^{0} + \sqrt{\tfrac{1}{2}}\pi^{0}\pi^{+}),$$

for which $C=+1$.

In eqs. (98) through (101) we have exhibited the seven possible charge states for $\pi^{+}\pi^{-}\pi^{0}$ and $\pi^{0}\pi^{0}\pi^{0}$ and seen by inspection that we have $C=(-1)^{I+1}$. We notice as a check that $3\pi^{0}$ occurs only for $I=1$ or 3, *i.e.*, for $C=+1$.

We consider at first only the predictions of the $\Delta I=\frac{1}{2}$ rule. Then for

$K^+ \to 3\pi$ and $K_2^0 \to 3\pi$ we can have only the 3π states $(I, I_3) = (1, +1)$, and $(1, 0)$, respectively. There are three independent 3π states with $I=1$, as we saw from the combinations $1(3\pi) = 1(\pi) \times 0(2\pi)$, $1(\pi) \times 1(2\pi)$, or $1(\pi) \times 2(2\pi)$. We could use the Clebsch-Gordan table to construct these states, as was done for the $(1, 0)$ states in eqs. (99*a*, *b*, *c*). However, it is more convenient to use another approach. (The functions we obtain for $(1, 0)$ are linear combinations of those found in eqs. (99).)

We have the three individual pion wave amplitudes $\boldsymbol{\pi}_1$, $\boldsymbol{\pi}_2$, and $\boldsymbol{\pi}_3$, each of which transforms like a vector ($I=1$) in I-spin space. We want to form a probability amplitude for 3π. This must be trilinear in $\boldsymbol{\pi}_1$, $\boldsymbol{\pi}_2$, and $\boldsymbol{\pi}_3$. We want that combination that transforms like a vector in I-spin space. There are three such combinations, which are, most simply, $\boldsymbol{A} \equiv \boldsymbol{\pi}_1(\boldsymbol{\pi}_2 \cdot \boldsymbol{\pi}_3)$, $\boldsymbol{B} \equiv \boldsymbol{\pi}_2(\boldsymbol{\pi}_3 \cdot \boldsymbol{\pi}_1)$, and $\boldsymbol{C} \equiv \boldsymbol{\pi}_3(\boldsymbol{\pi}_1 \cdot \boldsymbol{\pi}_2)$. The most general vector is then

$$V = A\boldsymbol{A} + B\boldsymbol{B} + C\boldsymbol{C}\,, \tag{102}$$

where A, B, and C are complex numbers.

The meaning of, for instance, $\boldsymbol{\pi}_2 \cdot \boldsymbol{\pi}_3$, can be expressed in two ways (which unfortunately differ by a factor of -1). We can use the Clebsch-Gordan table to find that combination of $\boldsymbol{\pi}_2$ and $\boldsymbol{\pi}_3$ that transforms like a scalar. That is

$$(\boldsymbol{\pi}_2 \cdot \boldsymbol{\pi}_3) = (0, 0)_{2\pi} = 1(\pi_2) \times 1(\pi_3) = (\sqrt{\tfrac{1}{3}}\,\pi_2^+\pi_3^- - \sqrt{\tfrac{1}{3}}\,\pi_2^0\pi_3^0 + \sqrt{\tfrac{1}{3}}\,\pi_2^-\pi_3^+\,. \tag{103}$$

Or, instead, we can use the spherical harmonics in eq. (1) (with the addition of a normalization factor) and write

$$\begin{aligned}\boldsymbol{\pi}_2 \cdot \boldsymbol{\pi}_3 &= \pi_{x_2}\pi_{x_3} + \pi_{y_2}\pi_{y_3} + \pi_{z_2}\pi_{z_3} = \\ &= \left(\frac{\pi_x + i\pi_y}{\sqrt{2}}\right)_2 \left(\frac{\pi_x - i\pi_y}{\sqrt{2}}\right)_3 + \left(\frac{\pi_x - i\pi_y}{\sqrt{2}}\right)_2 \left(\frac{\pi_x + i\pi_y}{\sqrt{2}}\right)_3 + \pi_{z_2}\pi_{z_3} = \\ &= -\,Y_1^1(2)\,Y_1^{-1}(3) - Y_1^{-1}(2)\,Y_1^1(3) + Y_1^0(2)\,Y_1^0(3) = -\,\pi_2^+\pi_3^- - \pi_2^-\pi_3^+ + \pi_2^0\pi_3^0\,,\end{aligned}$$

which is the same as (103) except for a common factor. We use eq. (103).

We can take x, y, and z components of the vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$; or we can take $+$, $-$, and 0 « components », since these are just linear combinations of the x, y, and z components. Thus

$$A^+ = \pi_1^+(\boldsymbol{\pi}_2 \cdot \boldsymbol{\pi}_3) = \pi_1^+ \frac{(\pi_2^+\pi_3^- + \pi_2^-\pi_3^+ - \pi_2^0\pi_3^0)}{\sqrt{3}}\,, \tag{104}$$

$$A^0 = \pi_1^0(\boldsymbol{\pi}_2 \cdot \boldsymbol{\pi}_3) = \pi_1^0 \frac{(\pi_2^+\pi_3^- + \pi_2^-\pi_3^+ - \pi_2^0\pi_3^0)}{\sqrt{3}}\,. \tag{105}$$

Instead of $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ we could take as our independent states the linear

combinations

$$(106)\qquad \boldsymbol{S} \equiv \boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C}=\boldsymbol{\pi}_1(\boldsymbol{\pi}_2\cdot\boldsymbol{\pi}_3)+\boldsymbol{\pi}_2(\boldsymbol{\pi}_3\cdot\boldsymbol{\pi}_1)+\boldsymbol{\pi}_3(\boldsymbol{\pi}_1\cdot\boldsymbol{\pi}_2)\,,$$

$$\boldsymbol{M}_1 \equiv \boldsymbol{B}-\boldsymbol{C}=\boldsymbol{\pi}_1\times(\boldsymbol{\pi}_2\times\boldsymbol{\pi}_3)\,,$$

and

$$\boldsymbol{M}_2 \equiv -\boldsymbol{A}+\boldsymbol{B}=(\boldsymbol{\pi}_1\times\boldsymbol{\pi}_2)\times\boldsymbol{\pi}_3\,.$$

The combination $\boldsymbol{S}$ is completely symmetric with respect to interchange of any two pions. The functions $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ have mixed symmetries. (For instance $\boldsymbol{M}_1$ is antisymmetric under interchange of 2 and 3 but has no other symmetry.)

We return to the general expression, eq. (102). We first write out the expression completely. Then we rearrange the terms so that π_1, π_2, π_3 always occur in order. We can then drop the subscripts 1, 2, and 3. For instance,

$$\pi_2^+\pi_1^-\pi_3^0=\pi_1^-\pi_2^+\pi_3^0\equiv\pi^-\pi^+\pi^0\equiv(-+0)\,.$$

Thus we have

$$V\equiv AA+BB+CC=A\boldsymbol{\pi}_1(\boldsymbol{\pi}_2\cdot\boldsymbol{\pi}_3)+B\boldsymbol{\pi}_2(\boldsymbol{\pi}_3\cdot\boldsymbol{\pi}_1)+C\boldsymbol{\pi}_3(\boldsymbol{\pi}_1\cdot\boldsymbol{\pi}_2)=$$
$$=\frac{A}{\sqrt{3}}\,\boldsymbol{\pi}(\pi^+\pi^-+\pi^-\pi^+-\pi^0\,\pi^0)+\frac{B}{\sqrt{3}}\,(\pi^+\boldsymbol{\pi}\pi^-+\pi^-\boldsymbol{\pi}\pi^+-\pi^0\boldsymbol{\pi}\pi^0)+$$
$$+\frac{C}{\sqrt{3}}\,(\pi^+\pi^-\boldsymbol{\pi}+\pi^-\pi^+\boldsymbol{\pi}-\pi^0\pi^0\,\boldsymbol{\pi})\,.$$

Taking components, we find

$$(107)\qquad (1,\,+1)_{3\pi}\equiv V^+=\sqrt{\tfrac{1}{3}}\,\{A[(++-)+(+-+)-(+00)]+$$
$$+B[(++-)+(-++)-(0+0)]+C[(+-+)+(-++)-(00+)]\}=$$
$$=\sqrt{\tfrac{1}{3}}\,\{(A+B)(++-)+(B+C)(-++)+(C+A)(+-+)-$$
$$-A(+00)-B(0+0)-C(00+)\}\,,$$

$$(108)\qquad (1,\,0)_{3\pi}\equiv V^0=\sqrt{\tfrac{1}{3}}\,\{A[(0+-)+(0-+)-(000)]+$$
$$+B[(+0-)+(-0+)-(000)]+C[(+-0)+(-+0)-(000)]\}=$$
$$=\sqrt{\tfrac{1}{3}}\,\{A[(0+-)+(0-+)]+B[(+0-)+(-0+)]+$$
$$+C[(+-0)+(-+0)]-(A+B+C)(000)\}\,.$$

We now turn to the predictions of the $\Delta I=\frac{1}{2}$ rule. We have, using our

usual notation (I, I_3), and the spurion $s(\Delta I, \Delta I_3)$,

$$K^+ \to (3\pi) + s\,,$$

i.e.,

$$(\tfrac{1}{2}, +\tfrac{1}{2}) \to (1, +1) + s(\tfrac{1}{2}, -\tfrac{1}{2})\,,$$

and

$$K^0 \to (3\pi) + s\,,$$

i.e.,

$$(\tfrac{1}{2}, -\tfrac{1}{2}) \to (1, 0) + s(\tfrac{1}{2}, -\tfrac{1}{2})\,.$$

Or, transposing both s and K, we have, from Table I,

(109) $$s(\tfrac{1}{2}, +\tfrac{1}{2}) = \tfrac{1}{2}(\bar{K}) \times 1(3\pi) = \\ = \sqrt{\tfrac{2}{3}}\,(\tfrac{1}{2}, -\tfrac{1}{2})(1, +1) - \sqrt{\tfrac{1}{3}}\,(\tfrac{1}{2}, +\tfrac{1}{2})(1, 0) = \sqrt{\tfrac{2}{3}}\,K^- V^+ - \sqrt{\tfrac{1}{3}}\,\bar{K}^0 V^0\,,$$

where V^+ and V^0 are given by (107) and (108).

In the term $\bar{K}^0 V^0$ we have contributions like $\bar{K}^0(0+-)$. This represents, after transposing, $K^0 \to \pi_1^0 + \pi_2^+ + \pi_3^-$. We are actually interested in $K_2^0 \to 3\pi$, rather than in $K^0 \to 3\pi$. Because of the relation $K^0 = (K_1^0 + K_2^0)/\sqrt{2}$, a pure K^0 beam is, in terms of intensities, half K_1^0 and half K_2^0. Only the K_2^0 half of the beam contributes to $K^0 \to 3\pi$ in the $I=1$ state. Therefore a pure K_2^0 beam would give twice the decay rate of a pure K^0 beam, into $I=1$. In terms of amplitudes we should therefore multiply the K^0-decay amplitude by $\pm\sqrt{2}$ to get the K_2^0-decay amplitude. (The choice of sign is arbitrary, since charge conservation prevents interference between K_2^0 and K^+-decay; the relative phase of K_2^0 and K^+ has no physical consequence.)

Finally we can write the decay rates, remembering that, for instance, $(++-)$ and $(-++)$ are distinguishable and do not interfere. After including a factor of 2 for K_2^0-decay, as discussed above, we have, from eqs. (109), (107), and (108),

(110) $$R(K^+ \to + + -) = \tfrac{2}{3} \cdot \tfrac{1}{3} \cdot \{|A+B|^2 + |B+C|^2 + |C+A|^2\}\,,$$

(111) $$R(K^+ \to +\ 0\ 0) = \tfrac{2}{3} \cdot \tfrac{1}{3} \cdot \{|A|^2 + |B|^2 + |C|^2\}\,,$$

(112) $$R(K_2^0 \to +-\ 0) = 2 \cdot \tfrac{1}{3} \cdot \tfrac{1}{3}\,\{2|A|^2 + 2|B|^2 + 2|C|^2\}\,,$$

(113) $$R(K_2^0 \to 0\ 0\ 0) = 2 \cdot \tfrac{1}{3} \cdot \tfrac{1}{3}\,\{|A+B+C|^2\}\,.$$

These equations contain the predictions of the $\Delta I = \tfrac{1}{2}$ rule. We can think of A, B, and C as functions of the momenta in the decay. Then the equations refer to a given configuration. (We consider only the rates and not the spectra. See Rosenfeld's notes for spectral considerations.)

From eqs. (110) through (113) we find

$$R(\mathrm{K}_2^0 \to +-0) = 2R(\mathrm{K}^+ \to +00)\,, \tag{114}$$

and

$$R(\mathrm{K}^+ \to ++-) - R(\mathrm{K}^+ \to +00) = R(\mathrm{K}_2^0 \to 000)\,. \tag{115}$$

Equations (114) and (115) hold for *any* choice of A, B, and C; in other words for any admixture of the symmetric $I=1$ state $\boldsymbol{S}$, given by $A=B=C$, and the mixed symmetry states $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$. These two equations give the best tests for the $\Delta I=\frac{1}{2}$ rule [9].

The symmetric $I=1$ state $\boldsymbol{S}$ plays a dominant role, empirically, as we shall see. We therefore write down the predictions for this state. Taking $A=B=C$ in eqs. (110) and (111), and then in eqs. (112) and (113), we obtain

$$R(\mathrm{K}^+ \to ++-) = 4R(\mathrm{K}^+ \to +0\ 0)\,, \tag{116}$$

$$R(\mathrm{K}_2^0 \to 0\ 0\ 0) = \tfrac{3}{2}R(\mathrm{K}_2^0 \to +-0)\,. \tag{117}$$

Notice that if the $\Delta I=\frac{1}{2}$ rule holds then $I=1$ is the only allowed 3π state. However, $I=1$ *can* be reached through either $\Delta I\ \frac{1}{2}$ or $\Delta I=\frac{3}{2}$. Equations (116) and (117) hold only for the symmetric $I=1$ state. We will find they are well satisfied experimentally; but of course this has not much bearing on the $\Delta I=\frac{1}{2}$ rule, since $\Delta I=\frac{3}{2}$ can reach this state. On the other hand, eqs. (114) and (115), which relate the charged and neutral decays, depend directly on the $\Delta I=\frac{1}{2}$ rule through the spurion eq. (109), and do not hold when $\Delta I=\frac{3}{2}$ is present.

Before giving the predictions when $\Delta I=\frac{3}{2}$ is included, we turn to the experiments. We include phase-space factors and will indicate their insertion by a double-stemmed arrow, $\Rightarrow$. From (116) we have, for the state $\boldsymbol{S}$,

$$\frac{R(\mathrm{K}^+ \to ++-)}{R(\mathrm{K}^+ \to +\ 0\ 0)} = 0.25 \Rightarrow 0.311\,. \tag{118}$$

Recent experimental values are summarized in ref. [10], and average to 0.298 ± 0.025. The agreement with (118) is excellent. We conclude that the symmetric $I=1$ state ($\boldsymbol{S}$) is important.

From (117) we expect, for $\boldsymbol{S}$,

$$\frac{R(\mathrm{K}_2^0 \to +-0)}{R(\mathrm{K}_2^0 \to 0\ 0\ 0)} = 1.5 \Rightarrow 1.82\,. \tag{119}$$

Results from Dubna presented at Geneva (1962) by ANIKINA *et al.* [2] give

$R(K_2^0 \to 000)/R(K_2^0 \to \text{all charged}) = 0.38 \pm 0.07$. LUERS *et al.* [11] have obtained

$$R(K_2^0 \to +-0)/R(K_2^0 \to \text{all charged}) = 0.134 \pm 0.018 \,. \tag{120}$$

Combining these two results, we obtain $R(K_2^0 \to 000)/R(K_2^0 \to +-0) = 2.83 \pm 0.52$. This result is in only fair agreement with eq. (119). On the other hand, the disagreement amounts to only two standard deviations, and does not shake our faith in the dominance of the state S.

We now turn to the prediction (114) of the $\Delta I = \frac{1}{2}$ rule. ALEXANDER *et al.* [10] have measured an absolute decay rate for $K_2^0 \to \pi^\pm + L^\mp + \nu$. When this is combined with the branching ratio (120) of LUERS *et al.*, they find

$$R(K_2^0 \to +-0) = (1.44 \pm 0.43) \times 10^6 \ \text{s}^{-1} \,. \tag{121}$$

This is to be compared with [12]

$$2R(K^+ \to +\ 0\ 0) = (2.78 \pm 0.22) \times 10^6 \ \text{s}^{-1} \,. \tag{122}$$

The agreement with (114) is very poor. ALEXANDER *et al.* quote 100/1 statistical odds against agreement.

We are thus motivated to look at the more complicated formulas that result when the $\Delta I = \frac{1}{2}$ rule does not hold. We must also ask whether it is reasonable to expect that the presence of $\Delta I = \frac{3}{2}$ could preserve the beautiful agreement of (116) with experiment and still give the expected disagreement with (114). The point here of course is that once we allow $\Delta I = \frac{3}{2}$ then we must allow $I = 2$ (for 3π) in $K^+ \to 3\pi$, as well as $I = 1$, and (116) should presumably not hold.

We now give up the $\Delta I = \frac{1}{2}$ rule and allow $\Delta I = \frac{3}{2}$. We still omit $\Delta I = \frac{5}{2}$ and $\frac{7}{2}$. (They will be included later!)

With $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ we can reach $I = 1$ and 2 in $K^+ \to 3\pi$, and $I = 1$ for $K_2^0 \to 3\pi$. (CP invariance rules out $I = 2$ for $K_2^0 \to 3\pi$.) The relation between K^+ and K_2^0 decay for $\Delta I = \frac{3}{2}$ going to $I = 1$ is given by a spurion equation similar to eq. (109), namely, from Table I,

$$s(\tfrac{3}{2}, +\tfrac{1}{2}) = \sqrt{\tfrac{1}{3}}\, K^- V^+ + \sqrt{\tfrac{2}{3}}\, \bar{K}^0 V^0 \,. \tag{123}$$

In the arbitrary constants A, B, and C in V^+ and V^0 we use subscript 1 for $\Delta I = \frac{1}{2}$ and subscript 3 for $\Delta I = \frac{3}{2}$. We have, then,

$$\boldsymbol{V}_1 = A_1 \boldsymbol{A} + B_1 \boldsymbol{B} + C_1 \boldsymbol{C} \,, \tag{124}$$

$$\boldsymbol{V}_3 = A_3 \boldsymbol{A} + B_3 \boldsymbol{B} + C_3 \boldsymbol{C} \,. \tag{125}$$

Thus for the $I = 1$ part of the wave function we have, using (109) with (124),

and (123) with (125),

$$\psi(I=1) = s(\tfrac{1}{2},\tfrac{1}{2}) + s(\tfrac{3}{2},\tfrac{1}{2}) = K^-[\sqrt{\tfrac{2}{3}}\,V_1^+ + \sqrt{\tfrac{1}{3}}\,V_3^+] + \bar{K}^0[-\sqrt{\tfrac{1}{3}}\,V_1^0 + \sqrt{\tfrac{2}{3}}\,V_3^0] =$$
$$= \sqrt{\tfrac{1}{3}}\,K^-\{(\sqrt{2}\,A_1 + A_3)\,A^+ + (\sqrt{2}\,B_1 + B_3)\,B^+ + (\sqrt{2}\,C_1 + C_3)\,C^+\} +$$
$$+ \sqrt{\tfrac{1}{3}}\,\bar{K}^0\{(-A_1 + \sqrt{2}\,A_3)\,A^0 + (-B_1 + \sqrt{2}\,B_3)B^0 + (-C_1 + \sqrt{2}\,C_3)\,C^0\}\,.$$

But

$$A^+ = \sqrt{\tfrac{1}{3}}[(\pi^+)\pi^+\pi^- + (\pi^+)\pi^-\pi^+ - (\pi^+)\pi^0\pi^0]\,,$$
$$B^+ = \sqrt{\tfrac{1}{3}}[\pi^+(\pi^+)\pi^- + \pi^-(\pi^+)\pi^+ - \pi^0(\pi^+)\pi^0]\,,$$
$$C^+ = \sqrt{\tfrac{1}{3}}[\pi^+\pi^-(\pi^+) + \pi^-\pi^+(\pi^+) - \pi^0\pi^0(\pi^+)]\,,$$
$$A^0 = \sqrt{\tfrac{1}{3}}[(\pi^0)\pi^+\pi^- + (\pi^0)\pi^-\pi^+ - (\pi^0)\pi^0\pi^0]\,,$$
$$B^0 = \sqrt{\tfrac{1}{3}}[\pi^+(\pi^0)\pi^- + \pi^-(\pi^0)\pi^+ - \pi^0(\pi^0)\pi^0]\,,$$
$$C^0 = \sqrt{\tfrac{1}{3}}[\pi^+\pi^-(\pi^0) + \pi^-\pi^+(\pi^0) - \pi^0\pi^0(\pi^0)]\,.$$

Combining these, we find

(126) $$\psi(I=1) = \tfrac{1}{3}K^-\{(\sqrt{2}\,A_1 + A_3 + \sqrt{2}\,B_1 + B_3)(++-) +$$
$$+ (\sqrt{2}\,B_1 + B_3 + \sqrt{2}\,C_1 + C_3)(-++) + (\sqrt{2}\,C_1 + C_3 + \sqrt{2}A_1 + A_3)(+-+) -$$
$$- (\sqrt{2}\,A_1 + A_3)(+00) - (\sqrt{2}\,B_1 + B_3)(0+0) - (\sqrt{2}\,C_1 + C_3)(00+)\} +$$
$$+ \tfrac{1}{3}\bar{K}^0\{(-A_1 + \sqrt{2}\,A_3)[(0+-) + (0-+)] +$$
$$+ (-B_1 + \sqrt{2}\,B_3)[(+0-) + (-0+)] + (-C_1 + \sqrt{2}\,C_3)[(+-0) + (-+0)] +$$
$$+ (A_1 - \sqrt{2}\,A_3 + B_1 - \sqrt{2}\,B_3 + C_1 - \sqrt{2}\,C_3)(000)\}\,.$$

We still need the $I=2$ wave function for 3π, for $K^+ \to 3\pi$. There are two possibilities, which we label with subscripts D and E. We have, from Table II,

(127) $$\psi_D(2,+1) \equiv 1(\pi)\times 1(2\pi) = \sqrt{\tfrac{1}{2}}\pi^+(1,0)_{2\pi} + \sqrt{\tfrac{1}{2}}\pi^0(1,+1)_{2\pi} =$$
$$= \sqrt{\tfrac{1}{2}}\pi^+(\sqrt{\tfrac{1}{2}}\pi^+\pi^- - \sqrt{\tfrac{1}{2}}\pi^-\pi^+) + \sqrt{\tfrac{1}{2}}\pi^0(\sqrt{\tfrac{1}{2}}\pi^+\pi^0 - \sqrt{\tfrac{1}{2}}\pi^0\pi^+) =$$
$$= \tfrac{1}{2}[(++-) - (+-+) + (0+0) - (00+)]\,,$$

and from Table VI (and Table II)

(128) $$\psi_E(2,+1) \equiv 1(\pi)\times 2(2\pi) = \sqrt{\tfrac{1}{3}}\pi^-(2,+2)_{2\pi} +$$
$$+ \sqrt{\tfrac{1}{6}}\pi^0(2,+1_{2\pi}) - \sqrt{\tfrac{1}{2}}\pi^+(2,0)_{2\pi} = \sqrt{\tfrac{1}{3}}\pi^-\pi^+\pi^+ + \sqrt{\tfrac{1}{6}}\pi^0[\sqrt{\tfrac{1}{2}}\pi^+\pi^0 + \sqrt{\tfrac{1}{2}}\pi^0\pi^+] -$$
$$- \sqrt{\tfrac{1}{2}}\pi^+[\sqrt{\tfrac{1}{6}}\pi^+\pi^- + \sqrt{\tfrac{2}{3}}\pi^0\pi^0 + \sqrt{\tfrac{1}{6}}\pi^-\pi^+] =$$
$$= \sqrt{\tfrac{1}{3}}\{(-++) - \tfrac{1}{2}[(++-) + (+-+)] + \tfrac{1}{2}[(0+0) + (00+)] - (+00)\}\,.$$

For the general case (for $I=2$) we have the superposition

(129) $$\psi(2, +1) = D\psi_D + E\psi_E ,$$

where D and E are complex numbers, and ψ_D and ψ_E are given by (127) and (128).

The corresponding spurion equation is not really needed, since we have only K^+ decay into $I=2$, and thus no coefficients relating K^+ and K_2^0 decay. However, for uniformity of notation we include the spurion. We have, from Table IV,

(130) $$s(\tfrac{3}{2}, +\tfrac{1}{2}) = \tfrac{1}{2}(\bar{K})\times 2(3\pi) = \sqrt{\tfrac{3}{5}}\,K^-(2, +1)_{3\pi} - \sqrt{\tfrac{2}{5}}\,\bar{K}^0(2, 0)_{3\pi} .$$

The term involving $\bar{K}^0$ corresponds to K_1^0 decay and is of no interest to us here. Omitting this term, and using (129), (127), and (128), we have

(131) $$\begin{aligned}\psi(I=2) &= \sqrt{\frac{3}{5}}\,K^-\{D\psi_D + E\psi_E\} = \\ &= \sqrt{\frac{3}{5}}\,K^-\left\{\left(\frac{1}{2}D - \sqrt{\frac{1}{12}}E\right)[(++-)-(00+)] - \right.\\ &\left. -\left(\frac{1}{2}D + \sqrt{\frac{1}{12}}E\right)[(+-+)-(0+0)] + \sqrt{\frac{1}{3}}E[(-++)-(+00)]\right\}.\end{aligned}$$

Finally we combine (131) and (126) to write

$$\psi = \psi(I=1) + \psi(I=2) .$$

We can now pick out the coefficients for $(++-)$, etc., and write the intensities. From (131), (126), and the above equation, and including the usual factor of 2 for K_2^0 decay, we have

(132) $$\begin{aligned}R(K^+ \to ++-) = {}&|\tfrac{1}{3}[\sqrt{2}(A_1+B_1) + (A_3+B_3)] + \\ &+ \tfrac{1}{2}\sqrt{\tfrac{1}{5}}(\sqrt{3}D - E)|^2 + |\tfrac{1}{3}[\sqrt{2}(B_1+C_1) + (B_3+C_3)] + \\ &+ \sqrt{\tfrac{1}{5}}E|^2 + |\tfrac{1}{3}[\sqrt{2}(C_1+A_1) + (C_3+A_3)] - \tfrac{1}{2}\sqrt{\tfrac{1}{5}}(\sqrt{3}D+E)|^2 ,\end{aligned}$$

(133) $$\begin{aligned}R(K^+ \to +00) = {}&|\tfrac{1}{3}[-\sqrt{2}A_1 - A_3] - \sqrt{\tfrac{1}{5}}E|^2 + |\tfrac{1}{3}[-\sqrt{2}B_1 - B_3] + \\ &+ \tfrac{1}{2}\sqrt{\tfrac{1}{5}}(\sqrt{3}D+E)|^2 + |\tfrac{1}{3}[-\sqrt{2}C_1 - C_3] - \tfrac{1}{2}\sqrt{\tfrac{1}{5}}(\sqrt{3}D - E)|^2 ,\end{aligned}$$

(134) $$\begin{aligned}R(K_2^0 \to +-0) = 2\,\{&|\tfrac{1}{3}(-A_1 + 2A_3)|^2(1^2+1^2) + \\ &+ |\tfrac{1}{3}(-B_1+\sqrt{2}B_3)|^2(1^2+1^2) + |\tfrac{1}{3}(-C_1+\sqrt{2}C_3)|^2(1^2+1^2)\},\end{aligned}$$

and

$$(135)\qquad R(K_2^0\to 000)=2\{|\tfrac{1}{3}[A_1+B_1+C_1-\sqrt{2}(A_3+B_3+C_3)]|^2\}.$$

Equations (132) through (135) are completely general for $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$. As a check we see that if we turn off the $\Delta I=\frac{3}{2}$ decay, *i.e.*, set $0=A_3=B_3==C_3=D=E$, we then get back our original eqs. (110) through (113).

In order to simplify the equations, we now make two assumptions. (We will later be able to verify that these were good assumptions.)

Assumption I. Assume that $\Delta I=\frac{1}{2}$ dominates. That is, neglect quadratic terms in A_3, B_3, C_3, D, and E, but keep linear terms in these quantities.

Assumption II. Assume that the dominant terms—that is $\Delta I=\frac{1}{2}$—go completely to the symmetric $I=1$ state, but that $\Delta I=\frac{3}{2}$ is completely free in this respect.

Assumption II is motivated by the good agreement of (116) with experiment. According to assumption II we have

$$(136)\qquad A_1=B_1=C_1.$$

We choose units so that

$$A_1\equiv 1.$$

Next, expand eqs. (132) through (135), dropping the quadratic terms according to assumption I. It is easy to see by inspection of (132) and (133) that if $A_1=B_1=C_1=1$, then the *linear* terms in D and E cancel identically, and in addition the linear terms in A_3, B_3, and C_3 occur only in the combination

$$(137)\qquad A_3+B_3+C_3\equiv 3\alpha_3.$$

We thus find (neglecting quadratic terms),

$$(138)\qquad R(K^+\to ++-)=\tfrac{1}{9}[24+24\sqrt{2}\,\mathrm{Re}\,\alpha_3],$$

$$(139)\qquad R(K^+\to +00)=\tfrac{1}{9}[\;6+\;6\sqrt{2}\,\mathrm{Re}\,\alpha_3],$$

$$(140)\qquad R(K_2^0\to +-0)=\tfrac{1}{9}[12-24\sqrt{2}\,\mathrm{Re}\,\alpha_3],$$

and

$$(141)\qquad R(K_2^0\to 000)=\tfrac{1}{9}[18-36\sqrt{2}\,\mathrm{Re}\,\alpha_3].$$

We see that we have

$$R(K^+\to ++-)=4R(K^+\to +00),$$

and

$$R(K_2^0 \to 000) = \tfrac{3}{2} R(K \to +-0);$$

i.e., eqs. (116) and (117) still hold! However, eq. (114) does *not* hold. We thus see that there is no incompatibility between the good agreement of experiment with (116), and the poor agreement of the experimental results (121) and (122) with (114), provided $\Delta I = \frac{3}{2}$ is present.

We must still verify that the $\Delta I = \frac{3}{2}$ terms are small, to justify our neglect of quadratic terms. From eqs. (139) and (140) we have

$$\text{(142)} \qquad \begin{cases} \sqrt{2}\,\mathrm{Re}\,\alpha_3 = \dfrac{2R(K^+ \to +00) - R(K_2^0 \to +-0)}{4R(K^+ \to +00) + R(K_2^0 \to +-0)}\,, \\[2ex] \qquad \Rightarrow \dfrac{2R(K^+ \to +00) - 0.97\,R(K_2^0 \to +-0)}{4R(K^+ \to +00) + 0.97\,R(K_2^0 \to +-0)}\,. \end{cases}$$

Putting in the experimental values from (121) and (122) we find

$$\text{(143)} \qquad \mathrm{Re}\,\alpha_3 = +\,0.136 \pm 0.053\,.$$

This is to be compared with $A_1 \equiv 1$.

We conclude from (143) that the neglect of quadratic terms is justified. Furthermore, we see that the amount of $\Delta I = \frac{3}{2}$ needed to satisfy the experiments is small. In fact, by comparison of eq. (143) with eq. (55) we see that the ratio of the amplitude for $\Delta I = \frac{3}{2}$ to that for $\frac{1}{2}$ that is required in $K \to 3\pi$ is about the « same » as that required in $K \to 2\pi$ to explain the existence of $K^+ \to \pi^+ + \pi^0$. Thus the ratio of the experimental results (121) and (122) is not actually in disagreement with the $\Delta I = \frac{1}{2}$ rule, but rather is « expected », from the well-known inexactness of the rule.

We now turn to the question of the possible presence of $\Delta I = \frac{5}{2}$ and $\frac{7}{2}$. The $\Delta I = \frac{5}{2}$ decay can lead to 3π states with $I = 2$ or $I = 3$. The $I = 2$ state cannot be reached by K_2^0 but only by K_1^0. Therefore the $\Delta I = \frac{5}{2}$ spurion equation relating K^+ and K_0 [analogous to the $\Delta I = \frac{3}{2}$ eq. (130)] is of no interest for $I = 2$. We need only the K^+ amplitude. Aside from normalization, we get the same answer as when we considered $I = 2$ in $K^+ \to 3\pi$ via $\Delta I = \frac{3}{2}$. There we found that if the $I = 2$ amplitude is small (compared to the $I = 1$ amplitude from $\Delta I = \frac{1}{2}$), so that quadratic terms are negligible, and if the $\Delta I = \frac{1}{2}$ amplitude goes to the *symmetric* $I = 1$ state, then the counting rates are not affected. This conclusion still holds. (This means that $I = 2$ final states are difficult to detect.)

The $\Delta I = \frac{7}{2}$ decay can lead only to the 3π state with $I = 3$. Thus we need consider only $I = 3$, from $\Delta I = \frac{5}{2}$ and $\frac{7}{2}$ transitions. Both K_2^0 and K^+ can go to $I = 3$, so that the spurion relations are important. These are given by

Table IV. We find

$$(144)\qquad s(\tfrac{5}{2}, +\tfrac{1}{2}) = \tfrac{1}{2}(\bar{K})\times 3(3\pi) = \sqrt{\tfrac{4}{7}}\,K^-(3, 1)_{3\pi} - \sqrt{\tfrac{3}{7}}\,\bar{K}^0(3, 0)_{3\pi}\,,$$

and

$$(145)\qquad s(\tfrac{7}{2}, +\tfrac{1}{2}) = \tfrac{1}{2}(\bar{K})\times 3(3\pi) = \sqrt{\tfrac{3}{7}}\,K^-(3, 1)_{3\pi} + \sqrt{\tfrac{4}{7}}\,\bar{K}^0(3, 0)_{3\pi}\,.$$

We find the 3π states in the usual way. There is only one state with $I=3$, given by $1(\pi)\times 2(2\pi)$. From Table VI (and Table II),

$$(146)\qquad (3, +1)_{\pi 3} = \sqrt{\frac{1}{15}}\,\pi^-(2, 2)_{2\pi} + \sqrt{\frac{8}{15}}\,\pi^0(2, 1)_{2\pi} + \sqrt{\frac{6}{15}}\,\pi^+(2, 0)_{2\pi} =$$

$$= \sqrt{\frac{1}{15}}\,\pi^-\pi^+\pi^+ + \sqrt{\frac{8}{15}}\,\pi^0\left\{\sqrt{\frac{1}{2}}\,\pi^+\pi^0 + \sqrt{\frac{1}{2}}\,\pi^0\pi^+\right\} +$$

$$+ \sqrt{\frac{6}{15}}\,\pi^+\left\{\sqrt{\frac{1}{6}}\,\pi^+\pi^- + \sqrt{\frac{2}{3}}\,\pi^0\pi^0 + \sqrt{\frac{1}{6}}\,\pi^-\pi^+\right\} =$$

$$= \sqrt{\frac{1}{15}}\,\{(-++) + (+-+) + (++-)\} + \sqrt{\frac{4}{15}}\,\{(+00) + (0+0) + (00+)\}\,.$$

$$(147)\qquad (3, 0)_{3\pi} = \sqrt{\frac{1}{5}}\,\pi^+(2, -1)_{2\pi} + \sqrt{\frac{3}{5}}\,\pi^0(2, 0)_{2\pi} + \sqrt{\frac{1}{5}}\,\pi^-(2, 1)_{2\pi} =$$

$$= \sqrt{\frac{1}{5}}\,\pi^+\left\{\sqrt{\frac{1}{2}}\,\pi^0\pi^- + \sqrt{\frac{1}{2}}\,\pi^-\pi^0\right\} + \sqrt{\frac{3}{5}}\,\pi^0\left\{\sqrt{\frac{1}{6}}\,\pi^+\pi^- + \sqrt{\frac{4}{6}}\,\pi^0\pi^0 + \sqrt{\frac{1}{6}}\,\pi^-\pi^+\right\} +$$

$$+ \sqrt{\frac{3}{5}}\,\pi^-\left\{\sqrt{\frac{1}{2}}\,\pi^+\pi^0 + \sqrt{\frac{1}{2}}\,\pi^0\pi^+\right\} =$$

$$= \sqrt{\frac{1}{10}}\,\{(+0-) + (+-0) + (0+-) + (0-+) + (-+0) + (-0+)\} +$$

$$+ \sqrt{\frac{4}{10}}\,(000)\,.$$

We now associate the complex numbers F_5 and G_7 with $s(\frac{5}{2}, \frac{1}{2})$ and $s(\frac{7}{2}, \frac{1}{2})$, and write, for ψ, (omitting $\Delta I = \frac{3}{2}$),

$$\psi = s(\tfrac{1}{2}, \tfrac{1}{2}) + F_5 s(\tfrac{5}{2}, \tfrac{1}{2}) + G_7\, s(\tfrac{5}{2}, \tfrac{1}{2}) = \sqrt{\tfrac{2}{3}}\,K^- V_1^+ - \sqrt{\tfrac{1}{3}}\,\bar{K}^0 V_1^0 +$$

$$+ F_5\left\{\sqrt{\tfrac{4}{7}}\,K^-(3, 1)_{3\pi} - \sqrt{\tfrac{3}{7}}\,\bar{K}^0(3, 0)_{3\pi}\right\} + G_7\left\{\sqrt{\tfrac{3}{7}}\,K^-(3, 1)_{3\pi} + \sqrt{\tfrac{4}{7}}\,\bar{K}^0(3, 0)_{3\pi}\right\}.$$

For V_1^+ and V_1^0 we use eqs. (107) and (108), with $A = B = C = 1$ (assumption that symmetric $I=1$ dominates for $\Delta I = \frac{1}{2}$). Using eqs. (146) and (147) and

collecting common terms, we have

$$\begin{aligned}(148)\qquad \psi = K^-\Bigg\{&\left(\frac{2\sqrt{2}}{3}+\sqrt{\frac{4}{105}}\,F_5+\sqrt{\frac{3}{105}}\,G_7\right)[(++-)+(+-+)+\\ &+(-++)]+\left(-\frac{\sqrt{2}}{3}+2\sqrt{\frac{4}{105}}\,F_5+2\sqrt{\frac{3}{105}}\,G_7\right)[(+00)+\\ &+(0+0)+(00+)]\Bigg\}-\overline{K}^0\Bigg\{\left(\frac{1}{3}+\sqrt{\frac{3}{70}}\,F_5-\sqrt{\frac{4}{70}}\,G_7\right)[(0+-)+\\ &+(0-+)+(+0-)+(-0+)+(+-0)+(-+0)]+\\ &+\left(-1+2\sqrt{\frac{3}{70}}\,F_5-2\sqrt{\frac{4}{70}}\,G_7\right)(000)\Bigg\}.\end{aligned}$$

Finally we can write down the counting rates, keeping only linear terms in F_5 and G_5. We now include the terms due to $\Delta I=\frac{3}{2}$, as appearing in eqs. (138) through (141). We thus obtain from (148) the relative counting rates

$$(149)\qquad R(K^+\to ++-) = \frac{1}{9}\left[24+24\sqrt{2}\,\mathrm{Re}\,\alpha_3+36\sqrt{2}\,\mathrm{Re}\left(\sqrt{\frac{4}{105}}\,F_5+\sqrt{\frac{3}{105}}\,G_7\right)\right],$$

$$(150)\qquad R(K^+\to +00) = \frac{1}{9}\left[6+6\sqrt{2}\,\mathrm{Re}\,\alpha_3-36\sqrt{2}\,\mathrm{Re}\left(\sqrt{\frac{4}{105}}\,F_5+\sqrt{\frac{3}{105}}\,G_7\right)\right],$$

$$(151)\qquad R(K_2^0\to +-0) = \frac{1}{9}\left[12-24\sqrt{2}\,\mathrm{Re}\,\alpha_3+72\,\mathrm{Re}\left(\sqrt{\frac{3}{70}}\,F_5-\sqrt{\frac{4}{70}}\,G_7\right)\right],$$

and

$$(152)\qquad R(K\to 000) = \frac{1}{9}\left[18-36\sqrt{2}\,\mathrm{Re}\,\alpha_3-72\,\mathrm{Re}\left(\sqrt{\frac{3}{70}}\,F_5-\sqrt{\frac{4}{70}}\,G_7\right)\right].$$

By inspection of (149) and (150) we see that for the ratio

$$R(K^+\to ++-)/R(K^+\to +00)\,,$$

to equal 4, in agreement with experiment [following eq. (118)], we need, within rather small experimental errors,

$$(153)\qquad \mathrm{Re}\left(\sqrt{\frac{4}{105}}\,F_5+\sqrt{\frac{3}{105}}\,G_7\right)=0\,.$$

The most reasonable conclusion is that

$$F_5 = 0\,, \qquad \text{and} \qquad G_7 = 0\,. \tag{154}$$

The unlikely possibility that the result (153) is due to an accidental cancellation, *i.e.*,

$$\sqrt{3}\,G_7 = -\sqrt{4}F_5\,, \tag{155}$$

can be checked by measurement of the ratio $R(K_2^0 \to 000)/R(K_2^0 \to +-0)$ According to (151) and (152) this ratio should be $\frac{3}{2}$ (plus phase-space corrections) if (154) holds, but not if (155) holds. The present experimental results are consistent with (154) but are not accurate enough to rule out (155). [See the discussion following eq. (119).]

In summary, the evidence from $K \to 3\pi$ branching ratios indicates

(*a*) Dominance of $\Delta I = \frac{1}{2}$.

(*b*) Dominance of the symmetric $I=1$ state.

(*c*) Roughly that amount of $\Delta I = \frac{3}{2}$ going to $I=1$ expected from $K^+ \to \pi^+\pi^0$.

(*d*) Negligible amounts of $\Delta I = \frac{5}{2}$ and $\frac{7}{2}$ going to $I=3$.

(*e*) Possibly small amounts of $I=2$ in K^+ decay from $\Delta I = \frac{3}{2}$ and $\frac{5}{2}$. These could be present to, say, 30 % in the amplitude (relative to $\Delta I = \frac{1}{2}$) and still be undetectable via K^+ branching ratios, since they give no effect in linear approximation, and the quadratic terms should give effects of $<10\%$. (If $I=2$ is present, the $\Delta I = \frac{3}{2}$ and $\frac{5}{2}$ contributions can be saparated only by comparing $K_1^0 \to 3\pi$ with $K^+ \to 3\pi$.)

5. – The $\Delta I = \frac{1}{2}$ rule for leptonic K-decays.

We begin with a summary of the decays we have studied so far, and also the strangeness-conserving decays, by means of a Puppi diagram, which is constructed as follows. A decay, for instance $n \to pe^-\bar{\nu}$, is written in transposed form, $p\bar{n} \to e^+\nu$. Then $p\bar{n}$ and $e^+\nu$ are called «vertices». Similarly, $\mu^+ \to e^+\nu\bar{\nu}$ becomes $\mu^+\nu \to e^+\nu$, $\mu^- + p \to n + \nu$ becomes $p\bar{n} \to \mu + \nu$, $\Lambda \to pe^-\nu$ becomes $p\bar{\Lambda} \to e\ \nu$. (We do not need to distinguish between ν_e and ν_μ). A given vertex is characterized by its quantum numbers for the strongly interacting particles. Thus the $p\bar{n}$ vertex has the same quantum numbers as π^+, and $p\bar{\Lambda}$ the same as K^+. We therefore call these the π^+ and K^+ vertices. Transitions are assumed to occur between any pair of vertices. (With each vertex we

may associate a « current ». Then transitions between two vertices are due to interaction between the two currents.)

Until recently the four vertices $e^+\nu$, $\mu^+\nu$, π^+, and K^+ seemed sufficient to summarize all known decays. One had a Puppi tetrahedron. [In addition one has the charge-conjugate diagram.] In our discussion we will need two additional vertices. Since a Puppi hexagon may become unwieldy, we use a « Puppi Table ». For each vertex give the total charge Q, strangeness S, isotopic spin I, and its third component I_3, for the *strongly interacting particles only*. Thus $Q=0$ for the $e^+\nu$ vertex (and so are S and I) since there are no strongly interacting particles. By this convention, Q is not conserved in $\pi^+\to\mu^++\nu$, although of course the total charge is conserved. For each vertex the baryon number is zero, and Q, I_3, and S are related through the famous formula

$$Q=I_3+\frac{S}{2}\,. \tag{156}$$

The two additional vertices, needed in our later discussion, will be named the $(\frac{3}{2},\frac{1}{2})$ and $(\frac{3}{2},\frac{3}{2})$ vertices, after their (I,I_3) values. The table follows.

Puppi table.

Vertex	Q	S	I	I_3	Particles
$e^+\nu$	0	0	0	0	$e^+\nu$
$\mu^+\nu$	0	0	0	0	$\mu^+\nu$
π^+	1	0	1	$+1$	π^+, $p\bar{n}$, $\Sigma^+\bar{\Lambda}$, $\sqrt{\frac{1}{2}}(\pi^+\pi^0-\pi^0\pi^+)$, ...
K^+	1	$+1$	$\frac{1}{2}$	$+\frac{1}{2}$	K^+, $p\bar{\Lambda}$, $\sqrt{\frac{2}{3}}\,n\bar{\Sigma}^--\sqrt{\frac{1}{3}}\,p\bar{\Sigma}^0$, $\sqrt{\frac{2}{3}}\,K^0\pi^+-\sqrt{\frac{1}{3}}\,K^+\pi^0$, ...
$\left(\frac{3}{2},+\frac{1}{2}\right)$	1	$+1$	$\frac{3}{2}$	$+\frac{1}{2}$	$\sqrt{\frac{1}{3}}\,n\bar{\Sigma}^-+\sqrt{\frac{2}{3}}\,p\bar{\Sigma}^0$, $\sqrt{\frac{1}{3}}\,K^0\pi^++\sqrt{\frac{2}{3}}\,K^+\pi^0$, ...
$\left(\frac{3}{2},+\frac{3}{2}\right)$	1	-1	$\frac{3}{2}$	$+\frac{3}{2}$	$\bar{n}\Sigma^+$, $\bar{K}^0\pi^+$, ...

The first three vertices take care of neutron β-decay, π-decay, μ-decay, and μ-capture, and (for example) predict $\Sigma^+\to\Lambda e^+\nu$. (An example of this decay has recently been reported by BLOCK *et al.* [2].)

The K^+ vertex is certainly present, since $K^+\to\mu^+\nu$ occurs. Transitions between the K^+ and π^+ vertices can give only $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$. (We have seen that both $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ are present but that $\Delta I=\frac{1}{2}$ dominates, in nonleptonic decays of strange particles.) If either of the two $I=\frac{3}{2}$ vertices is present, transitions to the π^+ vertex can occur with $\Delta I=\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$.

Transitions between the $(\frac{3}{2}, \frac{3}{2})$ vertex, which has $S=-1$, and the K^+ vertex ($S=+1$) can lead to decays with $\Delta S=2$. For instance $\Xi^- \to n\pi^-$ can take place via

$$K^+ = \overline{\Xi}^-\Lambda \to \overline{K}^0\pi^+ = (\tfrac{3}{2}, \tfrac{3}{2}),$$

or transposing,

$$\Xi^- \to \Lambda K^0\pi^- = n\pi^- . \tag{157}$$

(Here the « equals » sign represents a strong reaction that consrves the quantum numbers of the vertex, and the arrow represents the weak reaction.) Since the decay $\Xi \to n\pi$ has not yet been observed, there has been good reason to assume the $(\frac{3}{2}, \frac{3}{2})$ vertex to be absent.

Another argument (by OKUN) against the existence of the $(\frac{3}{2}, \frac{3}{2})$ vertex is provided by the smallness of the observed $K_1^0 - K_0^2$ mass difference. The transition $(\frac{3}{2}, \frac{3}{2}) \leftrightarrow (\frac{1}{2}, \frac{1}{2})$ allows $\overline{K}^0\pi^+ = (\frac{3}{2}, \frac{3}{2}) \to (\frac{1}{2}, \frac{1}{2}) = K^0\pi^+$ in first order; *i.e.*, $K^0 \leftrightarrow \overline{K}^0$ « rapidly ». Since we have $K^0 = (1/\sqrt{2})(K_1^0 + K_2^0)$ and $\overline{K}^0 = (1/\sqrt{2})(K_1^0 - K_2^0)$, rapid transitions $K^0 \leftrightarrow \overline{K}^0$ would correspond to rapid change of the relative phase of K_1^0 and K_2^0, *i.e.* to rapid time variation in $\exp[i(E_1 - E_2)t/\hbar]$, and thus to a large mass difference $m_1 - m_2$. If the $(\frac{3}{2}, \frac{3}{2})$ vertex is absent, $K^0 \leftrightarrow \overline{K}^0$ can only proceed in second order, via $K^0 \leftrightarrow \pi^+\pi \leftrightarrow \overline{K}^0$, leading to a « small » $K_1^0 - K_2^0$ mass difference, as seems to be observed.

The « $\Delta S/\Delta Q = +1$ » rule, for leptonic decays of strange particles follows from the exclusion of the $(\frac{3}{2}, \frac{3}{2})$ vertex. We see from the Puppi table that $\Delta S/\Delta Q = -1/-1 = +1$ for the leptonic decays $(\frac{1}{2}, \frac{1}{2}) \to L^+\nu$ (L^+ means e^+ or μ^+), and for $(\frac{3}{2}, \frac{1}{2}) \to L^+\nu$, but we have $\Delta S/\Delta Q = +1/-1 = -1$ for $(\frac{3}{2}, \frac{3}{2}) \to L^+\nu$.

We now turn to the three-body leptonic decays $K \to \pi L\nu$. We have the three possibilities

$$K^+ \to \pi^0 L^+\nu , \tag{158}$$

$$K^0 \to \pi^- L^+\nu , \tag{159}$$

$$K^0 \to \pi^+ L^- \overline{\nu} , \tag{160}$$

and the three reactions obtained from these by charge conjugation. The only possibilities are $\Delta I = \frac{1}{2}$ or $\frac{3}{2}$ (for the strongly interacting particles, always).

We can transpose all particles to the left side of the equations, and add a spurion s to the right side, to conserve I and I_3 (as well as Q and S). Reactions (158) and (159) have $I_3 = +\frac{1}{2}$ for the spurion, so that the spurion can have $I = \frac{1}{2}$ or $\frac{3}{2}$. Using Table I we find the amplitudes

$$s(\tfrac{1}{2}, +\tfrac{1}{2}) = \overline{L}^+\overline{\nu}\,\{\sqrt{\tfrac{2}{3}}\,K^0\pi^+ - \sqrt{\tfrac{1}{3}}\,K^+\pi^0\} , \tag{161}$$

and

$$s(\tfrac{3}{2}, +\tfrac{1}{2}) = \overline{L}^+\overline{\nu}\,\{\sqrt{\tfrac{1}{3}}\,K^0\pi^+ + \sqrt{\tfrac{2}{3}}\,K^+\pi^0\} . \tag{162}$$

(These correspond to the K^+ and $(\frac{3}{2}, \frac{1}{2})$ vertices in the Puppi table.) Reaction (160) has $I_3 = -\frac{3}{2}$ for the spurion, so that the spurion must have $I = \frac{3}{2}$. We then have

(163) $$s(\tfrac{3}{2}, -\tfrac{3}{2}) = \bar{L}^- \nu K^0 \pi^- .$$

corresponding to the $(\frac{3}{2}, \frac{3}{2})$ vertex (or its charge conjugate) in the Puppi table. We define the complex numbers a_{11}, a_{31}, and a_{33} corresponding to $s(\frac{1}{2}, \frac{1}{2})$, $s(\frac{3}{2}, \frac{1}{2})$, and $s(\frac{3}{2}, \frac{3}{2})$, and write

$$\psi \equiv a_{11} s(\tfrac{1}{2}, \tfrac{1}{2}) + a_{31} s(\tfrac{3}{2}, \tfrac{1}{2}) + a_{33} s(\tfrac{3}{2}, \tfrac{3}{2}) =$$
$$= \bar{L}^+ \bar{\nu} \left[\sqrt{\tfrac{2}{3}}\, a_{11} + \sqrt{\tfrac{1}{3}}\, a_{31}\right] K^0 \pi^+ + \bar{L}^+ \bar{\nu} \left[-\sqrt{\tfrac{1}{3}}\, a_{11} + \sqrt{\tfrac{2}{3}}\, a_{31}\right] K^+ \pi^0 + \bar{L}^- \nu\, a_{33} K^0 \pi^- .$$

Thus we have the transition amplitudes

(165) $$a(K^+ \to \pi^0 L^+ \nu) = -\sqrt{\tfrac{1}{3}}\, a_{11} + \sqrt{\tfrac{2}{3}}\, a_{31} \equiv a_+ ,$$

(166) $$a(K^0 \to \pi^- L^+ \nu) = \sqrt{\tfrac{2}{3}}\, a_{11} + \sqrt{\tfrac{1}{3}}\, a_{31} \equiv a ,$$

and

(167) $$a(K^0 \to \pi^+ L^- \nu) = a_{33} \equiv \bar{a} .$$

(The amplitude a_{33} corresponds to $\Delta S = -\Delta Q$.) Under the assumption of CP-invariance we have

(168) $$a(\bar{K}^0 \to \pi^+ L^- \bar{\nu}) = a(K^0 \to \pi^- L^+ \nu) \equiv a ,$$

and

(169) $$a(\bar{K}^0 \to \pi^- L^+ \nu) = a(K^0 \to \pi^+ L^- \bar{\nu}) \equiv \bar{a} .$$

[Equations (168) and (169) are not completely obvious. See remarks following eq. (176)]. Therefore for K_1^0 and K_2^0, since

$$K^0_{1,2} = \frac{1}{\sqrt{2}} (K^0 \pm \bar{K}^0) ,$$

we have

(170) $$a(K_1^0 \to \pi^- L^+ \nu) = \frac{1}{\sqrt{2}} [a(K^0 \to \pi^- L^+ \nu) + a(\bar{K}^0 \to \pi^- L^+ \nu)] = \frac{1}{\sqrt{2}} (a + \bar{a}) ,$$

(171) $$a(K_1^0 \to \pi^+ L^- \nu) = \frac{1}{\sqrt{2}} [a(K^0 \to \pi^+ L^- \nu) + a(\bar{K}^0 \to \pi^+ L^- \nu)] = \frac{1}{\sqrt{2}} (\bar{a} + a) ,$$

and similarly

(172) $$a(K_2^0 \to \pi^- L^+ \nu) = \frac{1}{\sqrt{2}} (a - \bar{a}) ,$$

(173) $$a(K_2^0 \to \pi^+ L^- \nu) = \frac{1}{\sqrt{2}} (\bar{a} - a) .$$

Thus we have the rates

$$R(\mathrm{K}_1^0 \to \pi^-\mathrm{L}^+\nu) = R(\mathrm{K}_1^0 \to \pi^+\mathrm{L}^-\nu) = \tfrac{1}{2}\left|\sqrt{\tfrac{2}{3}}\,a_{11} + \sqrt{\tfrac{1}{3}}\,a_{31} + a_{33}\right|^2, \tag{174}$$

$$R(\mathrm{K}_2^0 \to \pi^-\mathrm{L}^+\nu) = R(\mathrm{K}_2^0 \to \pi^+\mathrm{L}^-\nu) = \tfrac{1}{2}\left|\sqrt{\tfrac{2}{3}}\,a_{11} + \sqrt{\tfrac{1}{3}}\,a_{31} - a_{33}\right|^2, \tag{175}$$

and

$$R(\mathrm{K}^+ \to \pi^0\mathrm{L}^+\nu) = \left|-\sqrt{\tfrac{1}{3}}\,a_{11} + \sqrt{\tfrac{2}{3}}\,a_{31}\right|^2. \tag{176}$$

Before examining the predictions of eqs. (174), (175), and (176) we make some parenthetical remarks. First, time-reversal invariance requires that a_{11}, a_{31}, and a_{33} be all real, except for a common phase factor. (Final-state interactions are negligible here.)

Second, in eqs. (168) and (169) we wish to invoke *CP* invariance, *not C* invariance. In order to have interfering amplitudes we must have exactly the same configuration of charges, momenta, and spins. In the K rest frame the configuration can be specified by giving the linear momenta, $\boldsymbol{p}_i(i = \pi,\ \mathrm{L},\ \nu)$ and spins $\boldsymbol{\sigma}_i(i = \mathrm{L}, \nu)$. Under *P* the spin is unchanged, but the $\boldsymbol{p}_i$ are reversed. Then we should write that from *CP* invariance,

$$a \equiv a(\mathrm{K}^0 \to \pi^-\mathrm{L}^+\nu;\ \boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = a(\overline{\mathrm{K}}{}^0 \to \pi^+\mathrm{L}^-\bar{\nu};\ -\boldsymbol{p}_i;\ \boldsymbol{\sigma}_i),$$

and

$$\bar{a} \equiv a(\overline{\mathrm{K}}{}^0 \to \pi^-\mathrm{L}^+\nu;\ \boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = a(\mathrm{K}^0 \to \pi^+\mathrm{L}^-\bar{\nu};\ -\boldsymbol{p}_i;\ \boldsymbol{\sigma}_i).$$

Then

$$a(\mathrm{K}_1^0 \to \pi^-\mathrm{L}^+\nu;\quad \boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = \frac{1}{\sqrt{2}}(a + \bar{a}),$$

$$a(\mathrm{K}_2^0 \to \pi^-\mathrm{L}^+\nu;\quad \boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = \frac{1}{\sqrt{2}}(a - \bar{a}),$$

$$a(\mathrm{K}_1^0 \to \pi^+\mathrm{L}^-\nu;\ -\boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = \frac{1}{\sqrt{2}}(\bar{a} + a),$$

$$a(\mathrm{K}_2^0 \to \pi^+\mathrm{L}^-\nu;\ -\boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = \frac{1}{\sqrt{2}}(\bar{a} - a).$$

Finally, then, eqs. (174) and (175) should read

$$R(\mathrm{K}_1^0 \to \pi^-\mathrm{L}^+\nu;\ \boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = R(\mathrm{K}_1^0 \to \pi^+\mathrm{L}^-\bar{\nu};\ -\boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = \tfrac{1}{2}|a + \bar{a}|^2,$$

and

$$R(\mathrm{K}_2^0 \to \pi^-\mathrm{L}^+\nu;\ \boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = R(\mathrm{K}_2^0 \to \pi^+\mathrm{L}^-\bar{\nu};\ -\boldsymbol{p}_i;\ \boldsymbol{\sigma}_i) = \tfrac{1}{2}|a + \bar{a}|^2.$$

These equations should actually be modified once more. Since $\boldsymbol{p}_\pi + \boldsymbol{p}_L + \boldsymbol{p}_\nu = 0$, $\boldsymbol{p}_\pi$, $\boldsymbol{p}_L$, and $\boldsymbol{p}_\nu$ cannot form a pseudoscalar, and the entire configuration $-\boldsymbol{p}_\pi$, $-\boldsymbol{p}_L$, $-\boldsymbol{p}_\nu$ can be rotated until it coincides with $\boldsymbol{p}_\pi$, $\boldsymbol{p}_L$, $\boldsymbol{p}_\nu$. (This is allowed since the K spin is zero.) In this rotation the spins are also reversed. Thus we have $R(K_1^0 \to \pi^- L^+ \nu; \boldsymbol{p}_i; \boldsymbol{\sigma}_i) = R(K_1^0 \to \pi^+ L^- \bar\nu; \boldsymbol{p}_i; -\boldsymbol{\sigma}_i)$ and similarly for K_2^0 decay. Thus the *spectra* for $K_2^0 \to \pi^- L^+ \nu$ and $K_2^0 \to \pi^+ \Delta^- \bar\nu$ are the same, and our use of (168) and (169) is justified, as long as we do not measure spins.

Next we consider the predictions of eqs. (174), (175), and (176). We first sum over both signs of charge and let

$$R(K_1^0 \to \pi^- L^+ \nu) + R(K_1^0 \to \pi^+ L^- \nu) \equiv \Gamma_1, \tag{177}$$

$$R(K_2^0 \to \pi^- L^+ \nu) + R(K_2^0 \to \pi^+ L^- \nu) \equiv \Gamma_2, \tag{178}$$

and

$$R(K^+ \to \pi^0 L^+ \nu) \equiv \Gamma_+ . \tag{179}$$

The predictions become

$$\Gamma_1 = |\sqrt{\tfrac{2}{3}}\, a_{11} + \sqrt{\tfrac{1}{3}}\, a_{31} + a_{33}|^2 \equiv |a + \bar a|^2, \tag{180}$$

$$\Gamma_2 = |\sqrt{\tfrac{2}{3}}\, a_{11} + \sqrt{\tfrac{1}{3}}\, a_{31} - a_{33}|^2 \equiv |a - \bar a|^2, \tag{181}$$

$$\Gamma_+ = |-\sqrt{\tfrac{1}{3}}\, a_1 + \sqrt{\tfrac{2}{3}}\, a_{31}|^2 \equiv |a_+|^2 . \tag{182}$$

The predictions for some special cases follow:

1. *Pure* $\Delta I = \frac{1}{2}$ *rule.* (Includes $\Delta S = +\Delta Q$ rule.)

We have $a_{11} \neq 0$, $a_{31} = a_{33} = 0$. Then

$$\Gamma_1 = \Gamma_2 = 2\Gamma_+ . \tag{183}$$

2. $\Delta S = +\Delta Q$ *rule.* (Without $\Delta I = \frac{1}{2}$ rule.)

We have $a_{33} = 0$, $a_{11} \neq 0$, $a_{31} \neq 0$. Then

$$\Gamma_1 = \Gamma_2 \neq 2\Gamma_+ . \tag{184}$$

3. *No* $\Delta I = \frac{1}{2}$ *rule.* (For three-body decay.)

By this we mean $a_1 = 0$; $a_{31} \neq 0$, $a_{33} \neq 0$. At first sight we might expect that the existence of $K^+ \to \mu + \nu$ would guarantee $a_{11} \neq 0$, since we can write $(K^+\pi^0)_{I=\frac{1}{2}} = K^+ \to \mu + \nu$, where the «equals» sign corresponds to a strong

reaction. But conservation of angular momentum and parity forbids the strong reaction in this case. (Of course there are other possibilities.) Thus we should not assume, *a priori*, that $a_{11} \neq 0$, for three-body decay.

The no $\Delta I = \frac{1}{2}$ rule is easily seen to lead to a quadratic relation between the counting rates, namely

$$(\Gamma_1 - \Gamma_2)^2 = 4\Gamma_+(\Gamma_1 + \Gamma_2 - \Gamma_+) . \tag{185}$$

If we let

$$x \equiv \Gamma_1/\Gamma_2 , \tag{186}$$

and

$$y \equiv \Gamma_+/\Gamma_2 , \tag{187}$$

then (185) becomes

$$x = 1 + 2y \pm \sqrt{1 + 8y} . \tag{188}$$

4. *No* $(\frac{3}{2}, \frac{1}{2})$ *rule.*

We mean $a_{31} = 0$, $a_{11} \neq 0$, $a_{33} \neq 0$. Then we have the quadratic relation

$$(\Gamma_1 - \Gamma_2)^2 = 16\Gamma_+(\Gamma_1 + \Gamma_2 - 4\Gamma_+) , \tag{189}$$

which is equivalent to

$$x = 1 + 8y \pm 2\sqrt{8y} . \tag{190}$$

5. *Takeda rule.*

The intermediate-boson scheme of TAKEDA [12] allows all of a_{11}, a_{31}, and a_{33} to be nonzero, but imposes the constraint

$$a_{33} = \sqrt{3}\, a_{31} . \tag{191}$$

[Equation (191) is equivalent to eq. (48) of TAKEDA's paper. However, Takeda's eq. (48) has a typographical error—the factor $(\frac{1}{3})^{-\frac{1}{2}}$ should be replaced by $(\frac{1}{3})^{+\frac{1}{2}}$. (Private communication from G. TAKEDA). If we insert formula (191) into eq. (181) and compare the result with eqs. (182) and (180) we find the predictions

$$\Gamma_1 \neq \Gamma_2 = 2\Gamma_+ . \tag{192}$$

Remarkably, one of the predictions—namely $\Gamma_2 = 2\Gamma_+$—coincides with a prediction of the pure $\Delta I = \frac{1}{2}$ rule. [See eq. (183).]

We now turn to the experiments. The K^+ rates are obtained by combining branching ratios from emulsions and bubble chambers, and the K^+ lifetime from counter experiments [13]. The combined rates for $K \to e^+\pi^0\nu$ and $\mu^+\pi^0\nu$ give

(193) $$\Gamma_+(e^+, \mu^+) = (8.25 \pm 0.59) \cdot 10^6\ s^{-1}\,.$$

The rates for K_1^0 and K_2^0 are obtained as follows. Suppose one has a number N of K^0 produced at time $t=0$, by means of a reaction like

(194) $$K^+ + n \to K^0 + p\,,$$

or

(195) $$\pi^- + p \to K^0 + \Lambda\,.$$

At $t=0$ we have, for $\psi(t)$, the wave function in the rest system of the neutral K-meson,

$$\psi(0) = |K^0\rangle = \frac{|K_1^0\rangle + |K_2^0\rangle}{\sqrt{2}}\,.$$

For $t > 0$ we must include the oscillating time-dependent factor $\exp(-iE_1t/\hbar) \equiv \equiv \exp[-im_1t]$, and the decay factor $\exp[-\lambda_1 t/2]$, in the K_1^0 amplitude, and a similar factor for K_2^0, to get

$$\psi(t) = \frac{1}{\sqrt{2}}\,|K_1^0\rangle \exp[-im_1t - \lambda_1 t/2] + \frac{1}{\sqrt{2}}\,|K_2^0\rangle \exp[-im_2t - \lambda_2 t/2]\,.$$

We now calculate the time-dependent amplitude for decay into $\pi^-L^+\nu$ and $\pi^+L^-\nu$, using eqs. (170) to (173), to obtain

$$a(\pi^-L^+\nu) = \langle K_1^0|\psi(t)\rangle a(K_1^0 \to L^+) + \langle K_2^0|\psi(t)\rangle a(K_2^0 \to L^+) =$$
$$= \frac{\exp[-im_1t - \lambda_1 t/2]}{\sqrt{2}}\,\frac{(a+\bar{a})}{\sqrt{2}} + \frac{\exp[-im_2t - \lambda_2 t/2]}{\sqrt{2}}\,\frac{(a-\bar{a})}{\sqrt{2}}\,.$$

Similarly,

$$a(\pi^+L^-\bar{\nu}) = \frac{\exp[-im_1t - \lambda_1 t/2]}{\sqrt{2}}\,\frac{(\bar{a}+a)}{\sqrt{2}} + \frac{\exp[-im_2t - \lambda_2 t/2]}{\sqrt{2}}\,\frac{(\bar{a}-a)}{\sqrt{2}}\,.$$

The decay rate is given by the absolute square, so that the two decay rates (corresponding to a single K^0 at $t=0$) are

(196) $$R(L^\pm) = \tfrac{1}{4}\{|a+\bar{a}|^2 \exp[-\lambda_1 t] + |a-\bar{a}|^2 \exp[-\lambda_2 t] \pm \pm 2(|a|^2 - |\bar{a}|^2)\exp[-(\lambda_1+\lambda_2)t/2]\cos \Delta m t\}\,,$$

where the $+$ and $-$ signs in the cross term go with L^+ and L^-, respectively. In the cross term we have set equal to zero a term proportional to $\sin(\Delta m t) Im \bar{a}^* a$. Time-reversal invariance requires a and $\bar{a}$ to have a common phase factor, so that $\bar{a}^* a$ is real, and $Im \bar{a}^* a$ vanishes.

We see from eq. (196) that at $t=0$,

$$R(L^\pm) = \tfrac{1}{4}\{|a+\bar{a}|^2 + |a-\bar{a}|^2 \pm 2(|a|^2 - |\bar{a}|^2)\}, \tag{197}$$

i.e.,

$$R(L^+) = |a|^2, \qquad R(L^-) = |\bar{a}|^2.$$

Thus the ratio $R(L^-)/R(L^+)$ at $t=0$ gives the ratio $|\bar{a}|^2/|a|^2$.

If one adds the rates for L^+ and L^-, the cross term in eq. (194) cancels, and one obtains

$$R(L^+) + R(L^-) = \tfrac{1}{2}|a+\bar{a}|^2 \exp[-\lambda_1 t] + \tfrac{1}{2}|a-\bar{a}|^2 \exp[-\lambda_2 t] = \tag{198}$$

$$= \tfrac{1}{2}\Gamma_1 \exp[-\lambda_1 t] + \tfrac{1}{2}\Gamma_2 \exp[-\lambda_2 t].$$

Thus one can obtain Γ_1 and Γ_2 by studying the time dependence of $R(L^+)+R(L^-)$, without any knowledge of $m_1 - m_2 \equiv \Delta m$. In this case, however, the result is unchanged under the interchange of a and $\bar{a}$, as is evident from eq. (198).

ELY *et al.* [14], using K^0 produced by K^+ in propane through reaction (191), have studied the time dependence of decays into $\pi^- e^+ \nu$ and $\pi^+ e^- \nu$, using both, eqs. (196) and (198). They find, in disagreement with eqs. (183) or (184),

$$\frac{\Gamma_1(e^\pm)}{\Gamma_2(e^\pm)} = 11.9^{+7.5}_{-5.6}. \tag{199}$$

This is in bad agreement with the prediction of the $\Delta S = +\Delta Q$ rule. (They are not able to find the absolute rate for Γ_1 or Γ_2, since their sample is highly selected, so no comparison can be made with Γ_+.)

ALEXANDER *et al.* [10], using K^0 produced via reaction (195) in the 72-inch hydrogen chamber have studied the time dependence of $(\pi^\pm e^\mp \nu) + (\pi^\pm \mu^\mp \nu)$. No separation of charges was made, so that eq. (198) was used. Combining the decays into $e^\pm$ and $\mu^\pm$, they find

$$\frac{\Gamma_1(e^\pm, \mu^\pm)}{\Gamma_2(e^\pm, \mu^\pm)} = 6.6^{+6.0}_{-4.0}. \tag{200}$$

They also measure the absolute K_2^0 rates, and find

$$(201) \qquad \Gamma_2(e^\pm, \mu^\pm) = (9.31 \pm 2.49) \cdot 10^6 \, s^{-1} .$$

This is accomplished by using decays with sufficiently long K^0 flight time to insure that the K_1^0 have completely decayed ($\tau_1 \approx 0.9 \cdot 10^{-10}$ s) but the K_2^0 have not ($\tau_2 \approx 7 \cdot 10^{-8}$ s). Comparing (201) with (193) we see that the prediction, $\Gamma_2 = 2\Gamma_+$, of the $\Delta I = \frac{1}{2}$ rule, or of the Takada rule, is not satisfied.

CRAWFORD *et al.* [15] used K^0 produced via reaction (195) in the 10-inch hydrogen chamber. They found

$$(202) \qquad \frac{\Gamma_1(e^\pm, \mu^\pm)}{\Gamma_2(e^\pm, \mu^\pm)} = 3.5^{+3.9}_{-2.7} .$$

The chamber was too small to get rid of K_1^0 by attenuation in time, so that to measure $\Gamma_2(e^\pm, \mu^\pm)$ they had to assume a value for Γ_1/Γ_2. They assumed $\Gamma_1 = \Gamma_2$ [this is not in disagreement with (202)] and found

$$(203) \qquad \Gamma_2(e^\pm, \mu^\pm) = 20.4^{+7.2}_{-6.6} \cdot 10^6 \, s^{-1} ,$$

if $\Gamma_1 = \Gamma_2$. If instead one assumes $\Gamma_1/\Gamma = 9$, [this is taken as a compromise between (199) and (200)] one obtains from the same experiment

$$(204) \qquad \Gamma_2(e^\pm, \mu^\pm) = (8.5 \pm 2.8) \cdot 10^6 \, s^{-1} .$$

This agrees well with the result (201) of ALEXANDER *et al.* (whose result does not depend on Γ_1/Γ_2), and poorly with the prediction of the $\Delta I = \frac{1}{2}$ rule, or the Takeda rule.

Let us next see whether the «No $\Delta I = \frac{1}{2}$ rule» can be ruled out. We want to test eq. (188). The K_2^0 experiment of ALEXANDER *et al.* [10], combined with the K^+ results of other experiments [13], gives, from (201) and (193),

$$(205) \qquad y = \Gamma_+(e^+, \mu^+)/\Gamma_2(e^\pm, \mu^\pm) = (8.25 \pm 0.59)/(9.3 \pm 2.49) = 0.89 \pm 0.24 .$$

We insert this into (188) to *predict* (if $a_{11} = 0$),

$$(206) \qquad x = 1 + 2(.89) \pm \sqrt{1 + 8(.89)} =$$

$$= 2.78 \pm 2.85 = (5.63 \pm 0.83) \quad \text{or} \quad (0 \pm 0.15) ,$$

where we have included the statistical errors in the last step. The prediction (206) of the «No $\Delta I=\frac{1}{2}$ rule» is to be compared with eq. (200), the value obtained by ALEXANDER *et al.*, namely $x=6.6\pm5$. We see that the no $\Delta I=\frac{1}{2}$ rule (for three-body leptonic decays) cannot be ruled out by the present experimental data.

We next test the «No $(\frac{3}{2}, \frac{1}{2})$ rule», through its prediction (190), which becomes, according to (205),

$$x = 1 + 8(0.89) \pm 2\sqrt{8\,(0.89)} = \\ = 8.1 \pm 5.3 = (13.4 \pm 2.7) \quad \text{or} \quad (2.8 \pm 1.2)\,, \tag{207}$$

where experimental errors are only included in the final step. Neither of these predictions can be said to be in strong disagreement with the experimental result (200).

It is clear that more data are needed, to find the relative amounts of a_{11}, a_{31}, and a_{33}, and to see whether universality holds between e and μ.

Lastly, we must remark that part of our discussion has been oversimplified. Equations (180), (181), and (182) should be interpreted as giving the counting rates for a *specified configuration* of all the momenta and spins. Then a_{11}, a_{31}, and a_{33} are not constants, but are complicated functions of the configuration variables, the function depending on the dynamics of the decay. The comparison of experiment with the predictions of the «pure $\Delta I=\frac{1}{2}$ rule» [eq. (183)], the «$\Delta S=\Delta Q$ rule» [eq. (184)], and the «Takeda rule» [eq. (192)] are not affected by the fact that we have suppressed information on the spectra, since these predictions are such that they refer both to a given configuration, and to the total decay rates, and in fact to the sum over e and μ decays. However, predictions (188) and (190), of the «No $\Delta I=\frac{1}{2}$ rule» and the «No $(\frac{3}{2}, \frac{1}{2})$ rule», while they do hold for a *given configuration*, are *not* applicable to the total decay rates, nor to a sum of e and μ modes. The reason is that these (quadratic) predictions do not involve simple ratios. Therefore the «predictions» (206) and (207), and subsequent comparison with the experimental result (200), would make sense only if the form factors involved in a_{11}, a_{31}, and a_{33} were *all the same* function of the configuration and furthermore were the same for e and μ decay. Thus, only to the extent that the spectra correspond to phase-space alone—and to the extent that we neglect the $\mu-$e mass difference!—can the comparison of (206) and (207) with (200) be justified.

TABLE I. – $1 \times \frac{1}{2}$.

	$\left(\frac{3}{2}, \frac{3}{2}\right)$	$\left(\frac{3}{2}, \frac{1}{2}\right)$	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$\left(\frac{3}{2}, -\frac{1}{2}\right)$	$\left(\frac{1}{2}, -\frac{1}{2}\right)$	$\left(\frac{3}{2}, -\frac{3}{2}\right)$
$(1, 1)\left(\frac{1}{2}, \frac{1}{2}\right)$	1					
$(1, 1)\left(\frac{1}{2}, -\frac{1}{2}\right)$		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$			
$(1, 0)\left(\frac{1}{2}, \frac{1}{2}\right)$		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$			
$(1, 0)\left(\frac{1}{2}, -\frac{1}{2}\right)$				$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
$(1, -1)\left(\frac{1}{2}, \frac{1}{2}\right)$				$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
$(1, -1)\left(\frac{1}{2}, -\frac{1}{2}\right)$						1

TABLE II. – 1×1.

	(2, 2)	(2, 1)	(1, 1)	(2, 0)	(1, 0)	(0, 0)	(2, —1)	(1, —1)	(2, —2)
(1, 1) (1, 1)	1								
(1, 1) (1, 0)		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$						
(1, 0) (1, 1)		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$						
(1, 1) (1, —1)				$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
(1, 0) (1, 0)				$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$			
(1, —1)(1, 1)				$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
(1, 0) (1, —1)							$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
(1, —1)(1, 0)							$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
(1, —1)(1, —1)									1

TABLE III. – $\frac{3}{2}\times 1$.

	$\left(\frac{5}{2},\frac{5}{2}\right)$	$\left(\frac{5}{2},\frac{3}{2}\right)$	$\left(\frac{3}{2},\frac{3}{2}\right)$	$\left(\frac{5}{2},\frac{1}{2}\right)$	$\left(\frac{3}{2},\frac{1}{2}\right)$	$\left(\frac{1}{2},\frac{1}{2}\right)$	$\left(\frac{5}{2},-\frac{1}{2}\right)$	$\left(\frac{3}{2},-\frac{1}{2}\right)$	$\left(\frac{1}{2},-\frac{1}{2}\right)$	$\left(\frac{5}{2},-\frac{3}{2}\right)$	$\left(\frac{3}{2},-\frac{3}{2}\right)$	$\left(\frac{5}{2},-\frac{5}{2}\right)$
$\left(\frac{3}{2},\frac{3}{2}\right)(1,1)$	1											
$\left(\frac{3}{2},\frac{3}{2}\right)(1,0)$		$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$									
$\left(\frac{3}{2},\frac{1}{2}\right)(1,1)$		$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$									
$\left(\frac{3}{2},\frac{3}{2}\right)(1,-1)$				$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$						
$\left(\frac{3}{2},\frac{1}{2}\right)(1,0)$				$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$						
$\left(\frac{3}{2},-\frac{1}{2}\right)(1,1)$				$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$						
$\left(\frac{3}{2},\frac{1}{2}\right)(1,-1)$							$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$			
$\left(\frac{3}{2},-\frac{1}{2}\right)(1,0)$							$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$			
$\left(\frac{2}{3},-\frac{3}{2}\right)(1,1)$							$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$			
$\left(\frac{3}{2},-\frac{1}{2}\right)(1,-1)$										$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$	
$\left(\frac{3}{2},-\frac{3}{2}\right)(1,0)$										$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$	
$\left(\frac{3}{2},-\frac{3}{2}\right)(1,-1)$												1

TABLE IV. – $2\times\frac{1}{2}$.

	$\left(\frac{5}{2},\frac{5}{2}\right)$	$\left(\frac{5}{2},\frac{3}{2}\right)$	$\left(\frac{3}{2},\frac{3}{2}\right)$	$\left(\frac{5}{2},\frac{1}{2}\right)$	$\left(\frac{3}{2},\frac{1}{2}\right)$	$\left(\frac{5}{2},-\frac{1}{2}\right)$	$\left(\frac{3}{2},-\frac{1}{2}\right)$	$\left(\frac{5}{2},-\frac{3}{2}\right)$	$\left(\frac{3}{2},-\frac{3}{2}\right)$	$\left(\frac{5}{2},-\frac{5}{2}\right)$
$(2,\ 2)\left(\frac{1}{2},\ \frac{1}{2}\right)$	1									
$(2,\ 2)\left(\frac{1}{2},-\frac{1}{2}\right)$		$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$							
$(2,\ 1)\left(\frac{1}{2},\ \frac{1}{2}\right)$		$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$							
$(2,\ 1)\left(\frac{1}{2},-\frac{1}{2}\right)$				$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$					
$(2,\ 0)\left(\frac{1}{2},\ \frac{1}{2}\right)$				$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$					
$(2,\ 0)\left(\frac{1}{2},-\frac{1}{2}\right)$						$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$			
$(2,-1)\left(\frac{1}{2},\ \frac{1}{2}\right)$						$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$			
$(2,-1)\left(\frac{1}{2},-\frac{1}{2}\right)$								$\sqrt{\frac{4}{5}}$	$\sqrt{\frac{1}{5}}$	
$(2,-2)\left(\frac{1}{2},\ \frac{1}{2}\right)$								$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{4}{5}}$	
$(2,-2)\left(\frac{1}{2},\ \frac{1}{2}\right)$										1

TABLE V.

	$\left(\frac{7}{2}, \frac{7}{2}\right)$	$\left(\frac{7}{2}, \frac{5}{2}\right)$	$\left(\frac{5}{2}, \frac{5}{2}\right)$	$\left(\frac{7}{2}, \frac{3}{2}\right)$	$\left(\frac{5}{2}, \frac{3}{2}\right)$
$(3, 3)\left(\frac{1}{2}, \frac{1}{2}\right)$	1				
$(3, 3)\left(\frac{1}{2}, -\frac{1}{2}\right)$		$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{6}{7}}$		
$(3, 2)\left(\frac{1}{2}, \frac{1}{2}\right)$		$\sqrt{\frac{6}{7}}$	$-\sqrt{\frac{1}{7}}$		
$(3, 2)\left(\frac{1}{2}, -\frac{1}{2}\right)$				$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{5}{7}}$
$(3, 1)\left(\frac{1}{2}, \frac{1}{2}\right)$				$\sqrt{\frac{5}{7}}$	$-\sqrt{\frac{2}{7}}$
$(3, 1)\left(\frac{1}{2}, -\frac{1}{2}\right)$					
$(3, 0)\left(\frac{1}{2}, \frac{1}{2}\right)$					
$(3, 0)\left(\frac{1}{2}, -\frac{1}{2}\right)$					
$(3, -1)\left(\frac{1}{2}, \frac{1}{2}\right)$					
$(3, -1)\left(\frac{1}{2}, -\frac{1}{2}\right)$					
$(3, -2)\left(\frac{1}{2}, \frac{1}{2}\right)$					
$(3, -2)\left(\frac{1}{2}, -\frac{1}{2}\right)$					
$(3, -3)\left(\frac{1}{2}, \frac{1}{2}\right)$					
$(3, 3)\left(\frac{1}{2}, -\frac{1}{2}\right)$					

	$\left(\frac{7}{2},\frac{1}{2}\right)$	$\left(\frac{5}{2},\frac{1}{2}\right)$	$\left(\frac{7}{2},-\frac{1}{2}\right)$	$\left(\frac{5}{2},-\frac{1}{2}\right)$	$\left(\frac{7}{2},-\frac{3}{2}\right)$	$\left(\frac{5}{2},-\frac{3}{2}\right)$	$\left(\frac{7}{2},-\frac{5}{2}\right)$	$\left(\frac{5}{2},-\frac{5}{2}\right)$	$\left(\frac{7}{2},-\frac{7}{2}\right)$
	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$							
	$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$							
			$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{3}{7}}$					
			$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$					
					$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{2}{7}}$			
					$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$			
							$\sqrt{\frac{6}{7}}$	$\sqrt{\frac{1}{7}}$	
							$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{6}{7}}$	
									1

TABLE VI.

	(3, 3)	(3, 2) (2, 2)	(3, 1) (2, 1) (1, 1)
(2, 2) (1, 1)	1		
(2, 2) (1, 0) (2, 1) (1, 1)		$\sqrt{\frac{1}{3}}$ $\sqrt{\frac{2}{3}}$ $\sqrt{\frac{2}{3}}-\sqrt{\frac{1}{3}}$	
(2, 2) (1, —1) (2, 1) (1, 0) (2, 0) (1, 1)			$\sqrt{\frac{1}{15}}$ $\sqrt{\frac{1}{3}}$ $\sqrt{\frac{3}{5}}$ $\sqrt{\frac{8}{15}}$ $\sqrt{\frac{1}{6}}-\sqrt{\frac{3}{10}}$ $\sqrt{\frac{6}{15}}-\sqrt{\frac{1}{2}}$ $\sqrt{\frac{1}{10}}$
(2, 1) (1, —1) (2, 0) (1, 0) (2, —1)(1, 1)			
(2, 0) (1, —1) (2, —1)(1, 0) (2, —2)(1, 1)			
(2, —1)(1, —1) (2, —2)(1, 0)			
(2, —2)(1, —1)			

(3, 0)	(2, 0)	(1, 0)	(3, −1)	(2, −1)	(1, −1)	(3, −2)	(2, −2)	(3, −3)
$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$						
$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$						
$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$						
			$\sqrt{\frac{6}{15}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$			
			$\sqrt{\frac{8}{15}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$			
			$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$			
						$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
						$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
								1

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Strange-Particle Resonances.

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1. – Introduction.

Two years ago, at the Rochester Conference of 1960, ALSTON *et al.* [1] reported the observation of a resonant behavior in the interaction between the Λ hyperon and the pion. The existence of this resonance was quickly confirmed by other experiments [2-6]. Since then, several other resonances involving strange particles have been found. These, coupled with the discovery of the ρ, ω and η multipion resonances and the familiar pion-nucleon resonances, have opened a new avenue to the study of the strong interactions of elementary particles.

In my talks here, I do not wish to go into the broader questions raised or possibly even answered by these observations. Instead, I want to confine myself to a description of the present state of our experimental information concerning strange particle resonances. Furthermore, I shall discuss only those resonances for which the experimental evidence is so strong that their existence may be regarded as certain. This means that clear evidence for them has been obtained by several laboratories and in substantially different experiments. The resonances in this category are the $K^*_{\frac{1}{2}}$ (890), Y^*_1 (1385), Y^*_0 (1405), Y^{**}_0 (1520) and the $\Xi^*_{\frac{1}{2}}$ (1530). The subscript specifies the isospin of the resonance and the bracketed quantity the mass in MeV. There have been indications of the existence of other resonant states in individual experiments. Moreover, still other resonances whose existence was postulated on the basis of various theoretical considerations, were searched for and were not found. It is not clear what conclusions are to be drawn from these isolated cases of observation or nonobservation: at present, we have no way of predicting the matrix elements for the production of the resonant states.

With the exception of the Y^{**}_0 which can be produced *alone* in $\bar{K}\mathcal{N}$ collision, the strange particle resonant states are produced along with other par-

ticles. Including the decay of the resonant state there are therefore three or more particles in the final state. We start with a discussion of techniques which one can employ to study the properties of the resonances under such conditions.

2. – Kinematical and dynamical considerations.

For three-particle final states one of the most useful representations of the data is the Dalitz-Fabri (DF) plot first used in the study of the decay of the kaon [7, 8]. Consider a collision in which a total energy E becomes available in the c. m. system and three particles with masses m_1, m_2 and m_3 are produced. These have momenta $\boldsymbol{P}_i$ and total energies E_i ($i = 1, 2, 3$). Since

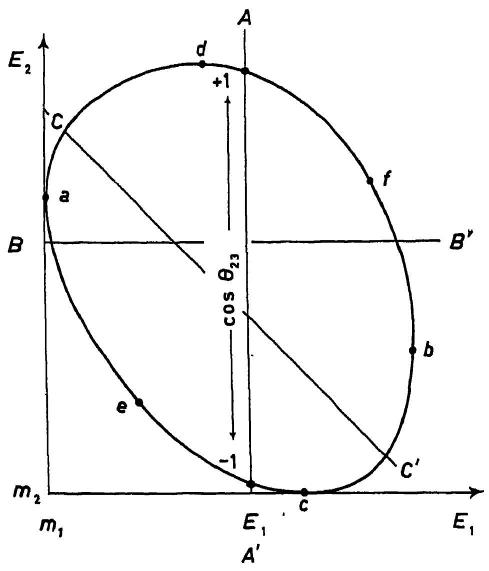

Fig. 1. – The Dalitz-Fabri plot.

$$E = E_1 + E_2 + E_3 , \tag{1}$$

individual events may be described by points in an E_1 *vs.* E_2 plot; see Fig. 1. The limits of the kinematically allowed region in the E_1, E_2 plane are readily established by the following reasoning. Let us, *for purposes of the argument*, imagine the production of the three particles as a two-step process: the production of particles m_1 and M_{23} and the subsequent decay of M_{23} into m_2 and m_3. Once E_1 is selected, M_{23} is specified by conservation of energy and momentum

$$\boldsymbol{P}_{23} = -\boldsymbol{P}_1 , \qquad E_{23} = E - E_1 , \tag{2}$$

hence

$$M_{23}^2 = E_{23}^2 - P_{23}^2 = E^2 + m_1^2 - 2EE_1 . \tag{3}$$

Next consider the decay of M_{23} into m_2 and m_3. Once M_{23} is specified, the total energies of the decay particles, ε_2 and ε_3, *in the M_{23} rest frame* are also specified by

$$\varepsilon_2 = (M_{23}^2 + m_2^2 - m_3^2)/2M_{23} ; \qquad \varepsilon_3 = (M_{23}^2 + m_3^2 - m_2^2)/2M_{23} , \tag{4}$$

as is the *magnitude* of the decay momentum π. Let θ_{23} be the angle between the direction of flight of M_{23} and the direction of π in the M_{23} rest frame. We next transform to the c.m. system

$$E_2 = \gamma(\varepsilon_2 + \beta\pi\cos\theta_{23}) = (E_{23}\varepsilon_2 + P_{23}\pi\cos\theta_{23})/M_{23}\,. \tag{5}$$

As a given choice of E_1 specifies E_{23}, P_{23}, M_{23}, ε_2 and π, the only free variable is $\cos\theta_{23}$. Thus along line AA' in Fig. 1, the cosine of the M_{23} decay angle θ_{23} varies linearly from -1 to $+1$. Similarly $\cos\theta_{13}$ varies linearly along line BB' and $\cos\theta_{12}$ along 45° lines such as CC'. At the boundary of the allowed region the three particles are therefore colinear.

In the absence of specific dinamical information we expect equal intervals of $\cos\theta_{23}$ to be equally probable. As the same can be said for $\cos\theta_{13}$ we expect equal areas within the allowed region of the DF plot to be equally populated. In other words, if, in the expression for the transition probability,

$$\omega = (2\pi/\hbar)\,|M|^2\,(\mathrm{d}N/\mathrm{d}E)\,, \tag{6}$$

we set $|M|^2 = \text{const}/(E_1E_2E_3)$ (the denominator results from the covariant normalization of the outgoing waves $\sim E_i^{-\frac{1}{2}}$), then we expect

$$(\mathrm{d}N/\mathrm{d}E) \sim (E_1E_2E_3)\,\mathrm{d}E_1\,\mathrm{d}E_2\,. \tag{7}$$

This is readily demonstrated: if we fix $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ then the total energy E may be varied by varying the angle θ between $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$. Thus

$$(\mathrm{d}N/\mathrm{d}E) \sim P_1^2\,\mathrm{d}P_1 P_2^2\,\mathrm{d}P_2[d(\cos\theta)/\mathrm{d}E]\,. \tag{8}$$

But

$$P_1\,\mathrm{d}P_1 = E_1\,\mathrm{d}E_1\,, \qquad P_2\,\mathrm{d}P_2 = E_2\,\mathrm{d}E_2\,, \tag{9a}$$

$$\mathrm{d}E/\mathrm{d}(\cos\theta) = \mathrm{d}\{\sqrt{m_1^2 + P_1^2} + \sqrt{m_2^2 + P_2^2} + \sqrt{m_3^2 + (\boldsymbol{P}_1 + \boldsymbol{P}_2)^2}\}/\mathrm{d}(\cos\theta)\,,$$

$$\mathrm{d}E/d(\cos\theta) = 2P_1P_2/E_3\,. \tag{9b}$$

Substituting (9a) and (9b) into (8) we indeed get (7).

While the DF plot exhibits directly any anisotropy of the decay of every two-particle system with respect to its own direction of motion, it completely suppresses all information regarding the orientation of the vectors $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ and $\boldsymbol{P}_3$ with respect to the direction of the incident particles which produced the interaction. This information is of great value, especially when it becomes necessary to determine the internal angular momentum of two of the three particles in the final state.

Consider, for example, the reaction

$$\mathrm{i} + \mathrm{I} \rightarrow 1 + 2 + 3 \, .$$

As this is most common, we consider the case when i, 1 and 2 are spinless while particles I and 3 have spin $\frac{1}{2}$. The final-state wave function may be written as a superposition of terms of the form

$$A^{m,\pm\frac{1}{2}}_{L,j,l}\, Y^{-m}_{L}(\Theta,\, \Phi)\, \mathscr{Y}^{m\pm\frac{1}{2}}_{j,l}(\theta,\, \varphi,\, s)\, , \tag{10}$$

where $\mathscr{Y}^{m\pm\frac{1}{2}}_{j,l}(\theta, \varphi, s)$ is the angular wave function of the m_2-m_3 system in its own rest frame, s is the spin variable of particle 3 and l is the orbital angular momentum of the system; $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$. Y^m_L is the angular wave function of the motion of m_1-M_{23} in the c.m. frame, L, the orbital angular momentum of m_1-M_{23} about the c.m. and m, its z-component. The z-axis for Θ, θ and the axis of quantization of the spins of I and 3 is taken to be the direction of the incident particle i; the $+$ or $-$ $\frac{1}{2}$ is chosen depending on whether the interaction occurs on I spinning along the direction of i or in the opposite direction. This is suggested by Fig. 2 where the x-direction has been chosen such that M_{23} is emitted in the x-z plane, $\Phi = 0$.

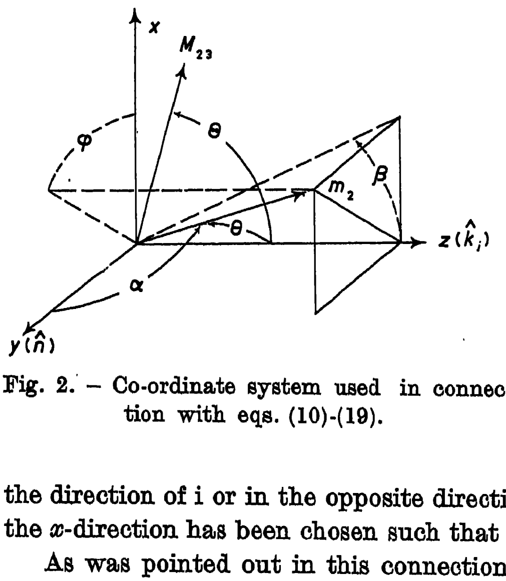

Fig. 2. – Co-ordinate system used in connection with eqs. (10)-(19).

As was pointed out in this connection by ADAIR [2], $A_{L,j,l}$ will, in general, have angular-momentum barrier factors such that

$$A_{L,j,l} \sim \frac{(P_1 R)^L}{(2L-1)!!} \cdot \frac{(\pi R)^l}{(2l-1)!!}\, , \tag{11}$$

provided $P_1 R \ll L$, $\pi R \ll l$; R is the «size» of the interaction region. These barrier factors lead to certain predictions about the density in the DF plot. At point a in Fig. 1 $\boldsymbol{P}_1 = 0$; hence only S-wave production of the M_{23} system can take place in the neighborhood of a. At b, $\boldsymbol{P}_1$ has its maximum value and therefore $\boldsymbol{\pi} = 0$; hence only M_{23} systems in the $s_{\frac{1}{2}}$ state can appear in the vicinity of b. Similar arguments apply to points c, d, e and f.

However, suppose that a resonant state exists with mass $M^*_{23} - i(\Gamma/2)$ which (possibly among other modes) can break up into m_2 and m_3. Let this state

have a given j and l and assume that for that particular j and l the formation of m_2 and m_3 proceeds dominantly through this resonant state. Then the production amplitudes for that j and l will contain a resonance factor

$$A_{L,j,l} \sim \left|\left(M_{23}^{*} - i\frac{\Gamma}{2}\right) - M_{23}\right|^{-1}, \tag{12}$$

which follows readily from second-order perturbation theory. Such a behavior will appear on the DF plot as an enhancement of the intensity centered along a line of constant E_1 satisfying (3) with $M_{23} = M_{23}^{*}$. The particular characteristic of such a resonance is that M_{23}^{*} remains constant as the bombarding energy and therefore E in (3) is varied.

When such a resonance is found experimentally it is, of course, crucial to establish its angular momentum and parity. Assuming that the intrinsic parities of 2 and 3 are known this means measuring j and l. One of the techniques has been given by ADAIR [9]. Let us suppose that the energy is sufficiently low such that only $L = 0$ production of the resonant state can take place and that nonresonant production of the 1, 2, 3 final state is negligible. Then the sum of terms of type (10) reduces to a single term

$$A_{0,j,l}^{\pm\frac{1}{2}} \frac{1}{\sqrt{4\pi}} \mathcal{Y}_{j,l}^{\pm\frac{1}{2}}(\theta, \varphi, s)\,. \tag{13}$$

The functions $\mathcal{Y}_{j,l}^{\pm\frac{1}{2}}$ are tabulated. Squaring (13) yields the M_{23} decay distribution *with respect to the direction of* i in the M_{23} rest frame. The results for the lowest angular momentum cases are

$$\begin{cases} j = \frac{1}{2}\,, & l = 0, 1\,, \quad I(\theta) \sim 1\,, \\ j = \frac{3}{2}\,, & l = 1, 2\,, \quad I(\theta) \sim (3\cos^2\theta + 1)\,, \\ j = \frac{5}{2}\,, & l = 2, 3\,, \quad I(\theta) \sim (5\cos^4\theta - 2\cos^2\theta + 1)\,. \end{cases} \tag{14}$$

However production in $L = 0$ can seldom be guaranteed, nor can the neglect of all except one j, l channel be always justified. Quite generally, let us rewrite the sum of terms of type (10) as follows

$$\sum_{J} \sum_{L,j,l} A_{J,L,j,l}^{\pm\frac{1}{2}} \sum_{m} C_{J,L,j}^{\pm\frac{1}{2},-m,m\pm\frac{1}{2}}\, Y_L^{-m} \mathcal{Y}_{j,l}^{m\pm\frac{1}{2}}\,, \tag{15}$$

where the quantities C are the Clebsch-Gordan coefficients which assure that, for a given L and j, the m components are added in such a superposition that the sum over m yields a wave function of total angular momentum J; $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{j}$. The motivation for this is, of course, the knowledge that isotropy of space

demands that the production amplitude $A^{\pm\frac{1}{2}}_{J,L,j,l}$ cannot depend on m (*). As an example which illustrates several points, consider the case when the outgoing waves $|J, L, j, l\rangle$ are $|\frac{1}{2}, 0, \frac{1}{2}, 0\rangle$, $|\frac{3}{2}, 0, \frac{3}{2}, 1\rangle$, $|\frac{1}{2}, 1, \frac{3}{2}, 1\rangle$ and $|\frac{5}{2}, 0, \frac{5}{2}, 2\rangle$ whose production amplitudes are A, B, C and D, respectively. Substituting the appropriate quantities into (15), squaring, integrating over φ and averaging over spins of I, we get

$$
\begin{aligned}
(16)\qquad I(\theta) = {} & |A|^2 + |B+2C\cos\Theta|^2(3\cos^2\theta+1) + 2|C|^2\sin^2\Theta(5-3\cos^2\theta) \\
& + |D|^2(5\cos^4\theta - 2\cos^2\theta + 1) + 4\,\mathrm{Re}[A^*(B+2C\cos\Theta)]\cos\theta + \\
& + 2\,\mathrm{Re}[A^*D](3\cos^2\theta - 1) + 8\,\mathrm{Re}[D^*(B+2C\cos\Theta)]\cos^3\theta\,.
\end{aligned}
$$

Any one of the four production amplitudes may be regarded as resonant, A, B and D represent S-wave production of $j=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ M_{23} systems while C represents P-wave production of a $j=\frac{3}{2}$ system. We observe

a) when all except one of A, B, or D are negligible we find again eq. (14), as expected;

b) when $A=D=0$, but B and $C\neq 0$, *i.e.* the $p_{\frac{3}{2}}$ state of M_{23} is produced in both S and P waves, then the $j=\frac{3}{2}$ decay distribution is reproduced only for production angles $\Theta=0$ or π, where the third term is negligible. In practice, the Adair analysis must be restricted to regions such that

$$\sin\Theta < 1/(L_{\max}+1)\,,$$

where $L_{\max}$ is the highest partial wave suggested by the production angular distribution;

c) suppose that a resonance is observed and a large coefficient of $\cos^2\theta$ is found in the decay of the resonance, but no significant $\cos^4\theta$ term. Before concluding that the resonance is in the $j=\frac{3}{2}$ state, one must prove that one is not dealing with a resonant $s_{\frac{1}{2}}$ state interfering with a $d_{\frac{5}{2}}$ background as given by the sixth term in (16). Since the phase of the resonant amplitude according to (12) varies rapidly with M_{23} in the resonant region while the background phase is expected to be roughly constant, the magnitude of the $\cos^2\theta$ coefficient is expected to vary rapidly with M_{23} if it is indeed caused by interference. Interference with states of opposite parity such as the fifth term of (16) are

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(*) The quantities $A^{\pm\frac{1}{2}}_{J,L,j,l}$ are related to the T-matrix elements $T_{J,L,j,l}$ through

$$A^{\pm\frac{1}{2}}_{J,L,j,l} = \sqrt{2l_i+1}\,C^{\pm\frac{1}{2},0,\pm\frac{1}{2}}_{J,l_i,\frac{1}{2}}\,T_{J,L,j,l}\,,$$

where l_i is the orbital angular momentum of the incident channel.

readily distinguished by the odd $(\cos\theta)$-dependence which is forbidden for pure states by conservation of parity.

While the Adair analysis makes specific predictions for various j it has not proved very useful primarily because, in practice, it must be restricted to $\cos\Theta$ regions near $+1$ and -1 where the data are usually inadequate by themselves, let alone after they are further subdivided for interference studies. If the resonant state has $j > \frac{1}{2}$ and is produced with polarization or alignment with respect to the normal to the production plane, then it will exhibit an anisotropy in its decay with respect to that direction. In contrast to the Adair argument one cannot predict *a priori* that such polarization or alignment must be present. However, if interference effects are absent, the observation of a nonvanishing coefficient of $\cos^{2n}\alpha$ where α is the angle of one of the decay products of the resonance with respect to the production normal, proves that $j \geqslant n + \frac{1}{2}$. Interference effects may give rise to such terms in the same way as in the previous case.

Once a final state wave function has been written, the co-ordinate system may, of course, be rotated such that the z-axis lies along the normal to the production plane as suggested by Fig. 2. We follow a more general course similar to that given by YANG and LEE [10] for the determination of the spin of the Λ. Let us suppose that *only* the resonant state is formed in the reaction, but place no restriction on the production partial waves. Then, if the resonant state has spin j, we may write the final-state wave function

$$\psi = \sum_{j_z} A^{j_z} \mathscr{Y}^{j_z}_{j,l}(\alpha, \beta, s)\ , \tag{17}$$

where A^{j_z} is the amplitude for the production of the resonant state with z-component of spin j_z, the z-direction being the normal to the production plane; the A^{j_z}'s are functions of the production angle Θ, of course. If we now substitute selected j, l values into (17), square, integrate around the normal and average over initial spins we find

$j = \frac{1}{2}$, $l = 0, 1$

$$I(\alpha) = \{|A^{\frac{1}{2}}|^2 + |A^{-\frac{1}{2}}|^2\}/2\ , \tag{18a}$$

$j = \frac{3}{2}$, $l = 1, 2$

$$I(\alpha) = \{[|A^{\frac{3}{2}}|^2 + |A^{-\frac{3}{2}}|^2]\, 3(1 - \cos^2\alpha) + [|A^{\frac{1}{2}}|^2 + A^{-\frac{1}{2}}|^2]\,(1 + 3\cos^2\alpha)\}/4\ . \tag{18b}$$

These equations do not distinguish the parities of the states. However, using $\psi^+\sigma_z\psi$, we may also calculate the polarization of particle 3 with respect to the normal to the production plane. Integrating over α and β and defining

the polarization of the resonant state by

$$(19a)\quad \left\{\begin{aligned} \mathscr{P}_{\frac{1}{2}} &= \frac{|A^{\frac{1}{2}}|^2-|A^{-\frac{1}{2}}|^2}{|A^{\frac{1}{2}}|^2+|A^{-\frac{1}{2}}|^2}\,,\\ \mathscr{P}_{\frac{3}{2}} &= \frac{|A^{\frac{3}{2}}|^2+3^{-1}|A^{\frac{1}{2}}|^2-3^{-1}|A^{-\frac{1}{2}}|^2-|A^{-\frac{3}{2}}|^2}{|A^{\frac{3}{2}}|^2+|A^{\frac{1}{2}}|^2+|A^{-\frac{1}{2}}|^2+|A^{-\frac{3}{2}}|^2}\,, \end{aligned}\right.$$

one finds for the z-component of the polarization of particle 3

$$(19b)\quad \left\{\begin{array}{lll} j=\frac{1}{2}\,, & l=0\,, & P_z=\mathscr{P}_{\frac{1}{2}}\,,\\ j=\frac{1}{2}\,, & l=1\,, & P_z=-\mathscr{P}_{\frac{1}{2}}/3\,,\\ j=\frac{3}{2}\,, & l=1\,, & P_z=\mathscr{P}_{\frac{3}{2}}\,,\\ j=\frac{3}{2}\,, & l=2\,, & P_z=-3\mathscr{P}_{\frac{3}{2}}/5\,. \end{array}\right.$$

In the case of interest to us, particle 3 is a hyperon whose polarization can be determined using its parity-violating decay. Since $|\mathscr{P}|\leq 1$, a sufficiently large hyperon polarization rules out the $j=l-\frac{1}{2}$ cases.

After this rather lengthly introduction we now turn to the description of the observed resonances.

3. – The $K^*_{\frac{1}{2}}(890)$ resonance.

This K^* resonance is the only boson strange-particle resonance whose existence may be regarded as certain at present. It was first observed in the reaction

$$(\text{A})\qquad K^-+p\to \overline{K}^0+\pi^-+p\,,$$

by Alston *et al.* [11] in the interaction of 1.15 GeV/c K^--mesons in the LRL 15 in. hydrogen bubble chamber. The original report was based on $\sim$ 20 events in the resonant peak. Figure 3 shows the FD plot of the same reaction based on a new experiment of Alston *et al.* [12] at 1.22 GeV/c. The invariant masses square $M^2_{\overline{K}^0\pi^-}$ *vs.* $M^2_{p\pi^-}$ are used as axes instead of E_p and E_{K^0}. They are, of course, linearly related through (3). The boundary is drawn for a c.m. energy $E=1.895$ GeV. It is clear that the $\overline{K}^0\pi^-$ system strongly prefers to be produced with an invariant mass near 0.9 GeV. It is therefore convenient to think of the interaction as a two-step process

$$(\text{B})\qquad K^-+p\to \overline{K}^{*-}+p\,,$$

followed by

$$(\text{C})\qquad K^{*-}\to \overline{K}^0+\pi^-\,.$$

The isotopic spin of the resonance could be $\frac{3}{2}$ or $\frac{1}{2}$. It can be deduced from the branching ratio to the alternate decay mode

$$\text{(D)} \qquad K^{*-} \to K^- + \pi^0 ,$$

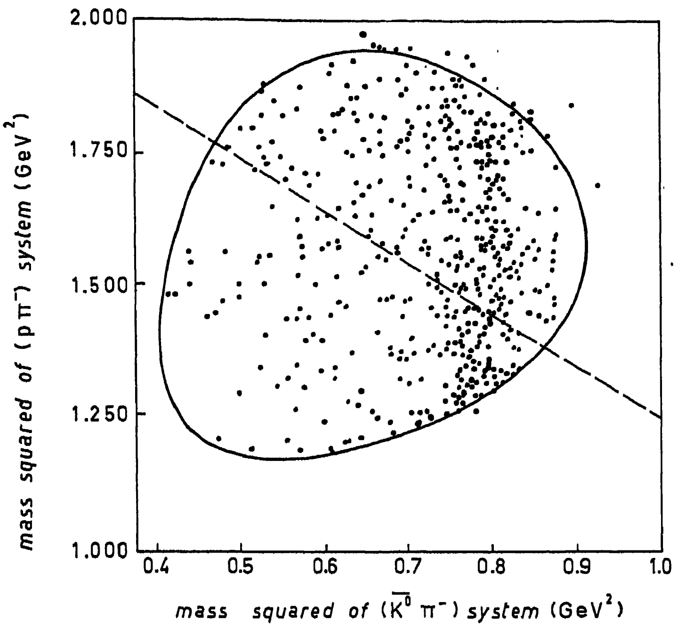

Fig. 3. – DF plot for $K^- + p \to \overline{K}^0 + \pi^- + p$ at 1.22 GeV/c; ref. [12].

by studying the $K^-\pi^0$ mass distribution in

$$\text{(E)} \qquad K^- + p \to K^- + \pi^0 + p .$$

Using the notation (I, I_z)

$$(20) \qquad (\tfrac{3}{2}, -\tfrac{1}{2}) = \{\sqrt{2}(1, 0)(\tfrac{1}{2}, -\tfrac{1}{2}) + (1, -1)(\tfrac{1}{2}, \tfrac{1}{2})\}/\sqrt{3} = $$
$$= \{\sqrt{2}K^-\pi^0 + \overline{K}^0\pi^-\}/\sqrt{3} ,$$

$$(21) \qquad (\tfrac{1}{2}, -\tfrac{1}{2}) = \{(1, 0)(\tfrac{1}{2}, -\tfrac{1}{2}) - \sqrt{2}(1, -1)(\tfrac{1}{2}, \tfrac{1}{2})\}/\sqrt{3} = $$
$$= \{K^-\pi^0 - \sqrt{2}\overline{K}^0\pi^-\}/\sqrt{3} .$$

The branching ratio $R = (K^{*-} \to K^- + \pi^0 / K^{*-} \to \overline{K}^0 + \pi^-)$ is thus expected to be 2 for $I = \frac{3}{2}$, $\frac{1}{2}$ for $I = \frac{1}{2}$. Experimentally [11] $R = 0.75 \pm 0.35$; thus $I = \frac{1}{2}$.

The DF plot of Fig. 3 is readily converted to a mass histogram. This is shown in Fig. 4. This histogram may be fitted by a distribution of the form

$$\mathrm{d}N(M_{K\pi}) = \left\{A + B\left[(M_{K\pi} - M^*_{K\pi})^2 + \left(\frac{\Gamma}{2}\right)^2\right]^{-1}\right\}\varrho(M_{K\pi})\,\mathrm{d}M_{K\pi}, \tag{22}$$

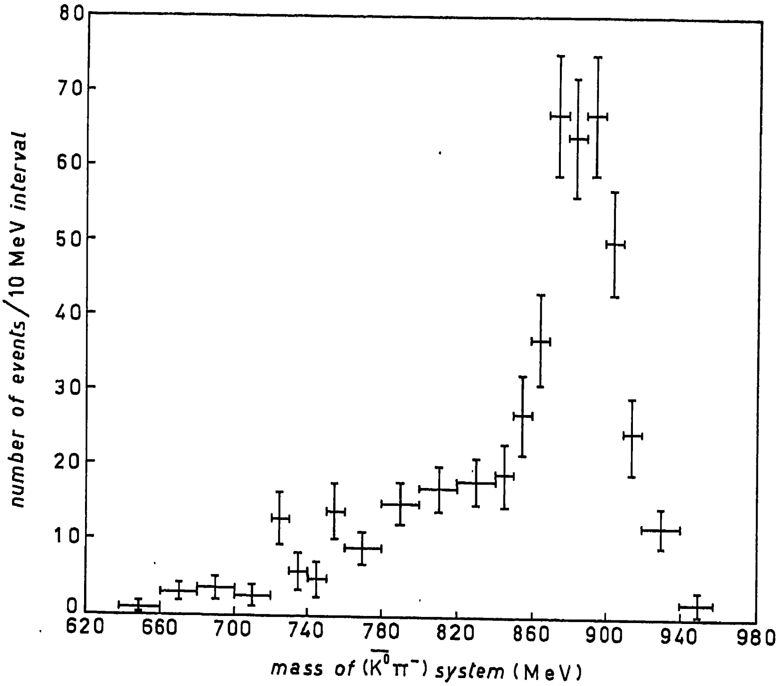

Fig. 4. – ($\overline{K}^0\pi^-$) mass histogram in $K^-+p \to \overline{K}^0+\pi^-+p$; ref. [12].

where $\varrho(M_{K\pi})$ is the phase-space distribution of $M_{K\pi}$ and A is the nonresonant (and presumably phase-space-like) background. The fit yields $M^*_{K\pi} = 0.890$ GeV, $\Gamma = 50$ MeV. Within the resonant region, defined by $0.86\ \text{GeV} < M_{K\pi} < 0.91\ \text{GeV}$, the ratio between the nonresonant and resonant production is $\sim 6\%$.

We turn next to the spin determination of the K* resonance. A glance at Fig. 3 shows that in this experiment the K* does not exhibit any appreciable decay anisotropy with respect to its direction of motion. The K* line appears to be populated with a roughly constant density of events. Figure 5 shows the angular distributions of K* decay in the resonant region in its rest frame with respect to the incident K^- direction, the normal to the production plane $\hat{n}$ and the K* direction of flight. No significant anisotropies appear in any distribution. This cannot be regarded as evidence for $j=0$, however. As suggested in the introduction, anisotropies with respect to the $\hat{n}$ and K* directions need not occur if the K* is produced without alignment or polar-

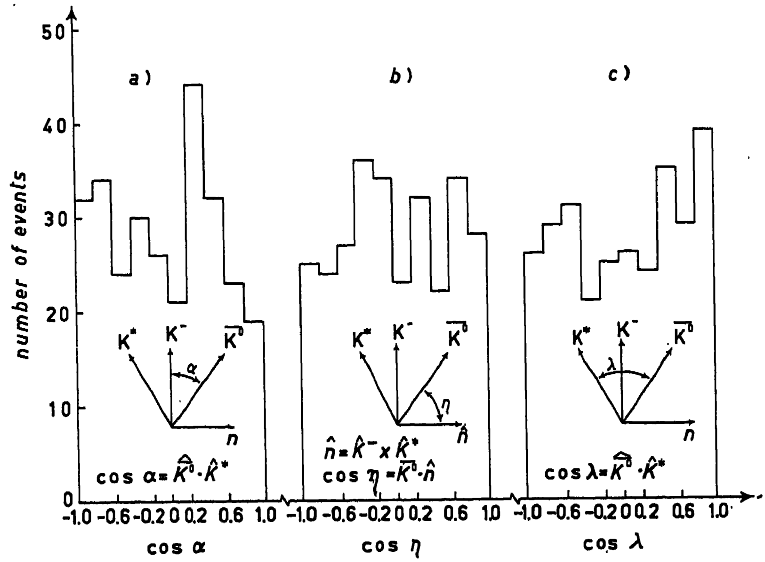

Fig. 5. – Angular distributions in K* decay at 1.22 GeV/c; ref. [12].

ization. The Adair analysis is not directly applicable since the K* decays into two bosons. Furthermore the K* production angular distribution (not shown) indicates that $L_{max} > 2$. Finally, if the intensity of the background is $\sim 6\%$ the interference terms could amount to 25%. In particular, the $I=\frac{3}{2}$, $J=\frac{3}{2}$ $\mathcal{N}^*$ resonance at 1.238 GeV crosses the K* resonance line in Fig. 3 at $M^2_{p\pi} = 1.53$ GeV. While there is no evidence for the $\mathcal{N}^*$ in these data the interference term could be appreciable.

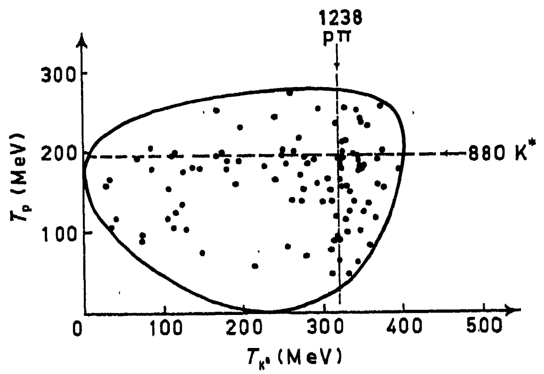

Fig. 6. – DF plot for $K^+ + p \rightarrow K^0 + \pi^+ + p$ at 1.96 GeV/c; ref. [13].

The effect of the $\mathcal{N}^*$ is seen quite clearly in the reaction

(F) $$K^+ + p \rightarrow K^0 + \pi^+ + p\,,$$

studied by Chinowsky *et al.* [13] at 1.96 GeV/c. The DF plot of their data is shown in Fig. 6. Both K* and $\mathcal{N}^*$ lines and a rather curious interference region are readily distinguished.

In the same experiment CHINOWSKY *et al.* [13] studied the reaction

(G) $$K^+ + p \to K^+ + \pi^- + p + \pi^+ .$$

Since this is a reaction with four particles in the final state, the DF plot is not applicable. However, for each event one can plot the invariant mass of the

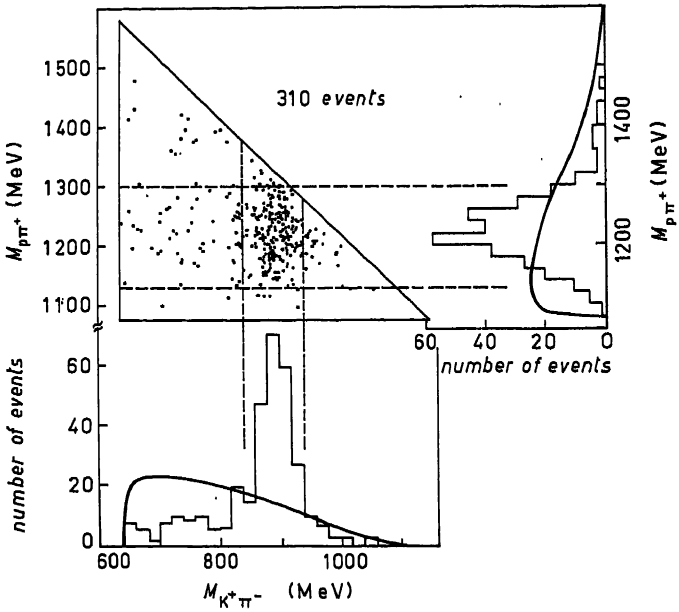

Fig. 7. – Mass plot for $(K^+\pi^-)$ *vs.* $(p\pi^+)$ in $K^+ + p \to K^+ + \pi^- + p + \pi^+$ at 1.96 GeV/c (310 events); ref. [13].

$K^+\pi^-$ system *vs.* that of the $p\pi^+$ system. This is done in Fig. 7. For a given choice of $M_{K^+\pi^-}$, $M_{p\pi^+}$ can vary from $(m_p + m_\pi)$—when the $p\pi^+$ system recoils without any internal kinetic energy—to $(E - M_{K^+\pi^-})$, when the $K^+\pi^-$ system remains at rest in the c.m. The points show a marked concentration in the $K^*\mathcal{N}^*$ region such that it seems appropriate to think of the reaction as

(H) $$K^+ + p \to K^{*0} + \mathcal{N}^{*++} ,$$

with the subsequent decays of both K^* and $\mathcal{N}^*$ yielding the four bodies in the final state. If the data are plotted on a $(K^+\pi^+)$ *vs.* $(p\pi^-)$ mass plot instead, no concentrations appear; since the K^* has $I = \frac{1}{2}$, the doubly charged K^* state cannot exist.

Figure 8 shows the production angular distribution of $\mathcal{N}^*$ in reaction (H). Most of the K* production is in the forward direction in the c.m. In view of the fact that this is the initial K$^+$ direction, it appears that small momentum transfers to the baryon are preferred in this process. The decay distribution of the K* with respect to the initial K$^+$ direction in the K* rest frame for those events where $\cos\Theta \geqslant +0.8$ is shown in Fig. 9. The histogram corresponds to an essentially pure $\cos^2\theta$ distribution. This result can be interpreted using the one-pion exchange picture, Fig. 10, as $K+\pi \to K+\pi$ scattering at point A. Since the exchanged π carries little momentum, the direction of the incident K$^+$ differs little ($\sim 10\%$) from the initial direction in the Kπ c.m. The data therefore show that when the Kπ c.m. energy corresponds to the K* mass the $K+\pi \to K+\pi$ scattering angular distribution is $\cos^2\theta$; therefore $j=1$ is indicated for the K*.

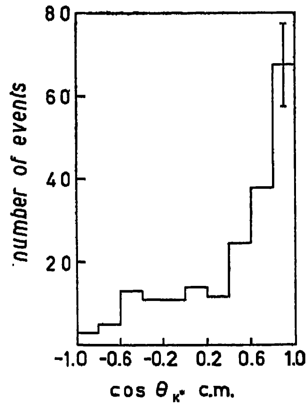

Fig. 8. – Angular distribution of K* and $K^+ + p \to K^* + \mathcal{N}^*_{33}$ at 1.96 GeV/c; $840 \leqslant M_{K^+\pi^-} \leqslant 940$ MeV, $1130 \leqslant M_{\pi^+ p} \leqslant 1300$ MeV; (201 events); ref. [13].

Another experiment which strongly suggests the same conclusion *but involves only symmetry arguments* was suggested by SCHWARTZ [14] and carried out by ARMENTEROS *et al.* [15]. Consider first the reaction

$$\text{(I)} \qquad \bar{p} + p \to K^0 + \bar{K}^0 .$$

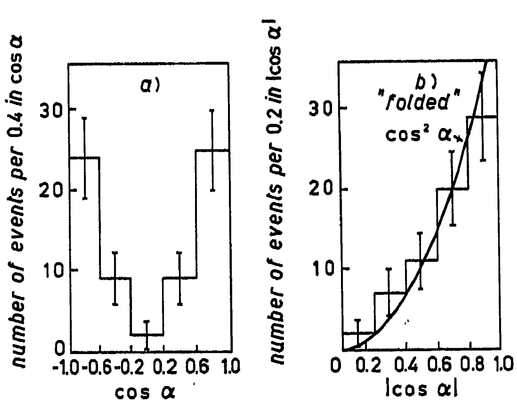

Fig. 9. – Decay angular distribution of the K* with respect to incident K$^+$ direction: $K^+ + p \to K^* + \mathcal{N}^*_{33}$; $1.0 \geqslant \cos\theta_{K^*} \geqslant 0.8$; (69 events;) ref. [13].

As indicated elsewhere [15] in this lecture series the $\bar{p}p$ system has parity $P = (-1)^{L+1}$ and charge-conjugation quantum number $C = (-1)^{L+S}$ where L and S are the orbital angular momentum and

TABLE I.

$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	etc.
0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}	etc.

the spin of the $\bar{p}p$ system, respectively. Thus we can prepare Table I which specifies J^{PC} for each $\bar{p}p$ state.

As far as the $K^0\bar{K}^0$ system is concerned, $P=(-1)^L$, *i.e.* 0^+, 1^-, 2^+, etc. Since J, P and C are conserved in reaction (I) we conclude that the $\bar{p}p$ system in

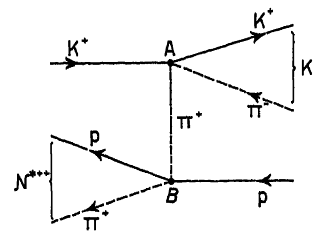

Fig. 10. – One-pion exchange diagram for $K^++p \to K^*+\mathcal{N}^*$.

a) $^1S_0$ does *not* yield $K^0\bar{K}^0$;

b) $^3S_1$ yields $K^0\bar{K}^0$ with $C=-1$;

c) $^1P_1$ and $^3P_1$ does *not* yield $K^0\bar{K}^0$;

d) $^3P_0$ and $^3P_2$ yields $K^0\bar{K}^0$ with $C=+1$, etc.

The $K^0\bar{K}^0$ wave functions for $C=+1$ and $C=-1$ are

(23) $$C=\pm 1\,, \qquad \psi_\pm = (K^0_a\bar{K}^0_b \pm \bar{K}^0_a K^0_b)/\sqrt{2}\,.$$

If we now decompose K^0 and $\bar{K}^0$ into K_1 and K_2 by

(24) $$K_1 = (K^0+\bar{K}^0)/\sqrt{2}\,, \qquad K_2 = (K^0-\bar{K}^0)/\sqrt{2}$$

we find

(25) $$\psi_+ = (K_{a1}K_{b1}-K_{a2}K_{b2})/\sqrt{2}\,, \qquad \psi_- = (K_{a1}K_{b2}-K_{a2}K_{b1})/\sqrt{2}\,.$$

Thus $C=+1$ yields either K_1 or K_2 pairs while $C=-1$ always yields K_1K_2 pairs. ARMENTEROS *et al.* [15] studied *stopping* $\bar{p}$ interactions in a hydrogen bubble chamber. They concentrated on interactions leading to one or two K_1's decaying via the $\pi^+\pi^-$ mode, possibly accompanied by other *neutral* particles

(α) $$p+\bar{p} \to K_1+(\text{neutrals})\,,$$

(β) $$\to 2K_1+(\text{neutrals})\,,$$

The momentum spectra of the K_1's in the two reactions are shown in Fig. 11*a* and *b*. One observes at once a large monochromatic group at ~ 800 MeV/c among the K_1's from (α) which has no counterpart whatever among the K_1's from (β). The total energies of these K_1's is 940 MeV; hence they originate from capture reaction (I). Even if the two K^0-mesons emitted in (I) were uncorrelated one would expect a peak of half the size among the example of (β). The result therefore proves that in the $\bar{p}p$ captures leading to $2K^0$'s, $C=-1$; *i.e.*, that the captures occur from even L states of the $\bar{p}p$ system. The most reasonable interpretation of this observation is that S-state capture dominates.

There is another monochromatic group of K_1's in the sample coming from (α) located at ~ 600 MeV/c. The corresponding peak among K_1's from (β) is hardly outside statistical fluctuations. The mass of the other particle for this K_1^0 peak corresponds to that of the K^*, presumably decaying into $K^0+\pi^0$.

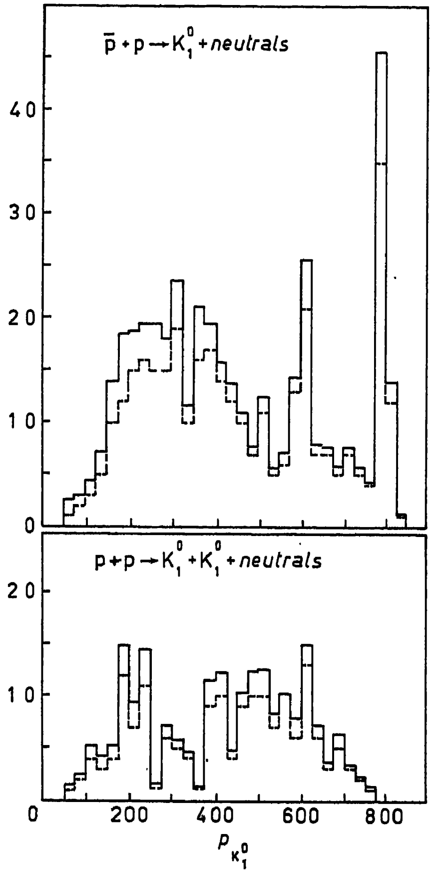

Fig. 11. – Momentum spectra of K_1^0's; $\bar{p}$ captures on protons ——— corrected for decay probabilities; - - - observed; ref. [15].

Let us suppose now that the K^* has spin 0. Since the K^* decays into $K+\pi$ and the pion is pseudoscalar, the parity of a spin-zero K^* relative to the K is odd; the list of possible KK^* angular-momentum and parity states is therefore 0^-, 1^+, 2^-, etc. Since captures occur only from $\bar{p}p$ S-states, 0^{-} is the only possibility for KK^* production and using Table I we conclude that in this case the KK^* system is produced with $C=+1$; however, since the π^0 from K^* decay has $C=+1$, this implies that the final $K^0\bar{K}^0$ system must also have $C=+1$. Therefore there should be twice as many events in the 600 MeV/c peak in the (β) class as there are in the (α) class. ARMENTEROS *et al.* [15] estimate 43 ± 14 events in the (α) peak and 13 ± 11 in the (β) peak Hence the spin-0 hypothesis appears quite unlikely. On the other hand if the K^* spin is 1 then the KK^* J^P list is 1^+, 0^-, 1^-, 2^- etc. and capture from both ^1S and ^3S states of the $\bar{p}p$ system can lead to KK^* production.

4. – The $\Xi_{\frac{1}{2}}^*(1530)$ resonance.

Figure 12 shows DF plots of 66 examples of reaction

$$\text{(J)} \qquad K^-+p \to \Xi^-+\pi^++K^0,$$

and 20 examples of reaction

$$\text{(K)} \qquad K^-+p \to \Xi^-+\pi^0+K^+,$$

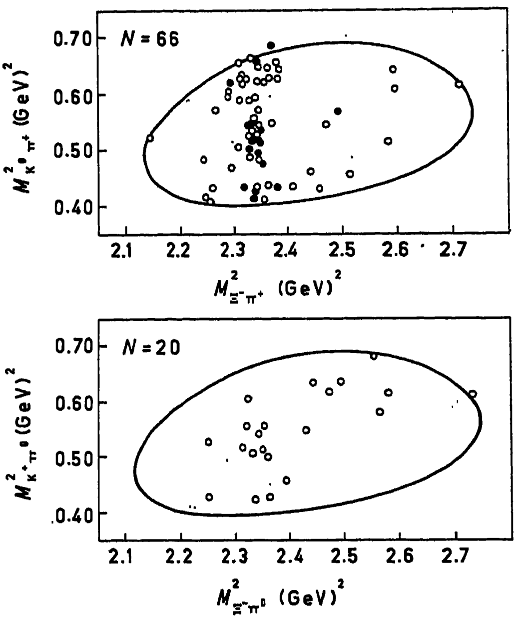

Fig. 12. – DF plot of $K^- + p \to \Xi^- + \pi^{\overset{0}{+}} + K^{\overset{+}{0}}$ at 1.80 GeV/c; ref. [17].

observed by PJERROU *et al.* [17] at 1.80 GeV/c. The solid points in the upper diagram correspond to events in which both the K^0 and the Λ from Ξ^--decay decayed by their *charged* decay modes. Such events can be measured with much greater precision than the others. An example of this event class is shown in Fig. 13. Both DF plots show a concentration of events which peaks at 1.530 GeV. The width Γ, deduced from the events with visible and K^0 decays, is not much greater than the experimental resolution of ~ 5 MeV; PJERROU *et al.* quote $\Gamma \leqslant 7$ MeV. The total $\Xi\pi K$ production cross-section at 1.80 GeV/c is $\sim 100\ \mu b$.

The results of BERTANZA *et al.* [18] for the same two reactions combined but at incident momenta of 2.24 and 2.50 GeV/c are shown in Fig. 14. The Ξ^* peak at $M^2_{\Xi\pi} = 2.35\ (\text{GeV})^2$ appears again. At these higher momenta K^* production also takes place. This makes the data somewhat more difficult to interpret. BERTANZA *et al.* [18] find $\Gamma < 30$ MeV.

To deduce the isotopic spin of the Ξ^* resonance one could compare the decay modes

(L) $$\left\{\begin{array}{lcl} \Xi^{*0} \to \Xi^- + \pi^+ & & \Xi^{*-} \to \Xi^- + \pi^0\,, \\ & \text{or} & \\ \quad\; \to \Xi^0 + \pi^0 & & \quad\;\; \to \Xi^0 + \pi^-\,, \end{array}\right.$$

However, Ξ^0's are hard to identify and to measure. For this reason the following argument using *production* amplitudes seems more reliable. The $\Xi\pi$ system can have $I = \frac{1}{2}$ or $\frac{3}{2}$. Adding the K, one gets a total isospin of 0 or 1 in the first case, 1 and 2 in the second. Since the initial K^-p system has only $I = 0$ and 1 components, we must deal with three production amplitudes, $a_{\frac{3}{2},1}$, $a_{\frac{1}{2},1}$ and $a_{\frac{1}{2},0}$ where the first index is the $\Xi\pi$ I spin, the second the total I spin. The I spin wave function of the final state is therefore

(26) $$a_{\frac{3}{2},1}\chi_1^{\frac{3}{2},\frac{1}{2}} + a_{\frac{1}{2},1}\chi_1^{\frac{1}{2},\frac{1}{2}} + a_{\frac{1}{2},0}\chi_0^{\frac{1}{2},\frac{1}{2}}\,,$$

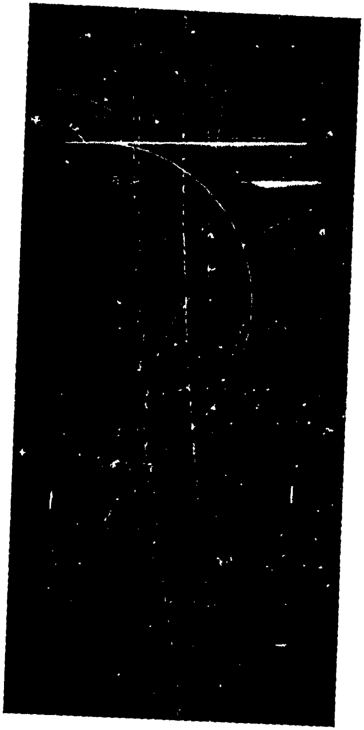

Fig. 13. Example of the reaction $K^- + p \rightarrow \Xi^- + \pi^+ + K^0$ at 1.80 GeV/c with charged Λ and K^0 decays.

where $\chi_r^{p,q}$ means a wave function obtained by vector addition of I-spins p and q to give r. If we write out these functions in terms of charge states, we

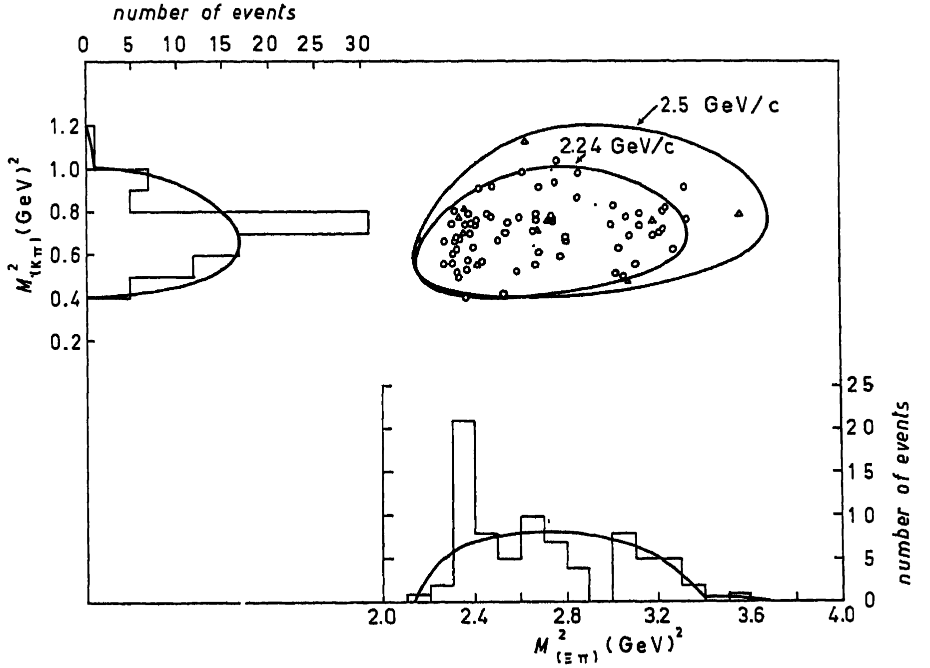

Fig. 14. – DF plot of $K^-+p\to\Xi^-+\pi^{\overset{+}{0}}+K^{\overset{0}{+}}$ at 2.24 and 2.50 GeV/c combined: ○ 68 events at 2.24 GeV/c; △ 11 events at 2.50 GeV/c; ref. [18].

find the total amplitudes for $\Xi^-\pi^+K^0$ and $\Xi^-\pi^0K^+$ production given by

$$\Xi^-\pi^0K^+ \to \{-\sqrt{2}\,a_{\frac{3}{2},1} + (a_{\frac{1}{2},1} - a_{\frac{1}{2},0})\}\,, \tag{27}$$

$$\Xi^-\pi^+K^0 \to \{a_{\frac{3}{2},1} + \sqrt{2}(a_{\frac{1}{2},1} + a_{\frac{1}{2},0})\}\,. \tag{28}$$

Now, the background in Fig. 12 is quite small; most of $\Xi\pi$ production proceeds via the resonance. Hence, if the I spin of the Ξ^* is $\frac{3}{2}$, eqs. (27) and (28) predict a unique $\Xi^-\pi^0/\Xi^-\pi^+$ production ratio of 2. However, we have seen that this ratio is, in fact $\frac{20}{66} = 0.3$ and actually $0.21 \pm .07$ if we restrict the data to the resonant region. Isospin $\frac{3}{2}$ appears to be ruled out by this argument. For a Ξ^* I spin of $\frac{1}{2}$, the production amplitudes $a_{\frac{1}{2},1}$ and $a_{\frac{1}{2},0}$ can interfere to give any production ratio. The experimental results can hardly be in contradiction with this hypothesis.

Some attempts to determine to Ξ^* spin have been made. However, the results are too uncertain to warrant detailed discussion in this survey.

5. – The $Y_0^{**}(1520)$ resonance.

Before discussing the two hyperon-pion $I=0$ resonances it is desirable to review briefly some of the information concerning the $(\overline{K}\mathcal{N})$ interaction at low energies. The following final states can be produced when K^--mesons interact with protons: (K^-p), $(\overline{K}^0n)$, $(\Lambda\pi^0)$, $(\Sigma^+\pi^-)$, $(\Sigma^-\pi^+)$, $(\Sigma^0\pi^0)$, $(\Lambda\pi^+\pi^-)$ and $(\Lambda\pi^0\pi^0)$. All the reactions are exothermic except $(\overline{K}^0n)$ which has a threshold of 90 MeV/c due to the np and $\overline{K}^0K^-$ mass differences. However, in the present discussion we shall disregard this complication and treat (K^-p) and $(\overline{K}^0n)$ as two charge states of the $(\overline{K}\mathcal{N})$ system.

Since the original suggestion of JACKSON *et al.* [19], the language of the zero-range scattering length S-wave approximation has been used in the description of these phenomena [20]. This language seems appropriate because the $(\overline{K}\mathcal{N})$ interaction is very strong and one does not expect that moderate changes of the $(\overline{K}\mathcal{N})$ total energy can seriously modify the wave function within the region of interaction. If this approximation is valid then

$$k \operatorname{ctg} \delta = (1/A)\,, \tag{29}$$

where k is the c.m. wave number, δ, the phase shift and A, the scattering length, assumed to be a constant. The K^-p interaction takes place in both isospin 0 and 1 channels; hence one needs two scattering lengths, A^0 and A^1, for these two channels, respectively. Now, in the $(\overline{K}\mathcal{N})$ case, inelastic interactions leading to the production of hyperons can occur down to zero energy. To take account of these processes the phase shifts, and consequently the scattering lengths, must be complex. The imaginary parts of A^0 and A^1 specify the inelasticity in these two channels. Production of the $(\Lambda 2\pi)$ final state at low energies is negligible because of phase space limitations. For this reason two further numbers are required to specify all the inelastic channels completely: the fraction ε of the $I=1$ inelastic processes which lead to the $(\Lambda\pi^0)$ final state and the phase φ between $(\Sigma\pi)$ production in the $I=0$ and $I=1$ channels. Let M^0 and M^1 be the complex production elements of the $(\Sigma\pi)$ systemin $I=0$ and 1. Then the isospin wave function of the produced $(\Sigma\pi)$ syrtem is

$$\begin{aligned} M^0\chi_0^{1,1}+M^1\chi_1^{1,1} &= \frac{M^0}{\sqrt{3}}\{\Sigma^+\pi^- - \Sigma^0\pi^0 + \Sigma^-\pi^+\} + \frac{M^1}{\sqrt{2}}\{\Sigma^-\pi^+ - \Sigma^+\pi^-\} = \\ &= \left\{\frac{M^0}{\sqrt{3}} - \frac{M^1}{\sqrt{2}}\right\}\Sigma^+\pi^- - \frac{M^0}{\sqrt{3}}\Sigma^0\pi^0 + \left\{\frac{M^0}{\sqrt{3}} + \frac{M^1}{\sqrt{2}}\right\}\Sigma^-\pi^+\,. \end{aligned} \tag{30}$$

The cross-sections for $(\Sigma\pi)$ production in the two isospin channels may thus

be deduced using

$$\sigma^0 = 3\sigma_{\Sigma^0\pi^0}; \qquad \sigma^1 = \sigma_{\Sigma^-\pi^+} + \sigma_{\Sigma^+\pi^-} - 2\sigma_{\Sigma^0\pi^0}. \tag{31}$$

The relative phase φ between M^0 and M^1 determines the relation between $\sigma_{\Sigma^+\pi^-}$ and $\sigma_{\Sigma^-\pi^+}$.

The most recent study of the low-energy K^-p interactions is due to HUMPHREY [21] and ROSS [22]. Using data from all channels and K^--mesons from rest up to 275 MeV/c they find two possible solutions which fit the data. These are given in Table II.

TABLE II.

I	$A_0=(-0.22\pm1.07)+i(2.74\pm0.31)$ fermi $A_1=(+0.02\pm0.33)+i(0.38\pm0.08)$ fermi	$\varphi=+96°$ $\varepsilon=0.40\pm0.03$	$P(\chi^2)=48\%$
II	$A_0=(-0.59\pm0.46)+i(0.96\pm0.17)$ fermi $A_1=(+1.20\pm0.06)+i(0.56\pm0.15)$ fermi	$\varphi=-50°$ $\varepsilon=0.39\pm0.02$	$P(\chi)^2=\ 8\%$

The φ's are evaluated at the K^0n threshold. $P(\chi^2)$ gives the probability of the fit. A recent analysis by AKIBA and CAPPS [23] has shown that, in spite of its lower χ^2 probability, solution II appears to be the correct one.

Let us now write out the differential cross-sections for elastic and charge-exchange scattering and the total inelastic cross-section of a K^-p system in full generality using partial waves.

$$\frac{d\sigma}{d\Omega}\bigg|_{\substack{el\\ce}} = \frac{1}{4k^2}\Big\{\Big|\sum_{l=0}^{\infty}[(l+1)(T^1_{l+}\pm T^0_{l+}) + l(T^1_{l-}\pm T^0_{l-})]P^0_l(\cos\theta)\Big|^2 + \\ + \Big|\sum_{l=1}^{\infty}[(T^1_{l+}\pm T^0_{l+}) - (T^1_{l-}\pm T^0_{l-})]\,P^1_l(\cos\theta)\Big|^2\Big\} \tag{32}$$

$$\sigma_{\text{in}} = \frac{2\pi}{k^2}\sum_{l=0}^{\infty}\big\{[(l+1)(\operatorname{Im}T^1_{l+} - |T^1_{l+}|^2) + l(\operatorname{Im}T^1_{l-} - |T^1_{l-}|^2)] + \\ + [(l+1)(\operatorname{Im}T^0_{l+} - |T^0_{l+}|^2) + l(\operatorname{Im}T^0_{l-} - |T^0_{l-}|^2)]\big\}. \tag{33}$$

These equations assume unpolarized target protons and neglect the Coulomb interaction and the $\bar{K}^0K^0$ and np mass differences. The $T^I_{l\pm}$ matrix elements refer to isospin I, orbital angular momentum l and $j=l\pm\frac{1}{2}$. They are of the form

$$T^I_{l\pm} = (\exp[2i\delta^I_{l\pm}]-1)/2i\,, \tag{34}$$

$\delta^I_{l\pm}$ being complex, of course; $\delta^I_{l\pm}=\beta^I_{l\pm}+i\alpha^I_{l\pm}$. Using (29) one readily finds

the S-wave T-matrix elements in the zero-range approximation

$$T_0^I = kA^I/(1 - ikA^I) \,. \tag{35}$$

If the absorption in a given I, l, and $j = l + \frac{1}{2}$ is complete, *i.e.* $\alpha_{l+}^I \gg 0$ or $T_{l+}^I = i/2$, then the partial absorption cross-section in that channel is $(j+\frac{1}{2})\pi/(2k^2)$. The same result is found for $j = l - \frac{1}{2}$. Hence

$$\sigma_{\max}(I, j) = (j + \tfrac{1}{2})\pi/2k^2 \,, \tag{36}$$

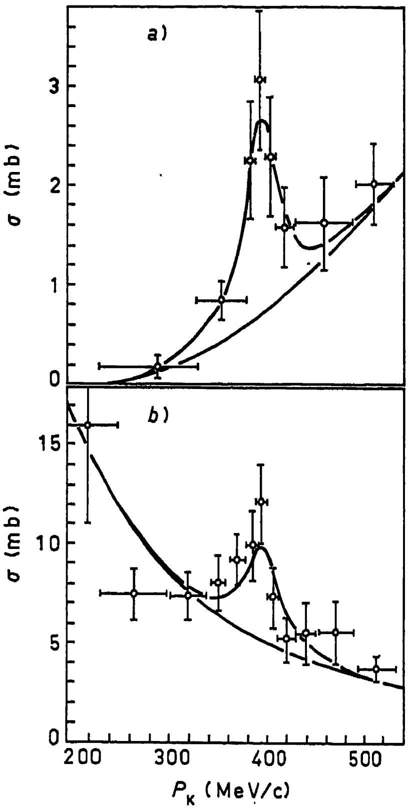

Fig. 15. – Momentum-dependence of the cross-sections of the reactions *a*) $K^- + p \to \Lambda + \pi^+ + \pi^-$ and *b*) $K^- + p \to \overline{K}{}^0 + n$; ref. [24].

The discovery of the Y_0^{**} (1520) was reported by FERRO-LUZZI, TRIPP and WATSON [24]. They studied the interactions of the K^--mesons with protons in the momentum band from 200 to 500 MeV/c. The results for the total $(\Lambda\pi^+\pi^-)$ production cross-section and the total charge exchange cross-section are shown in Fig. 15. Both show a resonant peak at 395 MeV/c corresponding to a total energy of 1.520 GeV in the c.m. and a $\Gamma = 16$ MeV. The elastic scattering cross-section and the other inelastic channels do not show such clear-cut bumps. Nevertheless, using eq. (31) and similar ones for the $(\Lambda 2\pi)$ final state they establish that the resonance occurs in the $I = 0$ state. In fact, summing the cross-sections of all inelastic channels in $I = 0$ one finds a total cross-section approaching π/k^2. According to (36), this implies $j \geq \frac{3}{2}$, since the resonance presumably has definite j and parity. From the sizes of the bumps the branching ratio of the decay of the resonance into $\overline{K}\mathcal{N}$ $\Sigma\pi$ and $\Lambda 2\pi$ is 3:5:1.

Figure 16 shows a study of the momentum-dependence of the coefficients A, B and C in

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2}(A + B\cos\theta + C\cos^2\theta) \,, \tag{37}$$

for elastic scattering and charge exchange. Note that the scales for the two

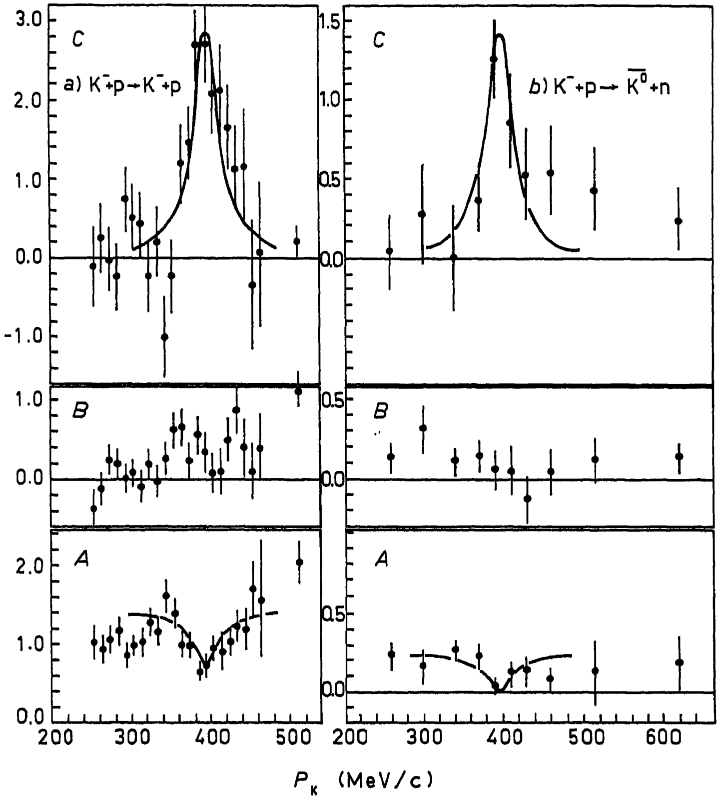

Fig. 16. – Momentum-dependence of the coefficients A, B and C in $(\mathrm{d}\sigma/\mathrm{d}\Omega)=(1/4k^2)(A+B\cos\theta+C\cos^2\theta)$ for elastic K^-p scattering and charge exchange; ref. [24].

are not the same. Only the C terms show a pronounced bump at the resonant energy. The A terms show a slight dip if anything, while the B terms are just small. There is no need for cubic or quartic $\cos\theta$ terms. Hence the Y_0^{**} spin is very likely $\frac{3}{2}$.

To complete the information of the Y_0^{**} one must determine the parity of the resonant state. Let us suppose, first of all, that the partial waves contributing to the angular distribution are nonresonant $I=0$ and 1 S-waves and and an $I=0$ resonant $P_{\frac{3}{2}}$-wave. Using (32) and abbreviating $T_0^{\pm}=T_0^1\pm T_0^0$,

$$(38)\qquad \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\substack{\mathrm{el}\\ \mathrm{ce}}}=\frac{1}{4k^2}\left\{[|T_0^{\pm}|^2+|T_{1+}^0|^2]+[4\,\mathrm{Re}\,T_0^{\pm *}T_{1+}^0]\cos\theta+[3|T_{1+}^0|^2]\cos^2\theta\right\}.$$

This does not fit at all: (38) demands equal C coefficients of (37) for elastic scattering and charge exchange and bumps in the A coefficients one third

the size of those in C. If we choose a resonant $D_{\frac{3}{2}}$-wave instead then

$$\left.\frac{d\sigma}{d\Omega}\right]_{\substack{el\\ ce}} = \frac{1}{4k^2}\left\{\left[\left|T_0^{\pm} - T_{2-}^0\right|^2\right] + 3\left[\left|T_{2-}^0\right|^2 + 2\,\mathrm{Re}\,T_0^{\pm *}T_{2-}^0\right]\cos^2\theta\right\}. \tag{39}$$

One sees readily that these angular distributions do not suffer from the above criticisms. However, one cannot as yet conclude that the resonance occurs in the $D_{\frac{3}{2}}$ state since the same angular distribution can result from a $P_{\frac{3}{2}}$ resonance in the presence of a $P_{\frac{1}{2}}$ background.

$$\left.\frac{d\sigma}{d\Omega}\right]_{\substack{el\\ ce}} = \frac{1}{4k^2}\left\{\left[\left|T_{1-}^{\pm} - T_{1+}^0\right|^2\right] + 3\left[\left|T_{1+}^0\right|^2 + 2\,\mathrm{Re}\,T_{1-}^{\pm *}T_{1+}^0\right]\cos^2\theta\right\}, \tag{40}$$

This is just a Minami transform of (39). But if (40) is correct then one must postulate that somewhere between the low-energy S-wave region studied by HUMPHREY and ROSS and 400 MeV/c, the S-wave has vanished and been replaced by a $P_{\frac{1}{2}}$ partial wave. Since the total cross-sections behave smoothle there must be a transition region where the S and $P_{\frac{1}{2}}$ amplitudes are comparably. In that region

$$\left.\frac{d\sigma}{d\Omega}\right]_{\substack{el\\ ce}} = \frac{1}{4k^2}\left\{\left[\left|T_0^{\pm}\right|^2 + \left|T_{1-}^{\pm}\right|^2\right] + 2\left[\mathrm{Re}\,T_0^{\pm *}T_{1-}^{\pm}\right]\cos\theta\right\}, \tag{41}$$

and one expects to see a large $\cos\theta$ coefficient. Since both channels are expected to be highly absorptive they are very unlikely to be out of phase. No such $\cos\theta$ is seen anywhere between zero and 400 MeV/c. Hence the Y_0^{**} breaks up into a $D_{\frac{3}{2}}$ $\overline{K}\mathcal{N}$ system.

More recently the same authors have shown by means of a polarization analysis of the Σ^+'s in the $(\Sigma^+\pi^-)$ channel of the Y_0^{**} resonance that the $\Sigma^+\pi^-$ system is also in $D_{\frac{3}{2}}$. Therefore the parity of the Σ relative to $\overline{K}\mathcal{N}$ is odd [25].

To avoid the impression that the Y_0^{**} is «somehow» different from the other resonances discussed before because it can be made unaccompanied by other particles we show, in Fig. 17, a DF plot of the reactions

$$K^- + p \to \Sigma^+ + \pi^- + \pi^0, \tag{M}$$

$$\to \Sigma^- + \pi^+ + \pi^0, \tag{N}$$

combined, plotting $M^2_{(\Sigma\pi)^0}$ *vs.* $M^2_{(\Sigma\pi)^\pm}$. These events were observed by ALSTON *et al.* [26] at 1.22 GeV/c. The Y_0^{**} appears as a sharp spike in the mass histogram at $M^2_{(\Sigma\pi)^0} = 2.3$ (GeV)^2. Another broader peak appears at $M^2_{(\Sigma\pi)^0} \sim 2.0$ (GeV)^2. This is the Y_0^* (1405) which we discuss next. The continuous lines are the

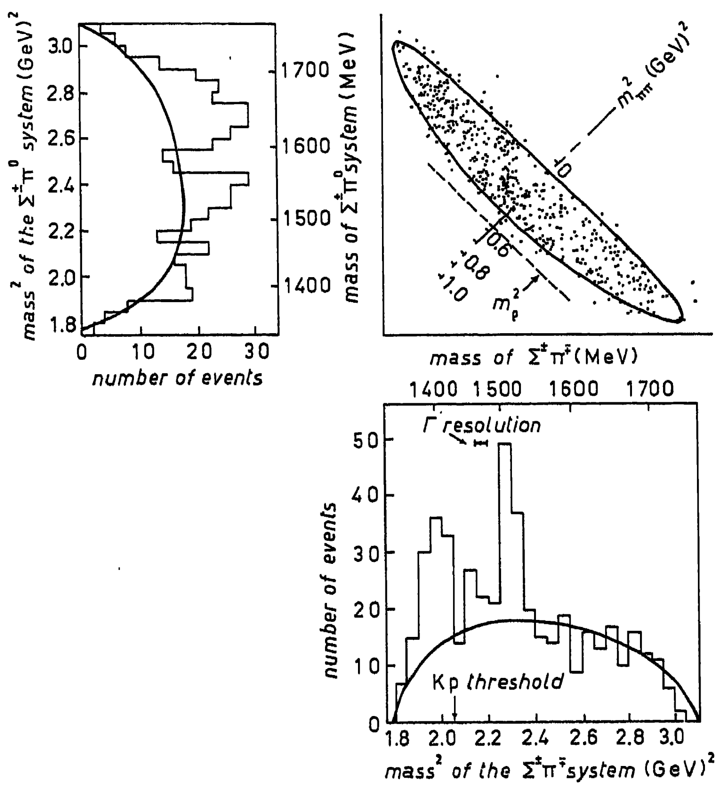

Fig. 17. - DF plot of reaction $K^-p \to \Sigma^\pm + \pi^\mp + \pi^0$ at $P_{K^-} = 1.22$ GeV/c, $E^* = 1.895$ GeV, (473 events); ref. [26].

mass distributions expected from the available phase space. A glance at the DF plot convinces one that the irregularities seen in the $M^2_{(\Sigma\pi)^\pm}$ histogram are actually just the projections of the peaks in $M^2_{(\Sigma\pi)^0}$ into the other charge axis.

6. – The Y_0^* (1405) resonance.

This resonance was discovered [27] in a study of the reactions

$$\text{(O)} \qquad K^- + p \to \Sigma^+ + \pi^- + \pi^+ + \pi^- ,$$

$$\text{(P)} \qquad K^- + p \to \Sigma^- + \pi^+ + \pi^+ + \pi^- ,$$

produced by 1.15 GeV/c K^--mesons. The results of a more recent study [26] of the same two reactions but with greater statistical weight is shown in

Fig. 18. The figure shows histograms of the invariant mass of all neutral and all doubly charged ($\Sigma\pi$) systems produced in the reactions. Twice as many entries appear among the neutral events as among the doubly-charged ones since two neutral ($\Sigma\pi$) systems may be chosen in each of reactions (O) and (P). The solid lines are the predictions of phase-space alone; the dotted lines result

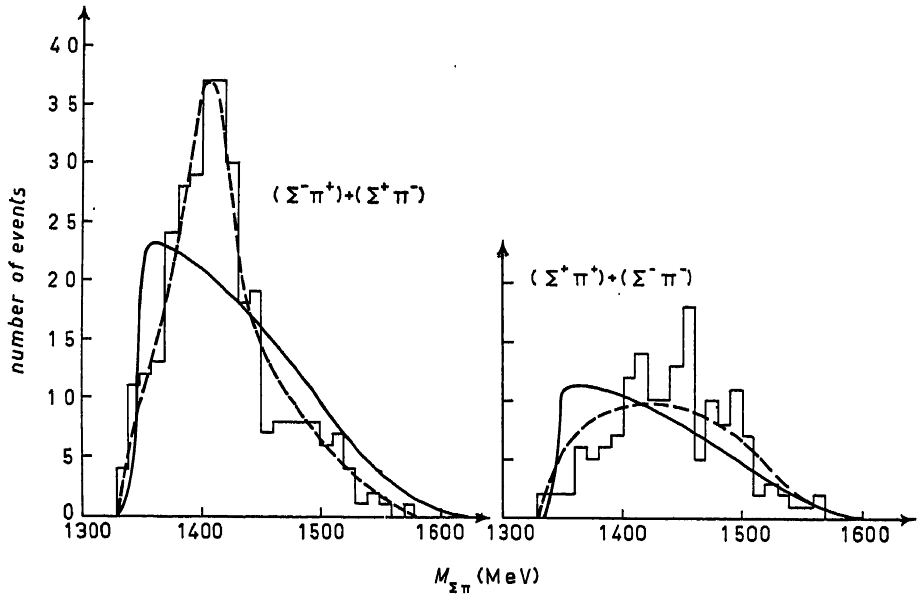

Fig. 18. – Mass histogram of neutral and doubly charged $\Sigma\pi$ systems in the reaction $K^-+p \rightarrow \Sigma^\pm+\pi^\mp+\pi^++\pi^-$ at P_K-1.22 GeV/c; 159 events; phase space: ——— without resonance; – – – with resonance at $M_{\Sigma^\pm\pi^\mp}=1405$ MeV, $\Gamma=50$ MeV; ref. [26].

if phase-space is modified by the assumption that in *each* event the Σ resonates with one of the pions of opposite charge. For this resonant state a mass of 1.405 GeV and a $\Gamma=50$ MeV was used. The fits are evidently good and suggest that the reactions (O) and (P) can be thought of as

$$\text{(Q)} \qquad K^-+p \rightarrow Y^{*0}+\pi^++\pi^-,$$

with subsequent decay of the Y^{*0} into $\Sigma^+\pi^-$ or $\Sigma^-\pi^+$. This is borne out further by the approximate equality of the production cross-sections of (O) and (P).

The isospin of the resonance can be readily deduced from reactions (M) and (N) and Fig. 17. Suppose that this Y^* has isospin 1; then, in the production process leading to $Y^*\pi$, the isospin wave function given by (30) applies, with Y^* taking the place of the Σ. The resonance is observed in the

charge state $Y^{*0}\pi^0$. But, according to (30) at least twice as many events leading to $Y^{*\pm}\pi^{\mp}$ should be produced and half of the $Y^{*\pm}$ should decay via $\Sigma^{\pm}\pi^0$, half via $\Sigma^0\pi^{\pm}$. The absence of a peak in the $M^2_{(\Sigma\pi)^\pm}$ histogram thus shows that this resonance has $I=0$. The same conclusion was reached by ALSTON *et al.* [27] through the study of the reaction

$$\text{(R)} \qquad K^- + p \to \Sigma^0 + \pi^0 + \pi^+ + \pi^- ,$$

which represents the remaining decay mode of the Y_0^* along with reactions (O) and (P).

Figure 19 shows a DF plot of the reactions

$$\text{(S)} \qquad \pi^- + p \to \Sigma^+ + \pi^- + K^0 ,$$

$$\text{(T)} \qquad \to \Sigma^- + \pi^+ + K^0 ,$$

observed by ALEXANDER *et al.* [28] at incident momenta of 1.89 and 2.04 GeV/c.

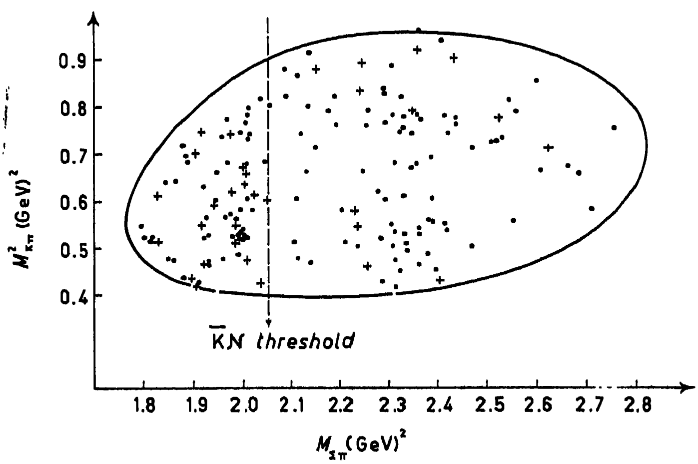

Fig. 19. – DF plot of the reaction $\pi^- + p \to \Sigma^{\pm} + \pi^{\mp} + K^0$ at 1.89 and 2.04 GeV/c combined: + $\Sigma^+\pi^-K^0$ (32 events); ● $\Sigma^-\pi^+K^0$ (127 events); ref. [28].

The K^* resonance and both the Y_0^* and Y_0^{**} resonances are clearly seen. ALEXANDER *et al.* called attention to the asymmetric appearance of the Y_0^* resonance curve. At the high-mass end, where the $(\Sigma\pi)$ invariant mass approaches the $\overline{K}\mathcal{N}$ threshold (1.433 GeV) the intensity seems to drop sharply.

Attempts to deduce the spin and parity of the Y_0^* (1405) from data such as those described above have proved unsuccessful to date. However, there exists fairly good circumstantial evidence which suggests that the resonance

is due to an $s_{\frac{1}{2}}$ bound state of the $I=0$ $\bar{K}\mathcal{N}$ system. How such a bound state might affect the properties of the $\Sigma\pi$ system has been discussed by DALITZ and TUAN [20] and SCHULT and CAPPS [29]. In particular, the latter authors predicted the existence of a resonance with the Y_0^* properties on the basis of a comparison of the intensities of various channels in K^- capture in hydrogen and deuterium. Table III gives the proportions of various $\Lambda\pi$ and $\Sigma\pi$ charge

TABLE III.

Hydrogen		Deuterium	
$\Sigma^-\pi^+$	$44\ \pm 1\%$	$\Sigma^-\pi^+n$	$21\ \pm 1\%$
$\Sigma^+\pi^-$	$20\ \pm 1\%$	$\Sigma^+\pi^-n$	$21\ \pm 1\%$
$\Sigma^0\pi^0$	$28\ \pm 2.5\%$	$\Sigma^0\pi^0 n$	$19\ \pm 1\%$
$\Lambda\pi^0$	$8\ \pm 1.5\%$	$\Lambda\pi^0 n$	$11\ \pm 1\%$
$\Lambda\pi^-\pi^-$	$0.13 \pm 0.06\%$	$\Sigma^-\pi^0 p$	$3.5 \pm 0.5\%$
		$\Sigma^0\pi^- p$	$3.5 \pm 0.5\%$
		$\Lambda\pi^- p$	$21\ \pm 1\%$
		$\Sigma^- p$	$0.5 \pm 0.2\%$
		$(\Lambda$ or $\Sigma^0)p$	$0.5 \pm 0.2\%$

states in such captures [30]. First of all, the fraction of K^- captures in ducterium leading to two baryons *only* is quite small. This suggests that the K^- seldom interacts with the deuteron as a whole. This being the case, one may regard the captures in deuterium as captures on one nucleon while the other does not participate in the primary capture act. SCHULT and CAPPS [29] have shown that final-state scattering between hyperon and spectator nucleon does not affect the proportion of various charge states in the primary capture act; *i.e.* that, apart from final-state scatters of the type $\Sigma^+n \rightarrow \Lambda p$ or $\Sigma^0 p$ and $\Sigma^0 p \rightarrow \Sigma^+ n$, the proportions of various charge states in captures on hydrogen and on the protons in deuterium may be compared directly. Now in hydrogen the $\Sigma^-\pi^+/\Sigma^+\pi^-$ ratio is 2.2 ± 0.1; in deuterium it is 1.00 ± 0.07. If one considers the depletion of Σ^+'s due to final-state scatters, the deuterium ratio at production is lower still. However, as compared to $\Sigma\pi$ systems produced on free protons, $\Sigma\pi$ systems produced in K^-d captures have a somewhat lower energy because the deuterium bond is broken and some energy is given to the spectator neutron. This difference in the $\Sigma\pi$ energy is ~ 10 MeV and results in a change of the c.m. momentum from $q = 0.90\ \text{fermi}^{-1}$ at K^-p threshold to $q = 0.85\ \text{fermi}^{-1}$. According to (30)

$$\frac{\Sigma^-\pi^+}{\Sigma^+\pi^-} = \frac{|(M^0/\sqrt{3}) + (M^1/\sqrt{2})|^2}{|(M^0/\sqrt{3}) - (M^1/\sqrt{2})|^2}. \tag{42}$$

We have seen that the Humphrey-Ross solution II requires a phase angle φ between M^0 and M^1 of $-50°$ at the $\overline{\mathrm{K}}^0\mathrm{n}$ threshold; the deuterium data suggest that 10 MeV below the $\mathrm{K}^-\mathrm{p}$ threshold $\varphi \geqslant 90°$. This large change of the relative phase between M^0 and M^1 for a change of c.m. momentum of $\sim 5\,\%$ is hard to understand unless one of the M's has a resonant behavior. A glance at Table III reveals that the $\Sigma^{\bar{0}}\pi^{\underline{0}}\mathrm{p}$ rates are quite small. K^- captures on neutrons take place in $I=1$ only. Therefore it is probable that the resonant behavior, if it exists, involves M^0.

Following a treatment given by DALITZ [31], we now explore the consequences of an $I=0$ $\mathrm{K}\mathcal{N}$ bound s-state in more detail. Let u/r and v/r be the radial wave functions of the $\overline{\mathrm{K}}\mathrm{n}$ and $\Sigma\pi$ systems involved here, respectively, and, in view of the small energies, let us use the Schrödinger equation. We again take the point of view that the interactions are so strong that the wave functions within the interaction region ($r \leqslant R$) are not affected by changes of the total energy of the system. Then at $r=R$, the following boundary conditions apply to the exterior solutions

$$(43)\qquad \begin{cases} u\Big|_R = \alpha \dfrac{\mathrm{d}u}{\mathrm{d}r}\Big|_R + \beta \dfrac{\mathrm{d}v}{\mathrm{d}r}\Big|_R , \\[2ex] v\Big|_R = \beta \dfrac{\mathrm{d}u}{\mathrm{d}r}\Big|_R + \gamma \dfrac{\mathrm{d}v}{\mathrm{d}r}\Big|_R . \end{cases}$$

Time reversal invariance demands that α, β and γ be real. Let us consider two cases: one, where the energy of the system is *above* the $\overline{\mathrm{K}}\mathcal{N}$ threshold *and* the incident system is $\overline{\mathrm{K}}\mathcal{N}$, while the outgoing system may be $\overline{\mathrm{K}}\mathcal{N}$ or $\Sigma\pi$; the other, where the energy is *below* the $\overline{\mathrm{K}}\mathcal{N}$ threshold and the incident and outgoing systems are $\Sigma\pi$.

a) $E > m_{\overline{\mathrm{K}}} + m_{\mathcal{N}}$

$$(44)\qquad u = A\,\{\exp[2i\delta_{\overline{\mathrm{K}}}]\exp[ikr] - \exp[-ikr]\} \qquad v = B\exp[iqr]\,,$$

where

$$(45)\qquad E = m_{\overline{\mathrm{K}}} + m_{\mathcal{N}} + (k^2/2\mu) = m_\Sigma + m_\pi + (q^2/2\mu_\Sigma)\,,$$

μ_{K} and μ_Σ being the reduced masses of the $\overline{\mathrm{K}}\mathcal{N}$ and $\Sigma\pi$ systems, respectively. Substituting into (43) and invoking the zero range approximation, $kR \approx 0$, $qR \approx 0$ one obtains

$$(46)\qquad A_{\overline{\mathrm{K}}} = (k \operatorname{ctg} \delta_{\overline{\mathrm{K}}})^{-1} = \alpha - \frac{q^2\gamma\beta^2}{1+q^2\gamma^2} + i\,\frac{q\beta^2}{1+q^2\gamma^2}\,.$$

Just above the $\overline{\mathrm{K}}\mathcal{N}$ threshold q varies slowly; to first approximation we may replace q by its value q_0 at the $\overline{\mathrm{K}}\mathcal{N}$ threshold

$$(q_0^2/2\mu_\Sigma) = m_{\overline{\mathrm{K}}} + m_{\mathcal{N}} - m_\Sigma - m_\pi \,, \tag{47}$$

$A_{\overline{\mathrm{K}}}$ is just the $\overline{\mathrm{K}}\mathcal{N}$ scattering length in $I=0$. So, if we set $A_{\overline{\mathrm{K}}} = A^0 = a^0 + ib^0$ and compare with (46)

$$\alpha = a^0 + b^0 q_0 \gamma \,, \qquad q_0 \beta^2 = b^0[1 + (q_0\gamma)^2] \,. \tag{48}$$

As a^0 and b^0 are fixed using the measured zero-range scattering length, γ is the only free variable.

b) $m_\Sigma + m_\pi < E < m_{\overline{\mathrm{K}}} + m_{\mathcal{N}}$

$$u = A \exp[-\lambda r] \,, \qquad v = B\,\{\exp[2i\delta_\Sigma]\exp[iqr] - \exp[-iqr]\} \,, \tag{49}$$

where

$$E = m_{\overline{\mathrm{K}}} + m_{\mathcal{N}} - (\lambda^2/2\mu_{\mathrm{K}}) \,. \tag{50}$$

Substituting (49) into (43)

$$A_\Sigma = (k \operatorname{ctg} \delta_\Sigma)^{-1} = \gamma - \beta^2 \frac{\lambda}{1 + \alpha\lambda} \,. \tag{51}$$

The $\Sigma\pi$ scattering length depends on λ, *i.e.* on the proximity of the $\overline{\mathrm{K}}\mathcal{N}$ threshold. If $\alpha < 0$, then for $\lambda = -1/\alpha$, A_Σ becomes infinite, and the $\Sigma\pi$ elastic scattering cross-section, generally given by

$$\sigma = 4\pi \frac{A_\Sigma^2}{1 + q^2 A_\Sigma^2} \,, \tag{52}$$

approaches $4\pi/q^2$. In the zero range approximation a negative value of α implies a bound $\overline{\mathrm{K}}\mathcal{N}$ state. We have seen that the Humphrey-Ross solution II has $\alpha < 0$, but the statistical error is large. Figure 20 shows plots of eq. (52) for the observed value of $b^0 = 0.96$ fermi and a^0 values displaced one and two standard deviations from the measured value of -0.59 fermi with γ as parameter. It appears that a^0 values from -1.0 to -1.5 fermi and γ values in the range from -0.5 to -1.0 fermi can reproduce the observed Y_0^* resonance including its steeper drop towards the $\overline{\mathrm{K}}\mathcal{N}$ threshold. If this inter-

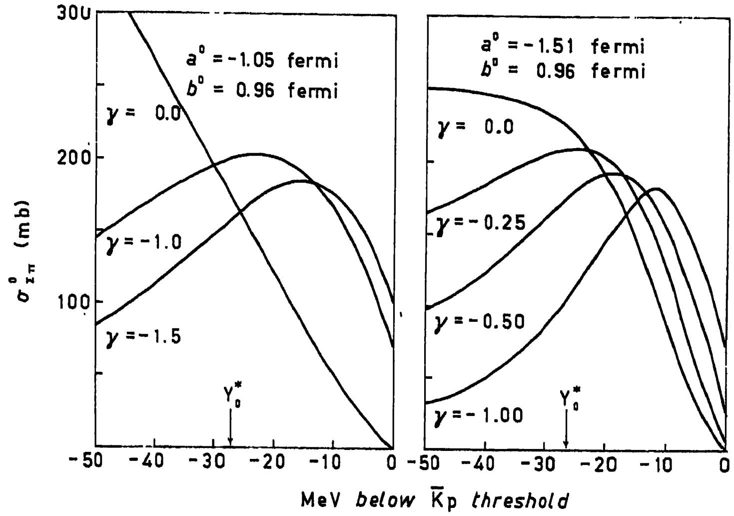

Fig. 20. – Calculation of the $I=0$ $\Sigma\pi$ elastic S-wave cross-section based on the zero-range approximation.

pretation is valid then this Y_0^* resonance is due to the coupling of the $\Sigma\pi$ state to a $\bar{K}\mathcal{N}$ $I=0$ $s_{\frac{1}{2}}$ bound state. An odd $K\mathcal{N}\Sigma$ parity was assumed in the calculation.

7. – The Y_1^* (1385) resonance.

The original discovery of the Y_0^* resonance was based on the data shown in Fig. 21. This DF plot was obtained from the study of the reaction

$$(U) \qquad K^- + p \to \Lambda + \pi^+ + \pi^- ,$$

using 1.15 GeV/c incident K^--mesons [1]. The DF plot indicates the presence of a pronounced $\Lambda\pi$ resonance such that reaction (U) may be regarded as

$$(V) \qquad \begin{cases} K^- + p \to Y_1^{*\pm} + \pi^\mp , \\ Y_1^{*\pm} \to \Lambda + \pi^\pm . \end{cases}$$

The peak of the resonance occurs at 1.385 GeV and $\Gamma \approx 60$ MeV. Its isospin is clearly one since the Λ isospin is zero. From Fig. 21 it is evident that the production rates of Y_1^{*+} and Y_1^{*-} are unequal. Since the Y_1^* has isospin one, this is readily understood as due to interference between the $I=0$ and

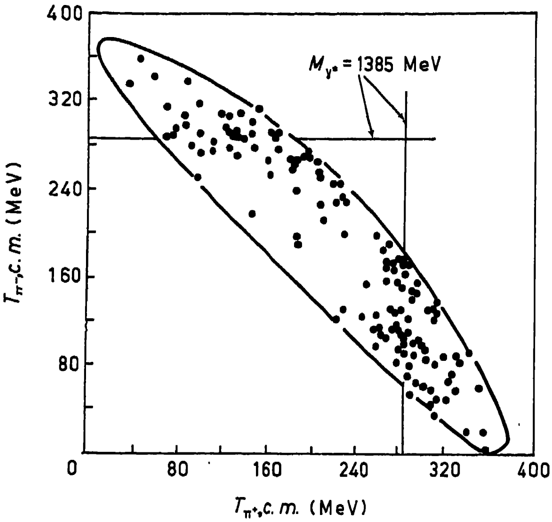

Fig. 21. – DF plot of the reaction $K^-+p \to \Lambda + +\pi^+ + \pi^-$ at 1.15 GeV/c; ref. [1].

$I=1$ incident channels (see eq. (42) with Y_1^* replacing Σ). The resonance has also been observed in the reactions [2]

(W) $K_2^0+p \to$

$$\to Y_1^{*0}+\pi^+ \to \Lambda+\pi^0+\pi^+ ,$$

and the capture reactions [4, 5]

(X) $K^-+d \to Y_1^{*-}+p ,$

$$K^-+{}^4He \to Y_1^{*-}+{}^3He .$$

In their original report Alston *et al.* [1] remarked on the similarity in position and shape of the Y_1^* resonance and the $\mathcal{N}_{\frac{3}{2}}^*$ pion-nucleon resonance. However, the determination of the spin and parity of the resonance has proved difficult. Let us review first the attempts to determine the spin by means of the Adair analysis. At low momenta only the lowest-production partial waves are expected to be involved and hence this region seems most suitable for the Adair analysis. However, at low momenta the Y_1^* bands overlap. Figure 22 shows the data for reaction (U) obtained by Berge *et al.* [3] at 850 MeV/c where the bands are just barely separated. The figure shows a considerable enhancement of the Y_1^* intensity in the region where the two bands approach.

This effect was explained by Dalitz and Miller [32] as due to the Bose statistics

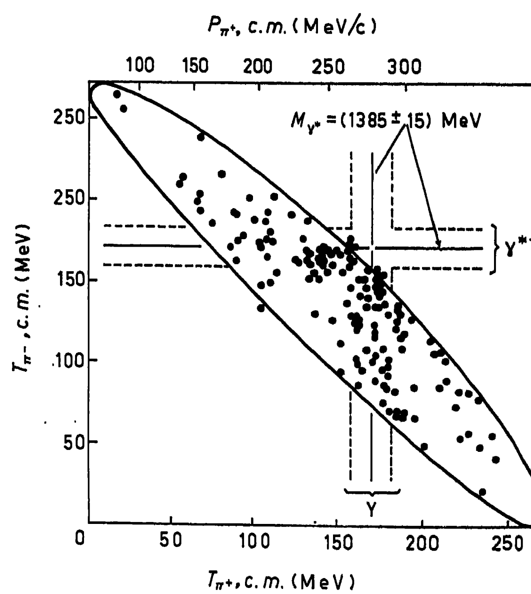

Fig. 22. – DF plot of the reaction $K^-+p \to \Lambda+\pi^++\pi^-$ at 0.85 GeV/c; ref. [3].

of the two final-state pions. The enhancement occurs in the region where the two pions have roughly equal and opposite momenta and the Λ has very low momentum. If we wish to use the Y_1^* concept then we must say that the Y_1^* prefers to decay such as to emit its decay pion in the direction opposite to that of the primary pion—in other words, along its own direction of flight. As we have seen in the introduction, such an asymmetry cannot occur in the decay of an isolated system in a pure state if the decay obeys conservation of parity. Hence, when the bands cross or approach closely on

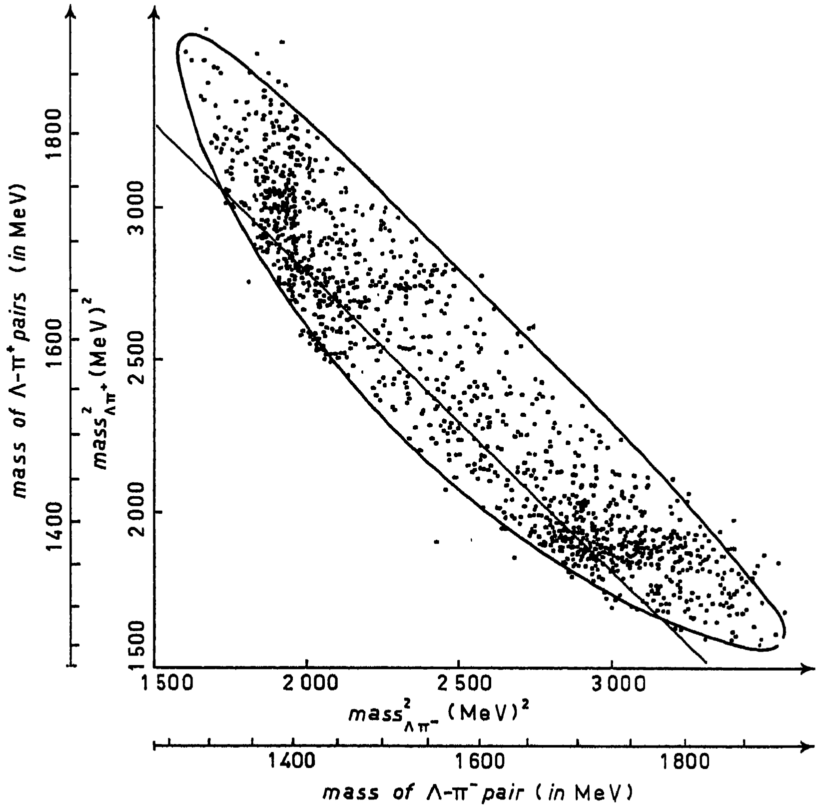

Fig. 23. – DF plot of the reaction $K^-+p\to\Lambda+\pi^++\pi^-$ at $P=1.51$ GeV/c, $E^*=2026$ MeV 1268 events; ref. [34].

the DF plot, the Y_1^* may not be regarded as a free system. The Adair distribution of the Y_1^* decay then becomes much harder to interpret.

At 1.11 GeV/c the Y_1^* production angular distribution already yields a dominant $\cos^4\Theta$ term [33]. As a result, the Adair analysis cannot be expected to be valid for $|\cos\Theta|<0.95$. In the region $|\cos\Theta|\geqslant 0.95$ the data are too meagre to have statistical significance.

A DF plot (plotted by a 7090 computer) of reaction (V) at 1.51 GeV/c is shown [34] in Fig. 23. At this momentum, in addition to $Y_1^{*\pm}$ production, one also observes the production of the ρ with $M_\rho = 0.75$ GeV.

$$\text{(Y)} \qquad K^- + p \to \Lambda + \rho \to \Lambda + \pi^+ + \pi^- .$$

The ρ resonance should peak along the diagonal line in Fig. 23. One observes in addition that the ρ exhibits constructive interference with Y_1^{*-}, destructive interference with Y_1^{*+}. The production of the ρ further complicates the analysis.

As was pointed out in the introduction, the resonant state may be produced polarized or aligned along the normal to the production plane. If it has $j > \frac{1}{2}$, it will then show an anisotropy with respect to the normal to the production plane in its decay. ELY *et al.* [33] have studied reaction (V) from this point of view and observed a distribution of the form $1 + c\cos^2\alpha$, with $c = +1.5 \pm 0.4$, where α is the angle between the Λ direction and the normal to the Y_1^* production plane in the Y_1^* rest frame. The presence of a nonvanishing coefficient of $\cos^2\alpha$ of course proves that the Y_1^* spin $j > \frac{1}{2}$, and most likely $j = \frac{3}{2}$.

A nonvanishing c coefficient may also be produced by the interference of a $j = \frac{1}{2}$ resonance with a background of higher j. If c is due to an interference between a resonance and a background, its magnitude should vary rapidly through the resonant region. ELY *et al.* [35] have searched for such a mass-dependent effect and have not seen it. It therefore appears unlikely that the $\cos^2\alpha$ term is due to interference.

Table IV gives c as reported by BUTTON-SHAFER *et al.* [34] for reaction (V) at higher momenta.

TABLE IV.

P_{K^-}	Y_1^{*+}	Y_1^{*-}
1.22 GeV/c	0.48 ± 0.36	0.52 ± 0.33
1.51 GeV/c	0.4 ± 0.6	0.4 ± 0.8

The coefficients are much smaller but they all have the same sign; furthermore they agree with the sign as reported by ELY *et al.* It is well to keep in mind that there is no need for observation at the various momenta to agree. In particular, the total K^-p total cross-section is known to have a strong $I = 0$ enhancement or resonance [36] at 1.05 GeV/c. The rapid variation of the magnitude of the $\cos^2\alpha$ coefficient above 1.11 GeV/c may be related to the behavior of the total K^-p cross-section in that region.

BERLEY *et al.* [37] have studied the reaction

$$\text{(Z)} \qquad \pi^- + p \to Y_1^{*\frac{0}{-}} + K^{\frac{0}{+}},$$

at 2 GeV/c. In their sample of events the coefficient c is 1.23 ± 0.77. In addition, they observed that the Λ's from Y_1^* decay were polarized with respect to the normal to the Y_1^* production plane with $|\alpha_\Lambda P| = .56 \pm .17$. If the Y_1^* spin is $\frac{3}{2}$, then the size of c imposes limits on the maximum polarization which may be observed. These limits are 0.47 for $p_{\frac{3}{2}}$ and 0.28 for $d_{\frac{3}{2}}$. The Λ polarization therefore suggets a $p_{\frac{3}{2}}$ Y_1^*-decay. In view of all this information taken together it appears likely at present, but not certain, that the Y_1^* resonance is in the $p_{\frac{3}{2}}$ state.

In conclusion we mention that reactions (M) and (N) which were used to determine the Y_0^* isospin may also be used to establish an upper limit on the branching ratio $(Y_1^* \to \Sigma\pi)/(Y_1^* \to \Lambda\pi)$. This limit at present is $2 \pm 2\,\%$ [38].

8. – Conclusion.

Table V summarizes the results on the properties of strange-particle resonances as known at present. The parities P specified in this Table are defined relative to the Λ. The $K\mathcal{N}\Sigma$ and $K\mathcal{N}\Lambda$ parities are assumed to be odd.

TABLE V.

	$K_{\frac{1}{2}}^*$	Y_1^*	Y_0^*	Y_0^{**}	$\Xi_{\frac{1}{2}}^*$
Mass	890 MeV	1385 MeV	1405 MeV	1520 MeV	1530 MeV
Γ	50 MeV	60 MeV	50 MeV	16 MeV	$\lesssim$ 7 MeV (?)
S	± 1	-1	-1	-1	-2
I	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$
j	1	$\frac{3}{2}$ (?)	$\frac{1}{2}$ (?)	$\frac{3}{2}$	(?)
P	—	+ (?)	— (?)	—	(?)
Decay Modes	$K\pi$	$\Lambda\pi$ $\Sigma\pi$ $2 \pm 2\%$	$\Sigma\pi$	$\overline{K}\mathcal{N}$, $\Sigma\pi$, $\Lambda 2\pi$ 3 : 5 : 1	$\Xi\pi$

In view of the rapid developments in this field it is most unlikely that this represents a complete list. In particular, good preliminary evidence exists for an $I=1$ $\overline{K}\mathcal{N}$ resonance at 1680 MeV [28] and a $I=0$, $\overline{K}\mathcal{N}$ resonance at 1810 MeV [36]. We hope that this report will not be seriously outdated by the time it reaches print.

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Form Factors of Elementary Particles.

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1. – Introduction.

In the general frame of strong interactions the problem of the electromagnetic form factors of the nucleons is of particular interest. Although there is no complete theory, an extensive amount of experimental data is available [1], and there exist a large number of theoretical calculations and models [2, 3]. The reason for the theoretical popularity of this problem of the form factors lies partly in the fact that among all the difficult problems concerning nucleons and pions, this is a particularly simple one: it selects only very few channels (exactly 4) from the very large number of channels connected by strong interactions and probes these alone.

This consideration may be made clear by comparing the absorption of a real photon by a nucleon with the elastic scattering of electrons on protons. In the first case (Fig. 1) the photon is capable of exciting an overwhelming number of possible final states containing any number of pions and other particles, with different quantum numbers: in particular, with different angular

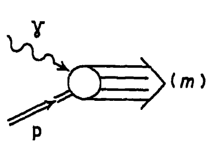

Fig. 1.

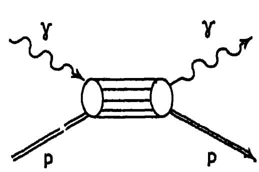

Fig. 2.

momenta obtained via electric and magnetic multipole absorption. Also in the case of proton Compton scattering the situation is not simple, because it can be easily seen that any possible final state (m) produced in the diagram of Fig. 1 can contribute as an intermediate state to the proton Compton

scattering (Fig. 2). Therefore, in order to calculate the latter process, one must have a great deal of information about the mechanisms of strong interactions.

At low-energies the approximate understanding and the evaluation of this process has been possible only by assuming that one particular intermediate state (the 3-3 resonance) dominates over all others.

On the other hand, the elastic scattering of electrons on nucleons (to the lowest order in the electromagnetic coupling) presents much simpler features. To illustrate this point consider:

$$\text{(1)} \qquad e+p \to e+p\,,$$

as described by the graph of Fig. 3.

Here p_i and p_f are the initial and final 4-momenta of the electron, $\mathfrak{P}_i$ and $\mathfrak{P}_f$ those of the proton. The 4-momentum of the intermediate photon is then

$$q = p_i - p_f = \mathfrak{P}_f - \mathfrak{P}_i\,.$$

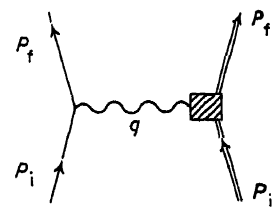

Fig. 3.

The components of 4-momentum p of a particle are given in terms of its total energy E and of its 3-momentum $\boldsymbol{p}$ by

$$p^0 = E\,, \qquad p^{1,2,3} = (\boldsymbol{p})^{1,2,3}\,.$$

The square of the four-momentum p is defined in our metric by

$$\text{(2)} \qquad p^2 = p^{02} - \boldsymbol{p}\cdot\boldsymbol{p} = m^2\,,$$

where m is the rest mass of the particle.

Because of (2) the 4-vectors will be represented by a mapping of the type shown in Fig. 4, where for simplicity the spatial part $\boldsymbol{p}$ is reported on only one axis.

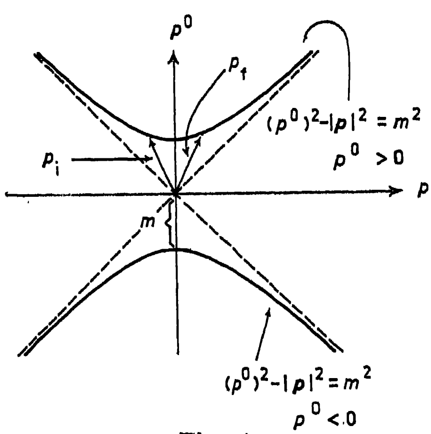

Fig. 4.

Both p_i and p_f terminate on the mass hyperboloid in the forward light cone, with positive energies as shown.

The vector q, as the difference between two vectors in the forward light cone of the same magnitude, has always a negative square (*i.e.*, $q^0 < |\boldsymbol{p}|$): $q^2 < 0$.

Let us now go back the case of process (1).

The electron scatters and emits radiation, which is seen from the proton

as the Møller potential of the electron. In the center-of-mass system of the initial particles, the energies e_i, e_f of the initial and final electrons are equal, and then

$$q:(0, \boldsymbol{q}); \qquad q^2 = -|\boldsymbol{q}|^2 < 0 .$$

We see that the intermediate photon in the c.m. system carries no energy, but only momentum, and when it is absorbed, it leaves the proton in the ground state. This process of absorption is then a very simple one, since it can be represented as the nucleon ground state expectation value of a simple known electro-dynamic operator (approximately, the static charge distribution at low energies). We see the difference with respect to the absorption of a real γ; in this process the photon transfers energy and momentum in equal measure, exciting resonant absorption frequencies of the nucleon (*i.e.* the channels of meson production). These many channels also contribute in the Compton scattering as shown by Fig. 2. In contrast only the few channels with the same quantum numbers of the proton are excited by the photon absorbing on the initial proton p_i in the box in Fig. 3 since only a single proton p_f emerges into the final state.

In order to explain further the very limited number of channels entering in the electron-proton scattering analysis, it is convenient to look at the so-called « crossed » reaction, *i.e.* at the electron-positron annihilation into a nucleon-antinucleon pair through the exchange of a single photon

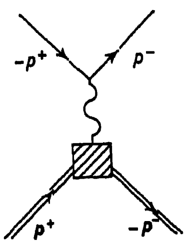

Fig. 5.

$$\text{(3)} \qquad e^+ + e^- \to p + \bar{p} ,$$

(Fig. 5) (or to the inverse reaction $p + \bar{p} \to e^+ + e^-$). Here p^-, p^+ are the 4-momenta of electron and positron, $\mathfrak{P}^+$, $\mathfrak{P}^-$ those of proton and antiproton respectively. This process is closely related to process (1), if one looks at the incoming positron as an outgoing electron propagating backward in time with negative energy, $-p^+$ (and similarly for the outgoing antiproton). As we shall see later, the amplitudes for (1) and (3) are closely related by a simple substitution rule, being represented by the same analytic function of energy and momentum transfer. In the case (3) the 4-momentum transfer to the nucleon is given by

$$q = p^- - (-p^+) = p^- + p^+ .$$

In the center-of-mass system ($E^- = E^+$)

$$q:(2E^-, \boldsymbol{O}); \qquad q^2 = 4(E^-)^2 > 0 .$$

Here $q^2 > 0$ is timelike since it is the *sum* of two timelike 4-vectors within the forward light cone.

The electromagnetic signal (virtual photon) seen by the proton carries no momentum, but only energy, appearing as a spatially uniform oscillating electric field. It transmits only one unit of angular momentum and no higher multipoles. To show this explicitly consider the electromagnetic current $j^\mu(x)$ generated by the « scattered » electron. It satisfies a current conservation law: $(\partial/\partial x^\mu) j^\mu(x) = 0$. By taking the Fourier transform of this equation with

$$j^\mu(x) = \frac{1}{(2\pi)^4}\int d^4q j^\mu(q) \exp[-iq\cdot x]\,,$$

one has equivalently $q_\mu j^\mu(q) = 0$. Since in the c.m. system $\boldsymbol{q} = 0$ it follows that $j^0(q) = 0$. The nonvanishing part of the current is then just the 3-vector $\boldsymbol{j}(q)$. This space vector carries one unit of angular momentum; hence the systems to which it can be coupled must also have $j = 1$. Furthermore, a polar vector such as $\boldsymbol{j}$ has negative parity; the systems coupled to it must therefore also have negative parity. Since a photon is odd under charge conjugation; *i.e.*, $C_\gamma = -1$, also these systems must have $C = -1$. The isospin of these systems can be 0 or 1, because these are the allowed values for the final $p\bar{p}$ system. Therefore the quantum numbers of the systems which connect the intermediate photon to the nucleon-antinucleon pair are

$$j = 1\,; \qquad p = -1\,; \qquad C = -1\,; \qquad I = 1 \text{ or } 0\,. \tag{4}$$

The configurations of the final $p\bar{p}$ system corresponding to the quantum numbers (4) are only the following

$$^3S_1 \qquad \text{and} \qquad ^3D_1\,.$$

By combining these two possibilities corresponding to the different possible spin orientations with the two possible values of the isospin (0 and 1) one arrives at the 4 channels allowed for the final state. The e^+-e^- annihilation, and also the e-p scattering since the same selection rules apply in the intermediate channels, must then be described by 4 scalar functions of the invariant q^2.

We notice that in nature there exist resonances which have just the quantum numbers (4). They are the ρ-meson (isospin $I=1$) and the ω-meson (isospin $I=0$). It is to be expected that these systems will be very important in the black box of Fig. 5, and that the experimental properties of the form factors are related to the properties of these mesons. We plan to elaborate and discuss this relation during these lectures. If further resonances are found with these same quantum numbers they too may be important in the analysis of the electromagnetic structure of nucleons.

2. – The Dirac theory and Feynman propagator.

By way of a brief review and in order to introduce notation, we outline calculation of the electron-proton scattering cross-section. We consider first the scattering of a Dirac electron by an electromagnetic field, following Feynman's propagator approach. In the calculation of the electron-proton scattering, this field is taken as given by the Møller potential of the physical proton.

In a relativistic notation the Dirac equation is written

$$\{i\nabla_x - e\,\gamma^\mu A_\mu(x) - m\}\,\psi(x) = 0\,, \tag{5}$$

where

$$\nabla_x \equiv \gamma^\mu \frac{\partial}{\partial x^\mu}\,, \tag{6}$$

with the sum always understood as taken over repeated indices $\mu = 0, 1, 2, 3$; m is the mass and we set $\hbar = c = 1$. The components of the co-ordinates are $x^\mu = (t, \boldsymbol{x})$ and of the 4-vector potential are $A^\mu = (\Phi, \boldsymbol{A})$. Our metric tensor has the components

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

and raises or lowers indices according to

$$V^\mu = g^{\mu\nu} V_\nu\,; \qquad V_\mu = g_{\mu\nu} V^\nu\,.$$

The four Dirac γ-matrices satisfy the anti-commutation relations

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\,. \tag{7}$$

This notation coincides with that of SCHWEBER [4].

We wish to integrate (5) and find the scattered electron wave arising from interaction with the field A^μ. In analogy with Huygens principle in optics we construct the Green's function or propagator which propagates the wave emerging from a unit point source. Since (5) is linear we may then superpose these waves, weighted by the strength of the source. In terms of the simple and known free Dirac propagator

$$(i\nabla_x - m)\,S_F(x - x') = \delta^4(x - x')\,, \tag{8}$$

we have for the formal solution of (5)

$$\psi(x) = \psi(x) + e\int d^4x'\, S_F(x-x')\,\gamma^\mu A_\mu(x')\psi(x')\,, \tag{9}$$

where $\psi(x)$ is the homogeneous solution,

$$(i\nabla_x - m)\,\psi(x) = 0\,, \tag{10}$$

representing the incident free wave. The desired solution of (8) for the propagator must incorporate the physical boundary conditions of the scattering problem.

In the Dirac hole theory the negative energy states are all filled so that a scattered electron can not fall into the negative energy sea, according to the exclusion principle. After scattering it can propagate forward in time in a positive energy state only. Therefore in solving (8) we construct the Green's function which propagates only positive energy free waves into the future, $t-t'>0$. This is the Feynman propagator and is conveniently expressed in terms of its Fourier transform

$$S_F(x-x') = \frac{1}{(2\pi)^4}\int d^4p\, S_F(p)\exp[-ip\cdot(x-x')]\,, \tag{11}$$

with

$$S_F(p) = \frac{p_\mu\gamma^\mu + m}{p^2-(m^2-i\varepsilon)} \equiv \frac{1}{p_\mu\gamma^\mu - m + i\varepsilon}\,,$$

where

$$p^2 \equiv p_\mu p^\mu = (p^0)^2 - |\boldsymbol{p}|^2\,.$$

The infinitesimal negative imaginary part $-i\varepsilon$ is added to the mass to insure that positive frequency waves only propagate into the future. This is most easily seen by performing the frequency integral in (11),

$$I_0(t-t') = \int\frac{dp_0}{2\pi}\,\frac{\exp[-ip_0(t-t')]}{p_0^2-[|\boldsymbol{p}|^2+m^2-i\varepsilon]}\,\{p_0\gamma^0 - \boldsymbol{p}\cdot\boldsymbol{\gamma} + m\}\,, \tag{12}$$

using Cauchy's theorem. The two simple poles in the complex p_0 plane are located as shown in Fig. 6. Therefore when we close the contour in the lower half-plane for $t>t'$ only the positive frequency pole at

$$\omega_p \equiv +\sqrt{|\boldsymbol{p}|^2+m^2}\,,$$

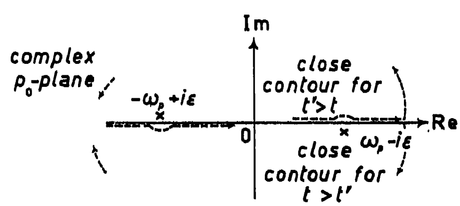

Fig. 6.

is enclosed as desired. However the negative energy root of the relativistic energy momentum relation at $-\omega_p$ will then necessarily be enclosed when we close the contour in the upper half-plane for $t<t'$. This leads to negative energy solutions of the Dirac equation propagating backward in time.

In order to show this explicitly and at the same time express the propagator in a very convenient form for introducing the scattering amplitude, or S-matrix, we introduce the solutions of the Dirac equation representing free particles of a given momentum. These can be written

$$\psi^{(+)}_{p,s}(x)=\sqrt{\frac{m}{EV}}\,u(p,s)\exp[-ip\cdot x]\,, \tag{13}$$

for positive energy solutions, where m is the mass, $p_0=E=+\sqrt{|\boldsymbol{p}|^2+m^2}$ is the energy, V the volume of the normalization box, and $u(p,s)$ is the free particle spinor for a positive energy solution of momentum $\boldsymbol{p}$, energy $E=p_0$, and spin $\boldsymbol{s}$:

$$(p_\mu\gamma^\mu-m)u(p,s)=0\,. \tag{14}$$

Similarly for the negative energy solutions we write

$$\psi^{(-)}_{p,s}(x)=\sqrt{\frac{m}{EV}}\,v(p,s)\exp[+ip\cdot x]\,, \tag{15}$$

where again $p_0=E=+\sqrt{|\boldsymbol{p}|^2+m^2}$, and now

$$(p_\mu\gamma^\mu+m)v(p,s)=0\,. \tag{16}$$

We shall not consider polarized particles in these lectures and therefore do not specify the spin values more precisely. Generally we are interested in summing over the two possible spin polarizations for a Dirac particle (electron) of a given momentum and sign of energy. The four spinor solutions for a given $\boldsymbol{p}$ can be represented very simply in the *rest system* by

$$u(0,\uparrow)=\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad u(0,\downarrow)=\begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad v(0,\uparrow)=\begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad v(0,\downarrow)=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \tag{17}$$

as eigenstates of σ_z. We may take

$$\beta \equiv \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

as diagonal in this representation (with

$$\gamma^{1,2,3} = \left(\begin{array}{c|c} 0 & \sigma^{1,2,3} \\ \hline -\sigma^{1,2,3} & 0 \end{array}\right),$$

in terms of the familiar 2×2 Pauli matrices) so that the adjoint spinors defined as $\bar{u} = u^\dagger \gamma^0$ and $\bar{v} = v^\dagger \gamma^0$ are (with $\uparrow$ and $\downarrow$ indicating $\pm s$)

$$(18) \qquad \begin{cases} \bar{u}(0, \uparrow) = (1\ 0\quad 0\ 0); & \bar{u}(0, \downarrow) = (0\ 1\ 0\quad 0); \\ \bar{v}(0, \uparrow) = (0\ 0\ {-1}\ 0); & \bar{v}(0, \downarrow) = (0\ 0\ 0\ {-1}). \end{cases}$$

The orthogonality and completeness relations in the rest system are easily constructed by inspection of (17) and (18):

$$(19) \qquad \begin{cases} \bar{u}(0, \mathring{s}) u(0, \mathring{s}') = \delta_{\mathring{s}\mathring{s}'}; & \bar{u}(0, \mathring{s}) v(0, \mathring{s}') = 0; \\ \bar{v}(0, \mathring{s}) v(0, \mathring{s}') = -\delta_{\mathring{s}\mathring{s}'}; & \bar{v}(0, \mathring{s}) u(0, \mathring{s}') = 0; \end{cases}$$

$$(20) \qquad \sum_{\pm \mathring{s}} \{u_\alpha(0, \mathring{s}) \bar{u}_\tau(0, \mathring{s}) - v_\alpha(0, \mathring{s}) \bar{v}_\tau(0, \mathring{s})\} = \delta_{\alpha\tau}\,.$$

To obtain (19) for spinors of arbitrary momentum $\boldsymbol{p}$ it is most efficient to Lorentz-transform to a system moving with $\boldsymbol{\beta} = -\boldsymbol{p}/E$. For the construction and discussion of the Lorentz transformation matrix $S(\beta)$ effecting this transformation the student is referred to a text (see SCHWEBER [4]). Here we need only the remarks that

(21) *a*) S exists, and

b) if $u(p, s) = S(\beta) u(0, \mathring{s})$, then $\bar{u}(p, s) = \bar{u}(0, \mathring{s}) S^{-1}(\beta)$; *i.e.*, u and $\bar{u}$ have inverse laws of Lorentz transformations;

c) for later reference we note that the transformation matrix S for a Lorentz transformation $x^{\nu'} = a^\nu{}_\mu x^\mu$ satisfies the algebraic identity

$$(22) \qquad S^{-1} \gamma^\mu S = a^\mu{}_\nu \gamma^\nu\,.$$

Using (19), (20), (21), and (22) we find for any solutions then

$$(23)\qquad \begin{cases} \bar{u}(p,s)\,u(p,s') = \bar{u}(0,\mathring{s})\,S^{-1}(\beta)\,S(\beta)\,u(0,\mathring{s}') = \delta_{s,s'}\,, \\ \bar{v}(p,s)\,v(p,s') = -\,\delta_{s,s'}\,, \\ \bar{v}(p,s)\,u(p,s') = \bar{u}(p,s)\,v(p,s') = 0\,, \end{cases}$$

$$(24)\qquad \sum_{\pm s}\big(u_\alpha(p,s)\bar{u}_\tau(p,s) - v_\alpha(p,s)\bar{v}_\tau(p,s)\big) = \sum_{\pm s}\sum_{\lambda,\varrho=1}^{4} S_{\alpha\lambda}(\beta)\,S^{-1}_{\varrho\tau}(\beta)\,\delta_{\lambda\varrho} = \delta_{\alpha\tau}\,.$$

The adjoint spinors satisfy Dirac equations

$$(25)\qquad \begin{cases} \bar{u}(p,s)\,(p_\mu\gamma^\mu - m) = 0\,, \\ \bar{v}(p,s)\,(p_\mu\gamma^\mu + m) = 0\,. \end{cases}$$

Proof:

$$\{(p_\mu\gamma^\mu - m)\,u(p,s)\}^+ = 0\,,$$

or

$$\bar{u}(p,s)\gamma^0(p_\mu\gamma^{\mu+} - m) = 0;$$

but

$$\gamma^0\gamma^{\mu+}\gamma^0 = \gamma^\mu;$$

therefore

$$\bar{u}(p,s)[p_\mu\gamma^\mu - m] = 0;$$

similarly for $\bar{v}(p,s)$.

In order to transform (23) and (24) into normalization statements on a product of spinor multiplied by a Hermitian conjugate we again appeal to the Dirac equation and the identity

$$(26)\qquad \bar{u}(p',s')\gamma^\mu(p_\mu\gamma^\mu - m)\,u(p,s) + \bar{u}(p',s')\,(p_\mu\gamma^{\mu\prime} - m)\gamma^\mu u(p,s) = 0\,.$$

Using the identity on Dirac matrices

$$(27)\qquad \gamma^\mu\gamma^\nu = \tfrac{1}{2}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) + \tfrac{1}{2}\,(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) = g^{\mu\nu} - i\sigma_{\mu\nu}\,,$$

with

$$\sigma^{\mu\nu} \equiv \frac{i}{2}\,(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\,,$$

we find

$$(28)\qquad \bar{u}(p', s')\gamma^\mu u(p, s) = \frac{(p+p')^\mu}{2m}\bar{u}(p', s')u(p, s) + \\ + \frac{i}{2m}\bar{u}(p', s')\sigma^{\mu\nu}(p'-p)_\nu u(p, s)\,.$$

For $p = p'$ and $\mu = 0$ this gives

$$(29)\qquad u^+(p, s')u(p, s) = \frac{E}{m}\delta_{ss'}\,.$$

Similarly from variations of (26) containing $v(p, s)$ as one or both factors there results

$$(30)\qquad \begin{cases} v^\dagger(p, s')v(p, s) = \dfrac{E}{m}\delta_{ss'}\,, \\ v^\dagger(-p, s')u(p, s) = 0\,. \end{cases}$$

The spin sum for positive and negative energy solutions separately may be extracted from (24) by using the Dirac eqs. (14) and (16); for example if we multiply (24) by

$$(31)\qquad \{\Lambda_+(p)\}_{\nu\alpha} \equiv \left(\frac{p_\mu\gamma^\mu + m}{2m}\right)_{\nu\alpha},$$

we find, since

$$(32)\qquad \Lambda_+(p)u(p, s) = u(p, s)\,, \qquad \Lambda_+(p)v(p, s) = 0;$$

$$(33)\qquad \sum_{\pm s} u_\alpha(p, s)\bar{u}_\tau(p, s) = \{\Lambda_+(p)\}_{\alpha\tau};$$

similarly

$$(34)\qquad \sum_{\pm s} v_\alpha(p, s)\bar{v}_\tau(p, s) = -\left(\frac{m - p_\mu\gamma^\mu}{2m}\right)_{\alpha\tau} \equiv -\{\Lambda_-(p)\}_{\alpha\tau}\,,$$

and

$$\Lambda_-(p)v(p, s) = v(p, s)\,, \qquad \Lambda_-(p)u(p, s) = 0\,.$$

Evidently $\Lambda_+(p)$ and $\Lambda_-(p)$ have all the properties of projection operators, *viz.*

$$(36)\qquad \begin{cases} \Lambda_+(p) + \Lambda_-(p) = 1\,, & \Lambda_-^2(p) = \Lambda_-(p)\,, \\ \Lambda_+^2(p) = \Lambda_+(p)\,, & \Lambda_-(p)\Lambda_+(p) = 0\,. \end{cases}$$

They are known as the positive and negative energy projection operators, respectively.

With these formal developments in hand we can now express the result of the frequency integral in (11) and (12) in terms of the actual free particle

solutions:

$$S_F(x-x') = -i\int \frac{d^3p}{(2\pi)^3} \exp[-ip\cdot(x-x')]\,\frac{m}{E}\,\Lambda_+(p)\theta(t-t') - \tag{37}$$

$$-i\int \frac{d^3p}{(2\pi)^3} \exp[+ip\cdot(x-x')]\,\frac{m}{E}\,\Lambda_-(p)\theta(t'-t) =$$

$$= -i\int \frac{V\,d^3p}{(2\pi)^3} \sum_{\pm s} \{\psi^{(+)}_{p,s}(x)\,\overline{\psi}^{(+)}_{p,s}(x')\theta(t-t') - \psi^{(-)}_{p,s}(x)\,\overline{\psi}^{(-)}_{p,s}(x')\theta(t'-t)\}\,,$$

where $\theta(x_0)$ is the unit step function, equal to unity for $x_0>0$ and vanishing for $x_0<0$:

$$\theta(x_0) = \begin{cases} 1, & x_0>0\,, \\ 0, & x_0<0\,. \end{cases} \tag{38}$$

The form (37) shows explicitly the appearance of positive energy Dirac solutions only in the future and of negative energy ones in the past. From (37) we can check that the Feynman propagator $S_F(x-x')$ propagates positive energy solutions into the future and negative energy ones into the past according to

$$\begin{cases} \theta(t-t')\,\psi^{(+)}(x) = i\int d^3x'\,S_F(x-x')\gamma^0\psi^{(+)}(x')\,, \\ \theta(t'-t)\,\psi^{(-)}(x) = -i\int d^3x'\,S_F(x-x')\gamma^0\psi^{(-)}(x')\,. \end{cases} \tag{39}$$

Equations (39) are just the mathematical statement of Huygen's principle for the free Dirac equation. The physical boundary condition allowing only positive energy solutions to appear in the future has led to negative energy ones propagating into the past. The factor γ^0 on the right-hand side appears for relativistic convenience and can be traced back to (8) which shows that S_F contains a factor γ^0 relative to a usual Green's function in the form of

$$i\nabla_x = i\gamma^0\frac{\partial}{\partial t} + i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,.$$

The proof of (39) runs as follows. Introduce S_F from (37) into the right-hand side of the first equation of (39),

This gives

$$\int \frac{V\,d^3p}{(2\pi)^3} \sum_{\pm s} \psi^{(+)}_{p,s}(x)\theta(t-t')\int d^3x'\left[\sqrt{\frac{m}{EV}}\,\overline{u}(p,s)\exp[ip\cdot x']\right]\cdot\gamma_0\cdot \tag{40}$$

$$\cdot\left[\sqrt{\frac{m}{EV}}\,u(p',s')\exp[-ip'\cdot x']\right].$$

The second term of S_F with the negative energy solutions does not contribute because of the $(2\pi)^3\delta^3(\boldsymbol{p}+\boldsymbol{p}')$ coming from x' integration and the orthogonality relation (30). The γ^0 in (40) changes $\bar{u}(p,s)$ into $u^\dagger(p,s)$ and, introducing (29), we are left with $\theta(t-t')\,\psi^{(+)}_{p,s}(x)$ as claimed.

3. – The S-matrix.

Let us return now to the exact solution, $\Psi(x)$, of the Dirac theory as given by eq. (9). The boundary condition on the Feynman propagator eq. (36) insures that only positive energy solutions propagate into the future. Using perturbation theory we construct an explicit expression for $\Psi(x)$ by an iteration procedure. Our aim is to determine the amplitude of the scattered wave in $\Psi(x)$ and we shall generally content ourselves with discussing the first or second terms in the iteration series in powers of $\alpha=1/137$ for electromagnetic interactions.

The zeroth approximation (no interaction) gives $\Psi_0(x)=\psi(x)$. The first order approximation $\Psi_1(x)$ is obtained by inserting $\Psi_0(x)=\psi(x)$ for $\Psi(x)$ in the right-hand side of (9), and so on, leading to the expansion

$$\Psi(x)=\psi(x)+\int\mathrm{d}^4y\,S_F(x-y)\,\gamma_\mu A^\mu(y)\,\psi(y)+ \\ +\int\mathrm{d}^4y\,S_F(x-y)\,e\gamma_\mu A^\mu(y)\int\mathrm{d}^4z\,S_F(y-z)\,e\gamma_\mu A^\mu(z)\,\psi(z)+\dots\,. \tag{41}$$

Let us see now what emerges as the scattered wave $\psi_{sc}(x)\equiv\Psi(x)-\psi(x)$, for $x_0=t\to+\infty$, if the solution $\psi(x)$ represents an incident positive energy free electron $\psi^{(+)}_{p_1,s_1}(x)$.

Since a physical external potential $A^\mu(y)$ acts always in a limited time region, the time $t=+\infty$ in (9) or (41) will be later than any y_0 for which the integrand does not vanish. Writing S_F in the form (37) one finds then that only the first term containing positive energy solutions contributes, leading to a scattered wave (we explicitly exhibit only the first order perturbation result)

$$[\psi_{sc}(x)]_{x_0\to+\infty}=\int\frac{V\,\mathrm{d}^3p}{(2\pi)^3}\sum_{\pm s}\psi^{(+)}_{p,s}(x)\left\{-ie\int\mathrm{d}^4y\,\bar{\psi}^{(+)}_{p,s}(y)\gamma_\mu A^\mu(y)\,\psi^{(+)}_{p_1,s_1}(y)+\dots\right\}. \tag{42}$$

In addition there is a scattered wave emerging at $x_0\to-\infty$ coming from the second term in (37):

$$[\psi_{sc}(x)]_{x_0\to-\infty}=\int\frac{V\,\mathrm{d}^3p}{(2\pi)^3}\sum_{\pm s}\psi^{(-)}_{p,s}(x)\left\{+ie\int\mathrm{d}^4y\,\bar{\psi}^{(-)}_{p,s}(y)\,\gamma_\mu A^\mu(y)\,\psi^{(+)}_{p_1,s_1}(y)+\dots\right\}. \tag{43}$$

The coefficient appearing within the brackets in eq. (42) is the *scattering amplitude* to first order in e for an electron incident with (p_i, s_i) to emerge after scattering by the field $A^\mu(y)$ with (p, s). The Lorentz-invariant integral d^4y is the continuous sum over all space-time points at which the interaction may occur after the incident electron enters the field and before it emerges. The Feynman diagram Fig. 7, gives a convenient pictorial representation of this interaction.

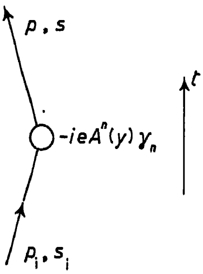

Fig. 7.

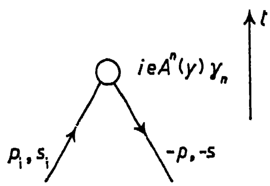

Fig. 8.

Similarly the coefficient in (43) is the scattering amplitude to first order in e for the incident electron to emerge propagating backward in time to $x_0 = -\infty$ in a negative energy state. This is interpreted as the amplitude for electron-positron annihilation since the *emergence* of an electron with charge $-|e|$ and with negative energy, $-\sqrt{|\boldsymbol{p}|^2 + m^2} < 0$, from the interaction appears as the *disappearance* of a plus charge $+|e|$ and of positive energy $+\sqrt{|\boldsymbol{p}|^2 + m^2} > 0$ into the interaction « vertex » at y in Fig. 8.

Similar expressions for electron-positron pair production and for positron scattering are obtained by replacing $\psi^{(+)}_{p_i, s_i}(y)$ in (42) and (43), respectively, by an incident negative energy solution propagating into the interaction of y; it represents the emerging final positron with $+|e|$ and $+\sqrt{|\boldsymbol{p}'|^2 + m^2}$.

4. – The electron-proton scattering cross-section.

The scattering cross-section for an electron by the potential $A^\mu(y)$ is given by the absolute square of the amplitude (42) appropriately normalized to unit incident flux and summed over final phase space.

For scattering in a static Coulomb field $A^\mu(y)$ reduces to

$$A^\mu(y) = (\Phi(\boldsymbol{y}), 0); \qquad \Phi(\boldsymbol{y}) = \frac{|e|}{4\pi|\boldsymbol{y}|}, \tag{44}$$

where $e^2/4\pi = \alpha = 1/137$ if the source of the field carries the charge of one proton. Equation (44) is the infinite mass limit of the potential created by

a physical proton of finite mass M_p scattering with the electron. As it recoils in the scattering the proton's current J^μ produces an A^μ according to Maxwell's equations. In the Lorentz gauge A^μ is given by a solution of

$$\Box A^\mu = J^\mu ,$$

or

$$A^\mu(y) = \int \mathrm{d}^4 z D_F(y-z) J^\mu(z) , \tag{45}$$

where

$$\Box_y D_F(y-z) = \delta^4(y-z) , \tag{46}$$

is the appropriate electromagnetic Green's function, or propagator. Fourier-transforming to momentum space as in (11) we have from (46)

$$D_F(y-z) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \exp[-iq\cdot(y-z)] D_F(q^2) , \tag{47}$$

with

$$-q^2 D_F(q^2) = 1 .$$

By analogy with (11) (m here vanishes) we define $D_F(q^2)$, as $q^2 \to 0$, by

$$D_F(q^2) = \frac{-1}{q^2 + i\varepsilon} , \tag{48}$$

where the infinitesimal positive $\varepsilon \to 0^+$ assures that only positive frequency waves or quanta are radiated into the future from the proton current, in accord with physical experience. To complete the determination of $A^\mu(y)$, the current $J^\mu(z)$ must be specified, and for this we appeal to the correspondence principle. As first discussed in the original quantum theory by Heisenberg the current operator of the Schrödinger equation is replaced by the transition current

$$(J^\mu_{\text{Schrödinger}})_{fi} = -ie\left[\Phi_f^* \frac{\partial}{\partial x_\mu} \Phi_i - \Phi_i \frac{\partial}{\partial x_\mu} \Phi_f^*\right] ,$$

for a particle starting in state i and ending up in state f. Similarly the transition current for a point proton, stripped of its strong interactions with mesons, and described to lowest order in α by a free Dirac equation, is

$$J^\mu(z) = e_p \bar\psi^{(+)}_{(p)_f}(z) \gamma^\mu \psi^{(+)}_{(p)_i}(z) , \tag{49}$$

where $\psi^{(+)}_{(p)}$ denote positive energy solutions of

$$(i\nabla - M_p)\psi_{(p)} = 0 , \tag{50}$$

for a proton. The A^μ produced by this J^μ,

$$A^\mu(y) = -\int \frac{d^4q}{(2\pi)^4} \frac{\exp[-iq\cdot y]}{q^2+i\varepsilon}\left\{\int d^4z \exp[iq\cdot z]\, e_p \bar{\psi}^{(+)}_{(p)_f}(z)\gamma^\mu \psi^{(+)}_{(p)_i}(z)\right\}, \tag{51}$$

is known as the Møller potential seen by the electron. Eq. (51) cannot be completely correct because it ignores all of the strong nonelectromagnetic interactions of the proton that give rise to its observed magnetic moment and electromagnetic form factors. Inclusion of the mesons and the electromagnetic currents they produce about a proton modifies (49) in a way which we shall discuss later. The free proton wave functions are given by expressions similar to (13), with $m \to M_p$ everywhere:

$$\left\{\begin{array}{l} \psi^{(+)}_{(p)}(x) = \sqrt{\dfrac{M_p}{\mathfrak{E}V}}\, u(\mathfrak{P}, S) \exp[-i\mathfrak{P}\cdot x]\,, \\ \text{with} \\ (\mathfrak{P}_\mu\gamma^\mu - M_p)\, u(\mathfrak{P}, S) = 0\,, \\ \text{and} \\ \mathfrak{P}_0 \equiv \sqrt{|\mathfrak{P}|^2 + M_p^2}\,. \end{array}\right. \tag{52}$$

Inserting (51) and (52) for the scattering amplitude S_{fi} in (42), we find to first order in α

$$S^{(1)}_{fi} = -i\iint d^4y[e\bar{\psi}^{(+)}_{p_f,s_f}(y)\gamma_\mu\psi^{(+)}_{p_i,s_i}(y)]\, D_F(y-z)[e_p\bar{\psi}^{(+)}_{(p)_f}(z)\gamma^\mu\psi^{(+)}_{(p)_i}(z)]\, d^4z = \tag{53}$$

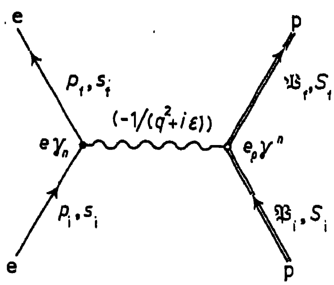

Fig. 9.

$$= -ie^2\sqrt{\frac{M_p^2}{\mathfrak{E}_f\mathfrak{E}_i}}\sqrt{\frac{m^2}{E_fE_i}}\frac{1}{V^2}(2\pi)^4\delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_i - p_i)\cdot$$
$$\cdot\bar{u}(p_f, s_f)\gamma_\mu u(p_i, s_i)\frac{1}{q^2+i\varepsilon}\bar{u}(\mathfrak{P}_f, S_f)\gamma^\mu u(\mathfrak{P}_i, S_i)\,,$$

where $q^2 = (p_i - p_f)^2 = (\mathfrak{P}_f - \mathfrak{P}_i)^2$ is the square of the invariant momentum transfer, $e_p = -e > 0$, and the kinematic labels are as indicated in Fig. 9.

In (53) the factor $(2\pi)^4\delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_i - p_i)$ ensures energy-momentum conservation and $-1/(q^2+i\varepsilon)$ is the amplitude for a «virtual photon» of momentum q_μ to propagate between the electron and proton currents. The factor

$$M_{fi} \equiv [\bar{u}(p_f, s_f)\gamma_\mu u(p_i, s_i)]\frac{1}{q^2+i\varepsilon}[\bar{u}(\mathfrak{P}_f, S_f)\gamma^\mu u(\mathfrak{P}_i, S_i)]\,, \tag{54}$$

is Lorentz-invariant, being the contraction of two four-vectors, according to (21) and (22), and can therefore be evaluated in any Lorentz frame as proves most convenient.

The path from (53) to a cross-section should be travelled at least once by all of you so here we go (with some haste):

If the spins of the final electron and proton are not measured and if the initial particles are unpolarized we must consider

$$(54)\qquad \overline{|M_{fi}|^2} = \tfrac{1}{2}\sum_{s_i}\tfrac{1}{2}\sum_{S_i}\sum_{s_f}\sum_{S_f}|M_{fi}|^2 = \tfrac{1}{4}\sum_{\text{all spins}}|M_{fi}|^2\,,$$

in forming the cross-section [the factors $\frac{1}{2}$ arise from $1/(2S+1)=\frac{1}{2}$ for $S=\frac{1}{2}$]. The spin sums are to be carried out by trace techniques, not by explicit multiplication of γ matrices. It is here that the projection operators (33) are of great merit. (Analogous ones for spin as well as energy projection can be constructed for problems in which polarizations are determined but we don't consider them here; see SCHWEBER [4].) The following identities are useful for evaluating (54);

(55) i) $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}$, or $(\gamma^0)^2=+1$, $(\gamma^i)^2=-1$; $i=1,2,3$;

$$\therefore\ \gamma^0\gamma^{\mu\dagger}\gamma^0=+\gamma^\mu \quad\text{and}\quad [\bar u(f)\gamma^\mu u(i)]^\dagger = u^\dagger(i)\gamma^{\mu\dagger}\gamma^0 u(f)=\bar u(i)\gamma^\mu u(f);$$

generally $[\bar u(f)Ou(i)]^\dagger=\bar u(i)\tilde O u(f)$ where $\tilde O=\gamma^0 O^\dagger\gamma^0$.

(56) ii) $\operatorname{Tr}1=4$, where Tr = Trace, or spur.

$$\operatorname{Tr}\gamma^\mu a_\mu\gamma^\nu b_\nu=4a\cdot b,$$

$$\operatorname{Tr}(\gamma^\mu a_\mu)(\gamma^\sigma b_\sigma)(\gamma^\tau c_\tau)(\gamma_\lambda d^\lambda)=4[a\cdot b\,c\cdot d-a\cdot c\,b\cdot d+a\cdot d\,b\cdot c]\,,$$

$$\operatorname{Tr}(\gamma^\mu a_{1\mu})(\gamma^\nu a_{2\nu})\ldots(\gamma^\tau a_{2n+1\,\tau})=0\,.$$

Introducing (55) and (56) in (54), and recalling (33), we find (indicating spinor indices explicitly to show formation of the trace)

$$(57)\qquad \overline{|M_{fi}|^2}=\frac{1}{4}\sum_{\text{spins}}\left|\frac{1}{q^2+i\varepsilon}\right|^2[\bar u_a(p_f,s_f)\gamma_{\mu_{ab}}u_b(p_i,s_i)\bar u_c(p_i,s_i)\gamma_{\nu_{cd}}u_d(p_f,s_f)]\cdot$$

$$\cdot[\text{similar factor for the proton}]=$$

$$=\frac{1}{4}\left|\frac{1}{q^2+i\varepsilon}\right|^2\operatorname{Tr}\{\Lambda_+(p_f)\gamma_\mu\Lambda_+(p_i)\gamma_\nu\}\cdot\operatorname{Tr}\{\Lambda_+(\mathfrak{P}_f)\gamma^\mu\Lambda_+(\mathfrak{P}_i)\gamma^\nu\}=$$

$$=\frac{1}{4(q^2)^2}\{p_{f_\mu}p_{i_\nu}+p_{f_\nu}p_{i_\mu}-g_{\mu\nu}[p_f\cdot p_i-m^2]\}\cdot\frac{1}{m^2M_p^2}\cdot$$

$$\cdot\{\mathfrak{P}_f^\mu\mathfrak{P}_i^\nu+\mathfrak{P}_f^\nu\mathfrak{P}_i^\mu-g^{\mu\nu}[\mathfrak{P}_f\cdot\mathfrak{P}_i-M_p^2]\}.$$

Evaluating (57) in the laboratory frame where $\mathfrak{P}_1^\rceil = (M_p, 0)$ and neglecting the electron rest mass for relativistic electrons we reduce (57) to

$$\overline{|M_{fi}|^2} = \frac{1}{16 E_1 E_f m^2 \sin^4(\theta/2)} \left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M_p^2}\sin^2\frac{\theta}{2}\right],$$

with

$$q^2 = -4E_1 E_f \sin^2\frac{\theta}{2}. \tag{58}$$

Proceeding to the cross-section we first compute the transition rate per unit volume; *i.e.*, the square of the scattering amplitude (53), summed over spins, divided by the space-time volume in which the transitions occur:

$$w_{fi} = \frac{1}{\int \mathrm{d}t \int \mathrm{d}^3x} |S_{fi}|^2 = \frac{e^4}{V^2} \frac{m^2 M_p^2}{E_1 E_f \mathfrak{E}_1 \mathfrak{E}_f} |M_{fi}|^2 \frac{1}{\int \mathrm{d}t \int \mathrm{d}^3x} [(2\pi)^4 \delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_1 - p_1)]^2 .$$

It will be observed (with appropriate horror by the mathematically sensitive who must at this point resort to normalizable packet solutions [5] instead of plane waves as we have used) that

$$\frac{1}{\int \mathrm{d}t \int \mathrm{d}^3x} [(2\pi)^4 \delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_1 - p_1)]^2 =$$

$$= \left[\frac{(2\pi)^4 \delta^4(0)}{\int \mathrm{d}t \int \mathrm{d}^3x}\right] (2\pi)^4 \delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_1 - p_1) = (2\pi)^4 \delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_1 - p_1),$$

and

$$w_{fi} = \frac{e^4}{V^4} \frac{m^2 M_p^2}{E_1 E_f \mathfrak{E}_1 \mathfrak{E}_f} (2\pi)^4 \delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_1 - p_1) \cdot |M_{fi}|^2 . \tag{59}$$

To obtain a cross-section from (59) we must multiply by the number of final electron and proton states $\left(\text{of a single given spin since we have already performed } \sum_{\text{spins}}\right)$ in the observed momentum interval

$$\iint_\tau \frac{V \mathrm{d}^3 p_f}{(2\pi)^3} \frac{V \mathrm{d}^3 \mathfrak{P}_f}{(2\pi)^3},$$

and divide by the incident particle flux $|J_{\text{inc}}|$ times the number of target particles per unit volume. In the laboratory system V^{-1} is the number of target protons per unit volume, since $u^\dagger(0)\,u(0) = 1/V$, by (52), is their density; and

$$|J_{\text{inc}}| = \frac{1}{V}\left|\frac{1}{E_1}\boldsymbol{p}_1 - \frac{1}{\mathfrak{E}_1}\boldsymbol{\mathfrak{P}}_1\right| = \frac{1}{V}, \qquad \text{since } \mathfrak{E}_1 = M_p, \qquad \text{and } \frac{p_1}{E_1} \simeq 1,$$

in the laboratory system. Hence

$$(60)\qquad \mathrm{d}\sigma = e^4\frac{m}{E_1}\int_{(\tau)}\frac{m\,\mathrm{d}^3p_t}{E_t(2\pi)^3}\,\frac{M_p\,\mathrm{d}^3\mathfrak{P}_t}{\mathfrak{E}_t(2\pi)^3}\,(2\pi)^4\,\delta^4(\mathfrak{P}_t+p_t-\mathfrak{P}_1-p_1)\,\overline{|M_{fi}|^2}\,.$$

Carrying out the final phase-space sum for an experiment which detects only the final electron, we use

$$\frac{\mathrm{d}^3p_t}{E_t} = p_t\,\mathrm{d}E_t\,\mathrm{d}\Omega\,,$$

and

$$\frac{\mathrm{d}^3\mathfrak{P}_t}{\mathfrak{E}_t} = 2\int_0^\infty \mathrm{d}\mathfrak{P}_t^0\,\mathrm{d}^3\mathfrak{P}_t\,\delta(\mathfrak{P}_t^{02}-|\mathfrak{P}_t|^2-M_p^2) = 2\int_{-\infty}^\infty \mathrm{d}^4\mathfrak{P}_t\,\delta(\mathfrak{P}_t^2-M_p^2)\,\theta(\mathfrak{P}_t^0)\,,$$

and find

$$(61)\qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2m^2M_pe^4}{4\pi^2E_1}\int_0^{M_p+E_1} p_t\,\mathrm{d}E_t\,\delta[2M_p(E_1-E_t)-2E_1E_t(1-\cos\theta)]\cdot\overline{|M_{fi}|^2} =$$

$$= \frac{e^4m^2}{4\pi^2}\,\frac{M_p^2\,\overline{|M_{fi}|^2}}{\{M_p+E_1(1-\cos\theta)\}^2} = \frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\left[\frac{\cos^2(\theta/2)-(q^2/2M_p^2)\sin^2(\theta/2)}{1+(2E_1/M_p)\sin^2(\theta/2)}\right].$$

5. – General form of electron-proton scattering.

We have seen in eq. (58) that the square of the invariant matrix element for electron scattering from a point Dirac proton, summed over spins, takes the form

$$(62)\qquad \overline{|M_{fi}|^2} = \frac{1}{4m^2t}\left[\frac{-1}{\sin^2(\theta/2)}+\left(1+\frac{t}{2M_p^2}\right)\right],$$

where $t = q^2$ is the square of the invariant momentum transfer, and θ is the scattering angle of the electron in the laboratory frame. This dependence upon scattering angle for fixed momentum transfer t is in fact the most general one, for *arbitrary* structure of the proton (or electron!) so long as we retain the approximation for one virtual photon exchange between the electron and proton currents as in Fig. 9. It can be proved directly by writing down the most general forms for the structure at the proton and electron vertices, in accord with the requirements of Lorentz invariance and of differential current conservation. An alternative and more transparent procedure has been given by GOURDIN and MARTIN [6], based on a theorem of Yang, to verify that the

most general form can be written

$$\overline{|M_{fi}|^2} \propto a(t) + \frac{b(t)}{\sin^2(\theta/2)}\,. \tag{63}$$

Equation (63) is a result of considerable importance since it severely limits the form of the scattering cross-section as a function of energy and angle, for fixed t. By (61) and (63)

$$\left(\frac{E_i}{E_f}\right)^2 \frac{d\sigma}{d\Omega} = a(t) + \frac{b(t)}{\sin^2(\theta/2)}\,, \tag{64}$$

and any deviation in the plot of $(E_i/E_f)^2 \sin^2(\theta/2)\, d\sigma/d\Omega$ *vs.* $\cos^2\theta/2$ from a straight line must be interpreted as a correction to the one-photon exchange approximation, or as a fundamental violation of Maxwell's equation for the virtual exchanged photon.

To prove (63) we work with invariant quantities, introducing

$$\begin{cases} s \equiv (\mathfrak{P}_i + p_i)^2 = (\mathfrak{P}_f + p_f)^2 = M_p^2 + 2p_i\cdot\mathfrak{P}_i\,, & \text{for } m \to 0\,, \\ t = (p_f - p_i)^2 = -2p_f\cdot p_i\,, & \text{for } m \to 0\,. \end{cases} \tag{65}$$

By momentum conservation

$$p_i\cdot\mathfrak{P}_f = p_f\cdot\mathfrak{P}_i = \tfrac{1}{2}(s + t - M_p^2)\,.$$

The key to the Gourdin-Martin method is to consider the reaction in the crossed channel, *i.e.*, the reaction seen by looking at Fig. 9, sideways

$$e^+ + e^- \leftrightarrow p + \bar{p}\,. \tag{66}$$

We need only substitute

$$\begin{cases} p_i \to p^-\,, & \mathfrak{P}_i \to -\mathfrak{P}^-\,, \\ p_f \to -p^+\,, & \mathfrak{P}_f \to \mathfrak{P}^+\,. \end{cases} \tag{67}$$

The invariant t now becomes the energy

$$t = [p^- - (-p^+)]^2 = (p^- + p^+)^2 = (\mathfrak{P}^- + \mathfrak{P}^+)^2 > 4M_p^2\,, \tag{68}$$

and $s = (p^- - \mathfrak{P}^-)^2 < 0$, the momentum transfer between electron and antiproton. We call this the t-channel. According to the substitution rule as mentioned in Section 1 the same analytic function of s and t represents the

reaction in both the t and the s channels (electron-proton scattering). We verify the substitution rule in perturbation theory by direct construction since according to (43), (15) and (16), the matrix element (54) becomes in the t channel

$$M_{fi}^{t} = \frac{1}{t}\,\{\bar{v}(p_+, s_+)\gamma_\mu u(p_-, s_-)\}\{\bar{u}(\mathfrak{P}_+, S_+)\gamma^\mu v(\mathfrak{P}_-, S_-)\}\,. \tag{69}$$

Squaring (69) and summing over spins gives (57) with the substitutions (67). The generalization of the substitution rule or so-called « crossing symmetry » to apply to all orders in the various interactions is a fundamental assumption in nonperturbative approaches to the S-matrix and will be retained here [7]. It assures us that (63), appropriately expressed in terms of the invariant variables s and t according to (65) and evaluated in the region $t > 0$, $s < 0$ instead of $t < 0$ and $s > 0$ represents $\sqrt{\overline{|M_{fi}^{(t)}|^2}}$.

The advantage of studying the crossed reaction (66) in the t channel is that we can go to the center-of-mass system, since M_{fi} is an invariant form, and recall from Section **1** that only very few channels participate.

In this system the intermediate photon carries one unit of angular momentum and odd parity since, as remarked above, eq. (4), the current is a polar vector, and therefore only channels with these quantum numbers participate in the scattering. But we know [6] that when the angular momentum of a system of two interacting particles is limited by an integral maximum value, $J^{\max}$, the angular distribution of the scattered particles in their center-of-mass system is a polynomial in the cos of the scattering angle with a maximum pover $\cos^{2J\max}\varphi_t$. For reaction (66) with $J_{\max} = 1$ this means

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{c.m.}}_{t\ \text{channel}} = \alpha(t)\,\overline{|M_{fi}^{(t)}|^2} \propto a_0(t) + a_1(t)\cos\varphi_t + a_2(t)\cos^2\varphi_t\,.$$

Furthermore $a_1(t) = 0$ since the $e\bar{e}$ or $p\bar{p}$ system must have a definite parity, -1, according to (4), and odd powers of $\cos\varphi_t$ originate only from the interference of waves of different parities. This leaves

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{c.m.}}_{t\ \text{channel}} \propto a_0(t) + a_2(t)\cos^2\varphi_t\,. \tag{70}$$

In terms of invariant variables we write from (67) ($m \to 0$)

$$t = 4E^2_{\text{c.m.}}\,,$$

$$s = M_p^2 - \frac{1}{2}t\left[1 - \sqrt{\frac{t - 4M_p^2}{t}}\cos\varphi_t\right],$$

or

$$\cos^2\varphi_t = \frac{4(s - M_p^2 + t/2)^2}{t(t - 4M_p^2)},$$

and therefore

$$\left(\frac{d\sigma}{d\Omega}\right)^{c.m.}_{t\ \text{channel}} = \alpha(t)\,|M_{fi}^{(t)}|^2 \propto a(t) + b(t)[s - M_p^2 + t/2]^2 . \tag{71}$$

The equivalence of (71) with (63) can now be shown by expressing the invariant $[s - M_p^2 + t/2]^2$ in terms of t and of the laboratory scattering angle θ in the S channel. From above (58) and the condition of energy conservation in (61) we recall that

$$t = -4E_1E_t\sin^2\frac{\theta}{2} = -2M_p(E_1 - E_t) . \tag{72}$$

Furthermore

$$s = M_p^2 + 2M_pE_1 ,$$

so that

$$\left(s - M_p^2 + \frac{t}{2}\right)^2 = (M_pE_1 + M_pE_t)^2 =$$

$$= M_p^2[(E_1 - E_t)^2 + 4E_1E_t] = M_p^2\left[\frac{t^2}{4M_p^2} - \frac{t}{\sin^2(\theta/2)}\right],$$

and, by (71),

$$|M_{fi}|^2 = A(t) + \frac{B(t)}{\sin^2(\theta/2)} . \qquad Q.E.D.$$

It is custumary to write the general form of the electron-proton scattering cross-section in terms of two particular invariant form factors $F_1(t)$ and $F_2(t)$ which are real for $t < 0$, as in (73) for scattering. These are associated in a way to be discussed in more detail in Section 7 with the electric and magnetic structure of the proton's interaction. Normalizing to $F_1(0) = 1$ and $F_2(0) = 1$ and introducing $\varkappa_p = 1.79$ for the anomalous proton magnetic moment we have

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\cdot\frac{1}{1 + (2E_1/M_p)\sin^2(\theta/2)}\cdot$$

$$\cdot\left\{\left[F_1^2(t) - \frac{\varkappa_p^2 t}{4M_p^2}F_2^2(t)\right]\cos^2\frac{\theta}{2} - \frac{t}{2M_p^2}[F_1(t) + \varkappa_pF_2(t)]^2\sin^2\frac{\theta}{2}\right\}. \tag{73}$$

Equation (73) is known as the Rosenbluth formula [8] (*) and is the most general form, according to our discussion above, consistent with one-photon exchange. An alternative pair of form factors defined by [9]

$$(74)\qquad \begin{cases} G_E(t) \equiv F_1(t) + \dfrac{t}{4M_p^2}\,\varkappa_p F_2(t)\,, \\ G_M(t) \equiv F_1(t) + \varkappa_p F_2(t)\,, \end{cases}$$

relative to F_1 and F_2 allows us to rewrite (73) in a convenient form containing squares of the two form factors only:

$$(75)\qquad \left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}} = \left(\frac{\alpha^2}{4E_1^2\sin^2(\theta/2)}\right)\cdot\left(\frac{1}{1+(2E_1/M_p)\sin^2(\theta/2)}\right)\cdot \\ \cdot\left[\frac{\text{ctg}^2(\theta/2)}{1-t/4M_p^2}\left\{G_E^2(t)-\frac{t}{4M_p^2}G_M^2(t)\right\}+2\left(\frac{-t}{4M_p^2}\right)G_M^2(t)\right].$$

The form of (75) not only allows measurement of the form factors as functions of t but also a check on the validity of our treatment of the photon exchange since

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}\left(\sin^2\frac{\theta}{2}\right)\left[1+\frac{2E_1}{M_p}\sin^2\frac{\theta}{2}\right]\frac{4E_1^2}{\alpha^2}\,,$$

must follow a straight line when plotted as a function of $\text{ctg}^2\,\theta/2$ for fixed t. A deviation from this behavior may be evidence either of a failure of the first Born approximation in $\alpha = 1/137$ or even perhaps of a profound modification in quantum electrodynamics. (In the current style one may question whether the exchanged photon [10] is a « Regge pole » and therefore propagates with variable angular momentum rather than with the one unit of J used in the above angular arguments).

6. – Corrections to the Rosenbluth form.

Before discussing the form factors we may inquire into possible deviations from the form of (75). So far no such experimental deviations have been found [11] up to momentum transfers of $|t| \leqslant 40\ (\text{fermi})^{-2} \approx (1.3\ \text{GeV}/c)^2$.

One possible source of correction might be the exchange of two or more photons between the electron and proton as in Fig. 10. At first sight it may

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(*) Our argument above shows this to be the most general form even in the presence of electron as well as proton structure. The claim to the contrary on p. 91 of ref. [3] is in error. The work of Nikishov referred to there is not.

be expected that corrections due to such graphs are small, being reduced by one power of $\alpha = 1/137$ relative to the one-photon exchange contribution. The situation is not so simple however because of the following reasons:

i) The form factors in (75) become progressively smaller as $|t|$ increases so that the first-order Born term in α is damped at large momentum transfers.

ii) If we slice Fig. 10 by a vertical line we see that it is the product of the Compton scattering of virtual photons by protons multiplied by electron Compton scattering. From the proton Compton scattering measurements it is known that there is a marked resonance for γ energies near 300 MeV, the resonance peak being a factor of roughly 10 larger than the calculated cross-section for a pure point proton [12]. The origin of this resonance appears to be in the formation of the 3-3 resonant state ($I = J = \frac{3}{2}$) of the nucleon by an incident photon of this energy as illustrated in Fig. 11. A similar enhancement of the virtual Compton amplitude in the 2-photon exchange diagram 10 might lead to an appreciable correction to the one-photon exchange result (75).

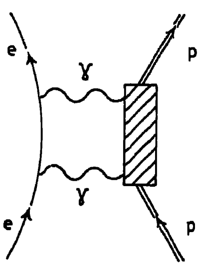


Fig. 10.

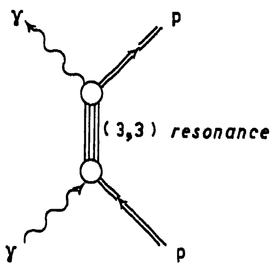


Fig. 11.

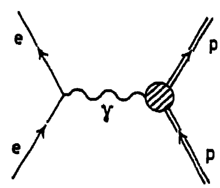


Fig. 12.

Why is this 3-3 resonant contribution possible for the two, but not one, photon exchange term? The reason is simply that the photon must form back again a proton with $I = J = \frac{1}{2}$ when it is absorbed and the resonant channel has the wrong quantum numbers. Also in the scattering center-of-mass system, the photon of Fig. 12 brings momentum but no energy. In Fig. 10 and 11 on the other hand one of the intermediate photons can bring arbitrary quantum numbers, *as well as energy*, which is then carried away again from the proton by the second photon; in between their emission and absorption however the proton can be *polarized*, *i.e.* excited to arbitrary resonant states of various multipole orders. These resonant effects may tend to equalize the one- and two-photon exchange terms.

To see what actually happens we write the scattering cross-section schematically as

$$\frac{d\sigma}{d\Omega} \propto |\alpha A_B + \alpha^2 A_C|^2 \approx \alpha^2 |A_B|^2 + 2\alpha^3 \operatorname{Re} A_B^* A_C\,, \tag{76}$$

where A_B denotes the first Born or one photon amplitude, Fig. 12, and A_C the two-photon, or virtual Compton amplitude, Fig. 10, respectively. The e^8 term is neglected in (76) as reduced by $\alpha^2 = (1/137)^2$; only the interference term is retained as the e^2 correction to the Rosenbluth cross-section. A_C is expected to be important near the resonance energy $\omega_R \approx 300$ MeV for Compton scattering. However near ω_R, A_C is predominantly imaginary, being just the shadow of the channel for photo meson production in the resonant 3-3 channel. Therefore A_C is approximately $\pi/2$ out of phase with A_B, a real potential scattering term in Born approximation, and the interference term in (76) is actually very small [13].

This point can also be understood by a simple classical picture. Consider a light wave, $a\cos\omega t$, incident on a proton which is represented as a charged oscillator with resonant frequency ω_r and damping constant γ. The constants ω_r and γ correspond to the total energy at resonance and the width of the (3, 3) resonance in the pion-nucleon interaction. The charge of the proton scatters the light wave with no change in phase, the amplitude for charge scattering being proportional to $\cos\omega t$. The dipole moment induced in the proton by the incident radiation gives rise to dipole radiation which is phase-shifted relative to the incident wave and is proportional to

$$\frac{(\omega_r^2-\omega^2)\cos\omega t+\gamma\omega\sin\omega t}{(\omega_r^2-\omega^2)^2+\gamma^2\omega^2}. \tag{77}$$

Only the first term in (77) interferes with the charge scattering term and this is zero at the resonance. Moreover, it is odd about the resonant frequency ω_r so that it is largely cancelled in the integration over the virtual photon spectrum of the scattered electron in Fig. (10).

The net result is that at electron energies $E_1 \leqslant 1$ GeV, the resonance in the Compton scattering does not contribute more than ≈ 1 percent to the total. This result is also in accord with latest experimental data [11] which are consistent with the form of (75).

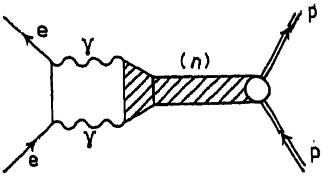


Fig. 13.

In addition to the (3, 3) resonance of the pion-nucleon system one might also consider possible resonant exchanges in the t-channel as illustrated in Fig. 13. The two photons can combine to form a variety of intermediate states (n) with $C=+1$ and different J values. For $J=0$ the contribution is suppressed and vanishes in the limit of $m\to 0$. The argument here is the same as that leading to the suppression of the $\pi\to e+\nu$ decay relative to the $\pi\to\mu+\nu$ for vector (axial) couplings. The possibility of an axial vector or tensor channel has been considered but no definite statements beyond kinematical arguments and speculations on the coupling strengths are possible [6, 14].

For an independent method of detecting two-photon exchange one can measure the polarization of the recoil proton in the scattering. To get rid of the interference term completely one can compare electron-proton and positron-proton scattering since the interference term in (76) changes sign when the electron is replaced by a positron. Therefore a deviation from the Rosenbluth form (75) in the *sum* of observed electron and positron scattering cross-sections can not be blamed on second Born corrections, without appealing to $\alpha^2 \sim 10^{-4}$ terms. If detected it would be of enormously disturbing significance as a test of the validity of quantum electrodynamics. A Regge-pole modification of the photon propagator would be of the same sign for both e^- and e^+ scattering, and would thus show up in their sum.

There are of course always the ordinary radiative corrections to elastic scattering which are present and depend on the resolution geometry of the particular experiment. In our discussion here we always assume that these are analysed [15] and removed from the data before comparing with (75).

So far we have talked in terms of electron-proton scattering exclusively. The crossed reaction (66) of proton–anti-proton annihilation into electron positron pairs is now actively pursued at CERN [16] and will provide important information in the kinematic region (68). The general form of the annihilation cross-section is given already by (70). Using (72) to relate the variables in the s and t channels we can express (70) in terms of the same form factors [17] in (74) and (75):

$$(78) \qquad \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\text{c.m.}}_{p\bar{p}\to e^-e^+} = \frac{\alpha^2}{4\sqrt{t(t-4M_p^2)}} \cdot \left\{\frac{4M_p^2}{t}\,|G_E(t)|^2 \sin^2\varphi_t + |G_M(t)|^2(1+\cos^2\varphi_t)\right\}.$$

Verification of the form of (78) together with information on the form factors for $t > 4M_p^2$ will be of very great interest. The role of this region of t in the theoretical analysis will be studied in Section **8**.

7. – Definition and interpretation of form factors.

The form factors $G_E(t)$ and $G_M(t)$, or equivalently $F_1(t)$ and $F_2(t)$, are the common meeting ground between the experimental measurements and theoretical analysis. An understanding of their observed t-dependence or at least a correlation with other processes in pion-nucleon physics is the goal of much hard work.

To help provide an intuitive framework with which to approach this question, we return to the proton current operator and discuss the physical meaning of the t dependence of the form factors.

The way in which F_1 and F_2 have been introduced in (73) corresponds to defining the electromagnetic current of the proton by

$$J_\mu^{(p)}(\mathfrak{P}_f, \mathfrak{P}_i) = e_p \bar{u}(\mathfrak{P}_f, S_f)\left[F_1(t)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{2M_p}\varkappa_p F_2(t)\right]u(\mathfrak{P}_i, S_i)\,, \tag{79}$$

where

$$\sigma_{\mu\nu} \equiv \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\,.$$

The form (79) is the most general possible one for the proton current if $J_\mu^{(p)}$ is to transform as a Lorentz four-vector and is to obey a differential conservation law

$$\frac{\partial}{\partial x_\mu} J_\mu^{(p)}(x) = 0\,,$$

or in momentum space,

$$(\mathfrak{P}_f - \mathfrak{P}_i)_\mu J^{\mu(p)}_{\mathfrak{P}_i\mathfrak{P}_f} = 0\,. \tag{80}$$

The proof of the generality of (79) can be constructed readily with the aid of (80), Lorentz covariance, and the Dirac equation, and is another way to establish the generality of the Rosenbluth cross-section (*). (We may also put J_μ in terms of the G_E and G_M with the aid of (74).)

If the proton had no extended spatial structure and were pointlike with a charge e_p and anomalous magnetic moment $\varkappa_p$, we would write the scattering amplitude as in (42) and (45), replacing the current source in (49) and (51) by

$$\Box_z A^\mu(z) = J^{\mu(1)}(z) + J^{\mu(2)}(z)\,,$$

where

$$J^{\mu(1)}(z) = e_p\, \bar\psi_{(p)}(z)\gamma^\mu\psi_{(p)}(z)\,, \tag{81}$$

and

$$J^{\mu(2)}(z) = -\,ie_p\frac{\varkappa_p}{2M_p}\frac{\partial}{\partial z^\nu}\left[\bar\psi_{(p)}(z)\,\sigma^{\mu\nu}\,\psi_{(p)}(z)\right]\,.$$

Repeating the volume integrations leading to (53) we find $\bar u(\mathfrak{P}_f, S_f)\gamma^\mu u(\mathfrak{P}_i, S_i)$ there replaced by

$$\bar u(\mathfrak{P}_f, S_f)\left[\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_p}\varkappa_p\right]u(\mathfrak{P}_i, S_i)\,. \tag{82}$$

(*) See for example ref. [3].

For a physical proton with a finite distribution of currents, (81) is replaced by

$$\Box_z A^\mu(z) = \int d^4w F_1(z-w) J^{\mu(1)}(w) + \int d^4w F_2(z-w) J^{\mu(2)}(w) , \tag{83}$$

where the two functions F_1 and F_2 describe the structure of the two currents. On grounds of relativistic covariance, F_1 and F_2 must be scalar functions of the invariant space-time interval $(z-w)^2$. The scattering amplitude (53) is now

$$S_{fi}^{(1)} = -\frac{ie^2}{V^2}\sqrt{\frac{m^2}{E_i E_f}}\sqrt{\frac{M_p^2}{\mathfrak{E}_i \mathfrak{E}_f}}\{\bar{u}(p_f, s_f)\gamma_\mu u(p_i, s_i)\}\cdot\frac{1}{q^2}\cdot \tag{84}$$

$$\cdot\int d^4z \exp[i(p_f - p_i)\cdot z]\int d^4w \exp[i(\mathfrak{P}_f - \mathfrak{P}_i)\cdot w]\{\bar{u}(\mathfrak{P}_f, S_f) O^\mu(z-w) u(\mathfrak{P}_i, S_i)\},$$

where

$$O^\mu(z-w) \equiv F_1(z-w)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_p}\varkappa_p F_2(z-w) .$$

Displacing the origin of the d^4w integration to z and integrating first over w and then over z we obtain

$$S_{fi}^{(1)} = -\frac{ie^2}{V^2}\sqrt{\frac{m^2}{E_i E_f}}\sqrt{\frac{M_p^2}{\mathfrak{E}_i \mathfrak{E}_f}}(2\pi)^4\delta^4(\mathfrak{P}_f + p_f - \mathfrak{P}_i - p_i)\cdot \tag{85}$$

$$\cdot\{\bar{u}(p_f, s_f)\gamma_\mu u(p_i, s_i)\}\frac{1}{q^2}J^{\mu(p)}(\mathfrak{P}_f, \mathfrak{P}_i) ,$$

with $J^{\mu(p)}(\mathfrak{P}_f, \mathfrak{P}_i)$ given by (79) and

$$F_{1,2}(t) \equiv \int d^4v \exp[iq\cdot v]F_{1,2}(v); \qquad t = q^2 . \tag{86}$$

The entire structure of the proton current is contained in the $F_{1,2}(t)$. For a point interaction $F(v) = \delta^4(v)$ and, by (86), $F_{1,2}(q^2) = 1$, a constant independent of q^2. In this case (79) reduces to (82). Thus a variation of $F_{1,2}(q^2)$ with q^2 corresponds to a finite structure.

For a geometric picture of this structure we go to the center-of-mass system where $q^\mu = (0, \boldsymbol{q})$ and

$$F(t) = \int d^3v\int dv_0 F(v_0^2 - |\boldsymbol{v}|^2)\exp[-i\boldsymbol{q}\cdot\boldsymbol{v}] \equiv \int d^3v\, \mathscr{F}(|\boldsymbol{v}|^2)\exp[-i\boldsymbol{q}\cdot\boldsymbol{v}] . \tag{87}$$

Expanding for small $|\boldsymbol{q}|^2$ we have

$$F(\boldsymbol{q}) = \int d^3v\,\mathscr{F}(|\boldsymbol{v}|^2) - \frac{1}{6}|\boldsymbol{q}|^2\int d^3v|\boldsymbol{v}|^2\mathscr{F}(|\boldsymbol{v}|^2) + \ldots \equiv 1 - \frac{1}{6}|\boldsymbol{q}|^2\langle r^2\rangle;$$

where $\int d^3v\,\mathscr{F}(|\boldsymbol{v}|^2) = F(0) = 1$ and the mean square radius is defined by the second term. Returning to an arbitrary Lorentz frame we have, since $F(q^2)$ is a scalar,

$$F(q^2) = 1 + \tfrac{1}{6}q^2\langle r^2\rangle + \ldots . \tag{88}$$

In the center-of-mass system $\langle r^2 \rangle$ is just the mean square co-ordinate weighted over $\mathscr{F}(|\boldsymbol{v}|^2)$; in general it is just $6\times$ the slope of $F(q^2)$ at $q^2=0$:

$$(89)\qquad \langle r^2 \rangle \equiv 6 \frac{\mathrm{d}}{\mathrm{d}q^2} F(q^2) \bigg|_{q^2=0} .$$

The most recent analyses and measurements [1, 2, 11] on the proton form factors show the following behaviour of G_E and G_M *vs.* $-t(>0)$:

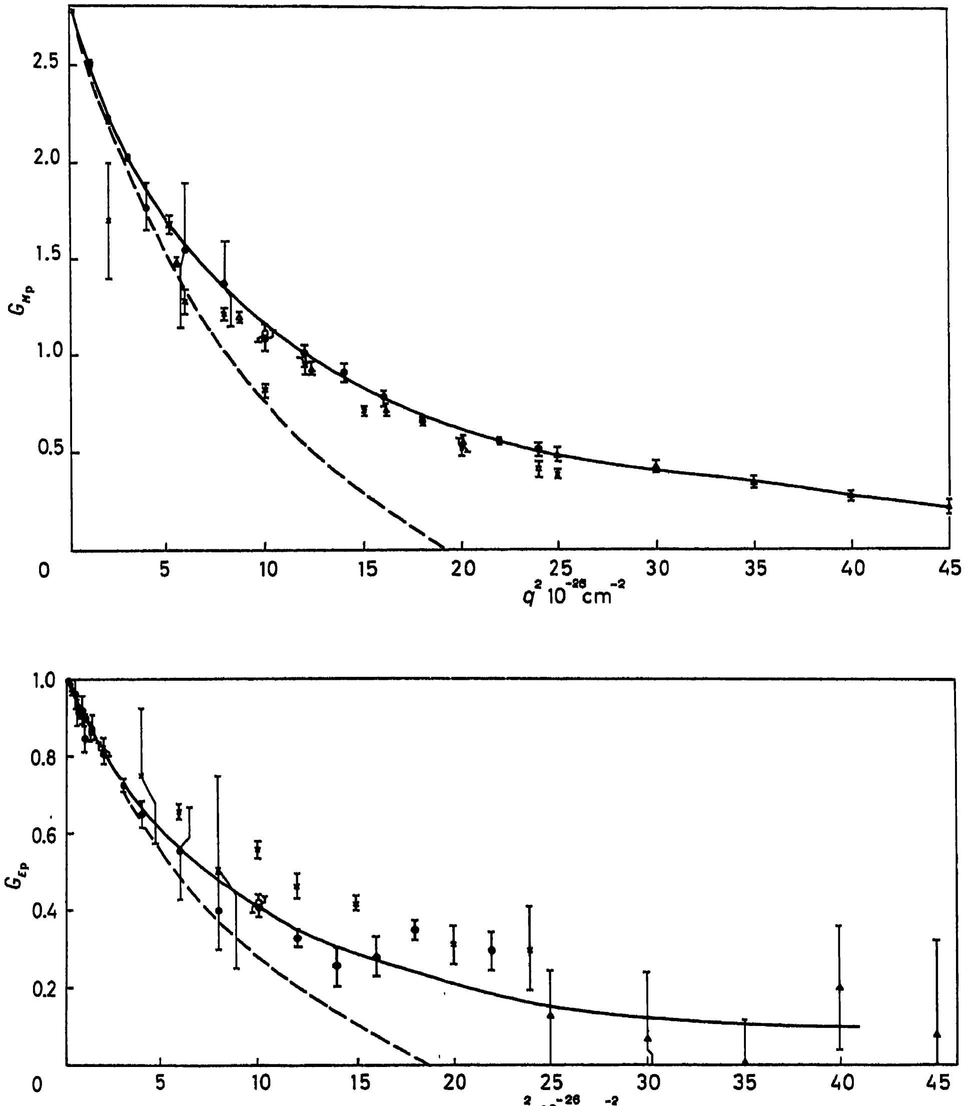


Fig. 14. – See HAND *et al.* [1].

Out to $-t = 2\ \text{fermi}^{-2}$, ($1\ \text{fermi}^{-1} = 197\ \text{MeV}/c$), F_1 and F_2 differ by less than 3% and correspond to a radius, by (88), of

$$\langle r^2 \rangle = 0.58\ (\text{fermi})^2 . \tag{90}$$

To measure the neutron form factors one has to scatter electrons from deuterons and subtract the known contribution of the proton. However this procedure introduces additional theoretical difficulties (*). It also magnifies the experimental errors since the largest part of the contribution to scattering from the deuteron is due to the proton. The validity of the one photon exchange approximation can be verified by a straight-line behaviour analogous to (63) for the proton [19].

Experimentally it is found that the anomalous moment form factor $F_2^n(t)$ for the neutron has a somewhat larger slope than $F_2^p(t)$, and that $F_1^n(t)$ remains very close to zero. This is consistent with the measurements on the neutron-electron interaction by FERMI and by RABI (**) and their collaborators which first showed

$$\langle r_1^2 \rangle_n \equiv 6 \frac{d}{dq^2} F_1^n(q^2) \bigg|_{q^2=0} < (0.1\,\text{fermi})^2 . \tag{91}$$

This observation has remained a formidable theoretical puzzle and challenge until today!

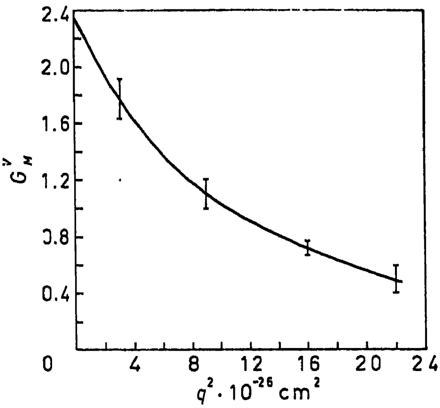
In order to relate the observed form-factor behaviours to the known vector resonances, ρ and ω, with $I=1$ and $I=0$ respectively, we find it convenient to form the sum and difference of the proton and neutron form factors, which correspond to exchange of definite I-spin between the electron and nucleon lines. The isovector combinations for $I=1$ exchange are

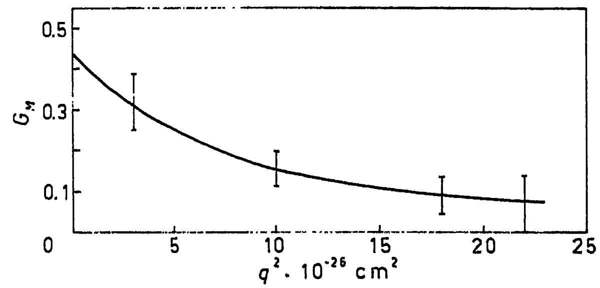
$$\left\{\begin{aligned}
&G_E^V \equiv \frac{1}{2}[G_E^p - G_E^n] \equiv F_1^V + \frac{t}{4M_p^2} F_2^V ,\\
&G_M^V \equiv \tfrac{1}{2}[G_M^p - G_M^n] \equiv F_1^V + F_2^V ,\\
&\text{where}\\
&F_1^V \equiv \tfrac{1}{2}(F_1^p - F_1^n) ,\\
&F_2^V \equiv \tfrac{1}{2}(\varkappa_p F_2^p - \varkappa_n F_2^n) .\\
&\text{The isoscalar ones for } I=0 \text{ exchange are}\\
&G_E^S \equiv \frac{1}{2}(G_E^p + G_E^n) \equiv F_1^S + \frac{t}{4M_p^2} F_2^S ,\\
&G_M^S \equiv \tfrac{1}{2}(G_M^p + G_M^n) \equiv F_1^S + F_2^S .
\end{aligned}\right. \tag{92}$$

(*) Such as binding corrections and exchange current terms [18].

(**) For literature and discussion see ref. [3].

It is tempting and natural to attempt to correlate the observed isovector form factors with the ρ-meson and the isoscalar ones with the ω-meson which carry the same quantum numbers. Whether the ρ and ω are to be interpreted as « elementary » vector particles forming an intermediary current tying the photon to the nucleon, or as dynamical resonances formed from virtual pions and nucleon–





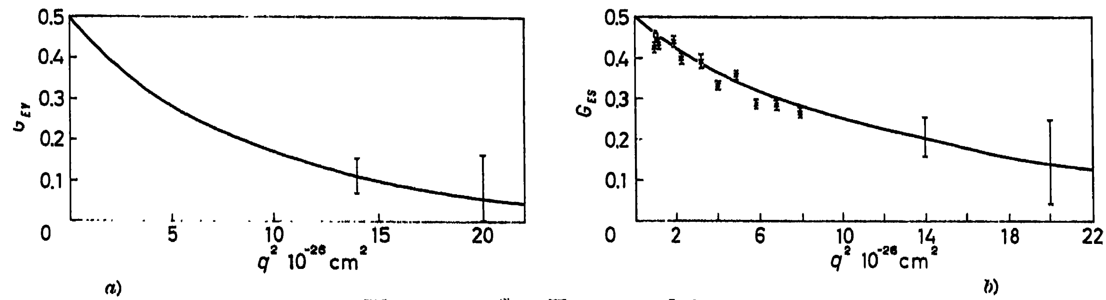


Fig. 15. – See HAND *et al.* [1].

anti-nucleon pairs is a separate question. If these resonances which are produced and observed in high-energy pion and nucleon strong-coupling processes also appear as important channels in the electromagnetic interaction, we shall have a significant advance in our ability to correlate different measurements and to identify the dominant strong interaction characteristics.

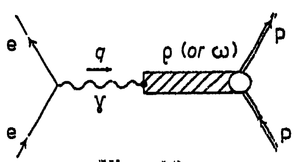


Fig. 16.

In terms of a very simple model we may picture the electromagnetic interaction as shown in Fig. 16. If we assume point couplings of the γ to ρ and of the ρ to the nucleon line, then in computing the scattering amplitude, the γ propagator $1/q^2$ of eq. (85) is replaced by

$$\frac{1}{q^2}\left\{C_\rho \frac{-m_\rho^2}{q^2-m_\rho^2}\right\}, \tag{93}$$

where m_ρ is the ρ mass and C_ρ defines a coupling constant of the ρ to the γ and nucleon. Equation (93) has the advantage of giving a simple and definite prediction for the form factors, corresponding to a radius according to (89) of

$$\langle r^2 \rangle_{V,S} = \frac{6}{m^2_{\rho,\omega}} C_{\rho,\omega} \, . \tag{94}$$

Comparing with (90), we see that

$$C_{\rho,\omega} \sim 1.4 \, .$$

This is a definite deviation from unity, independent of remaining uncertainties in the detailed form-factor behaviours at larger momentum transfers, and shows our picture to be too simple. The necessary corrections may come from other « resonances » or from nonresonant contributions in these channels which correct the propagator (93) or give structure to the vertices, $C_\rho(t)$. We may take the optimistic view that a significant portion of the observed form-factor behaviour *is* consistent with a simple picture of the interaction being dominated by the ρ and ω resonances and press on to a dispersion-theoretic attack on calculating the G_E and G_M, or F_1 and F_2, in terms of a few dominant channels, notably including the ρ and ω.

8. – Dispersion theory approach to calculations of nucleon form factors.

At present, dispersion theory provides the most hopeful procedure for attempting calculations of the electromagnetic form factors of nucleons [20].

In order to write dispersion relations for the form factor one must know the analytic properties of $F(q^2)$ in the complex q^2 plane. ($F(q^2)$ here represents any of the four form factors.) The physical region for $F(q^2)$ in the q^2 plane is along the negative real axis $q^2 < 0$, corresponding to electron proton scattering, and along the positive real axis for $q^2 > 4M_p^2$, the threshold for $p\bar{p}$ production by an e^-e^+ pair. The same function of q^2 describes the form factor in both regions shown in Fig. 17 according to the crossing symmetry.

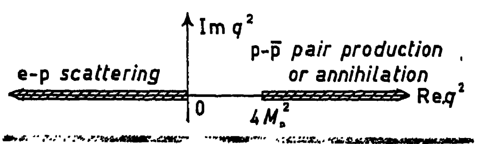


Fig. 17.

The necessary properties for the analytic continuation, $F(z)$ of $F(q^2)$, defined by

$$F(q^2) = \lim_{z \to q^2 + i\varepsilon} F(z) \, , \tag{95}$$

are

i) $F(z)$ is analytic in the closed upper half z plane, except for a branch cut along the positive real axis, $z > 4m_\pi^2$, where m_π is the mass of the pion; and

ii) $F(z)$, or perhaps $F(z)/z$, vanishes as $z \to \infty$ anywhere in the upper half plane.

These properties have not been established by rigorous arguments [21] (for the physically observed mass ratio $m_\pi/M_p = 0.15 < (\sqrt{2}-1)$). The most fruitful approach has been to investigate the analytic properties of the perturbation theoretic expressions for $F(z)$ and statements i) and ii) have been shown to be true to any order of perturbation theory [22]. Consider just the two Feynman graphs in the second order perturbation expansion of the pion-nucleon coupling, with the kinematics as assigned in Fig. 18.

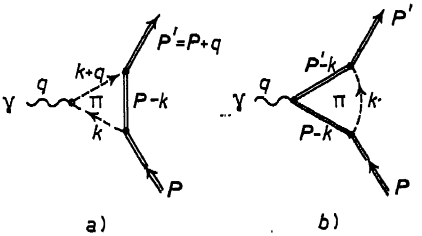


Fig. 18.

In the absence of the pion-nucleon coupling the photon would interact with the nucleon at a point, leading to $F(q^2) = 1$ which clearly satisfies the analyticity properties. We can write down the amplitude for the diagram (*a*) using Feynman rules and have, upon omitting all inessential (spinor) factors and numerators which do not alter the analytic structure,

$$(96) \qquad F(q^2) \sim \int d^4k \, \frac{1}{(\mathfrak{P}-k)^2 - M_p^2 + i\varepsilon} \cdot \frac{1}{k^2 - m_\pi^2 + i\varepsilon} \cdot \frac{1}{(k+q)^2 - m_\pi^2 + i\varepsilon} \, .$$

$F(q^2)$ is just the product of three Feynman propagators with the three internal lines with the momenta, as labelled, conserved at each vertex and with the infinitesimal $+i\varepsilon$ appended to assure the appropriate boundary conditions as in (11) and (48). The pion propagator differs from the photon's (48) only by the finite mass term.

With the help of the familiar and useful identity

$$\frac{1}{a_1 a_2 a_3} = 2! \int\limits_0^\infty \frac{dx_1\, dx_2\, dx_3\, \delta(1 - x_1 - x_2 - x_3)}{[a_1 x_1 + a_2 x_2 + a_3 x_3]^3} \, ,$$

and a linear shift of the momentum integration variable we rewrite (96) as

$$F(q^2) \sim \int\limits_0^\infty dx_1\, dx_2\, dx_3\, \delta(1 - x_1 - x_2 - x_3) \int \frac{d^4k'}{(k'^2 + \Delta)^3} \, ,$$

where

$$k' = k + l \, ,$$

$$l = q x_2 - \mathfrak{P} x_3 \, , \qquad \text{with} \qquad \mathfrak{P}^2 = M_p^2 = (\mathfrak{P} + q)^2 \, ,$$

and

$$\Delta \equiv q^2\,[x_2(1-x_2) - x_2x_3] - M_p^2x_3^2 - m_\pi^2(1-x_3) + i\varepsilon\;.$$

The integral on d^4k' gives

$$F(q^2) \sim \int_0^1 dx_3 \int_0^{1-x_3} dx_2\,\frac{1}{\Delta}\,,$$

which may be rewritten in terms of $x \equiv 1-x_3$ and $y \equiv x_2/x$ as

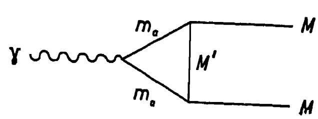
$$F(q^2) \sim \int_0^1 x\,dx \int_0^1 dy\,\frac{1}{q^2x^2y(1-y) - M_p^2(1-x)^2 - m_\pi^2 x + i\varepsilon}\,. \tag{97}$$

The *positive* infinitesimal imaginary part in the denominator originates from the boundary conditions on the Feynman propagators in (96) and shows that the singularities of $F(q^2)$ as a function of complex q^2 lie in the lower half of the complex q^2 plane. We may thus make the desired continuation of $F(q^2)$ in accord with (95), observing that $F(z)$ is analytic except for singularities when the denominator vanishes at

$$z = \frac{M_p^2(1-x)^2 + m_\pi^2 x}{x^2y(1-y)}\,. \tag{98}$$

Since $0<x,\,y<1$ according to (97), this relation can be satisfied for any $z=q^2>4m_\pi^2$. In conclusion, the second order perturbation diagram of Fig. 18*a* leads to $F(z)$ which is analytic in the entire complex q^2 plane except for a continuous line of poles, or branch cut along the positive real axis starting at $q^2=4m_\pi^2$. A similar discussion of Fig. 18*b* leads to a similar conclusion with the cut starting at $q^2=4M_p^2$. Similar results may be established in any order of perturbation theory with the cut starting in each case (*) at a

(*) As discussed in ref. [22] there occur *anomalous thresholds* for the cuts for special ranges of mass relations between the various particles. Consider for example the graph



and suppose M is a proton and m_a the pion. For a physical intermediate state $M' > M$

threshold value of q^2 equal to the square of the mass of the lightest real physical system that can connect the virtual photon of mass q^2 to the nucleon line. It is also found in perturbation theory that convergence property ii) on p. 32 is satisfied by $F(z)$, or in some cases by $F(z)/z$.

From perturbation-theory diagrams we have then (more than) the desired properties i) and ii) for $F(z)$. Assuming at first that ii) applies for $F(z)$ itself we can therefore write, by Cauchy's theorem, the contour integral

$$F(z) = \frac{1}{2\pi i}\oint_c \frac{F(\omega)}{\omega - z}\, d\omega\,, \tag{99}$$

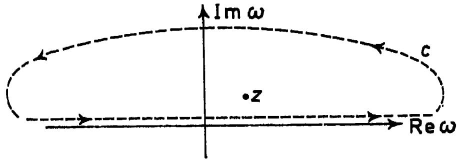


Fig. 19.

where c is the contour shown in Fig. 19. Since $F(z)$ vanishes at ∞, the contribution of the semicircle at infinity is zero. Then since the quantity of interest is $F(q^2)$, we reduce (99) to

$$F(q^2) = \lim_{\varepsilon\to 0}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{F(q'^2)}{q'^2 - q^2 - i\varepsilon}\, dq'^2\,, \tag{100}$$

where the integration is now along the real axis, and the contour has been made to approach it from the upper half plane. Using the symbolic identity

$$\lim_{\varepsilon\to 0}\frac{1}{x - i\varepsilon} = P\,\frac{1}{x} + i\pi\,\delta(x)\,, \tag{101}$$

and $M^2 < M'^2 + m_a^2$. Then the cut starts at the normal threshold

$$q^2 > 4m_a^2\,.$$

Consider next the case where the graph describes the deuteron's electromagnetic structure. We have then

$$m_a = M' = M_p \qquad \text{and} \qquad M = M_D = 2M_p - \varepsilon\,,$$

and $M^2 > m_a^2 + M'^2$ (because the deuteron binding energy ε is small). In this case one finds that the cut can start earlier than expected (*anomalous threshold*), and runs from

$$q^2 = \frac{1}{M'^2}\left[(m_a + M')^2 - M^2\right]\cdot\left[M^2 - (m_a - M')^2\right] = 16 M_D\,\varepsilon\,.$$

Thus when we consider the electromagnetic size of a compound system such as the deuteron with $M^2 > m_a^2 + M'^2$, the size of the electromagnetic structure is determined by the binding energy and not by the pion mass. It agrees with the size given by the wave function $\exp[-2\sqrt{\varepsilon M}\,r]$.

where p means principal value, this becomes

$$F(q^2) = \frac{1}{\pi i} \, \mathrm{P}\!\!\int\limits_{-\infty}^{\infty} \frac{F(q'^2)}{q'^2 - q^2} \, dq'^2 . \tag{102}$$

Now taking the real part of this equation gives

$$\operatorname{Re} F(q^2) = \frac{1}{\pi} \, \mathrm{P}\!\!\int\limits_{-\infty}^{\infty} \frac{\operatorname{Im} F(q'^2)}{q'^2 - q^2} \, dq'^2 ,$$

or equivalently

$$F(q^2) = \frac{1}{\pi} \int\limits_{-\infty}^{\infty} \frac{\operatorname{Im} F(q'^2)}{q'^2 - q^2 - i\varepsilon} \, dq'^2 . \tag{103}$$

The limit $\varepsilon \to 0$ is to be understood from now on.

If there is a region of the real axis in which $\operatorname{Im} F(q^2) = 0$, the above equation may be used to define the analytic continuation of $F(z)$ into the lower half-plane. To be precise, define

$$F(z) = \frac{1}{\pi} \int\limits_{-\infty}^{\infty} \frac{\operatorname{Im} F(q'^2)}{q'^2 - z} \, dq'^2 , \tag{104}$$

for an arbitrary z. Now let z approach the real axis, from above and below, in a region where $\operatorname{Im} F(q'^2) = 0$. The same value is obtained, so eq. (104) in fact defines the analytic continuation of $F(z)$ into the lower half-plane, across this region of the real axis. If, on the other hand, z is allowed to approach the real axis from above and below at a point where Im F does not vanish, one obtains from (103) and (104),

$$\lim_{\varepsilon \to 0} \{F(z)|_{z=q^2+i\varepsilon} - F(z)|_{z=q^2-i\varepsilon}\} = 2i \operatorname{Im} F(q^2) ,$$

where by (95)

$$\operatorname{Im} F(q^2) = \operatorname{Im} \{\lim_{z \to q^2+i\varepsilon} F(z)\} .$$

Thus the analytic function $F(z)$, defined in the entire plane, has a branch cut across which the discontinuity of the function is $2i \operatorname{Im} F(q^2)$. Elsewhere on the real axis $F(q^2)$ is real. Real form factors, for $q^2 < 0$ and real as in

electron-proton scattering, correspond to a Hermitian current in (79) and to a unitary scattering amplitude (84) as required for probability conservation in the elastic electron-proton interaction. For timelike $q^2 > 4m_\pi^2$ an interacting $e\bar{e}$ pair can form a $\pi^+\pi^-$ pair, or other strongly interacting systems as q^2 increases above their thresholds. The possibility of the annihilation process proceeding into such additional channels depletes the probability of forming a $p\bar{p}$ system. Thus the probability to form a $p\bar{p}$ system only is less than unity and for $q^2 > 4m_\pi^2$ the interaction currents are not Hermitian and $F(q^2)$ may develop an imaginary part.

According to (98) and the physical arguments following it, $\operatorname{Im} F(q^2) \neq 0$ and the branch cut in (104) exists for $q^2 > 4m_\pi^2$, so tat finally,

$$F(q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\operatorname{Im} F(q'^2)}{q'^2 - q^2 - i\varepsilon}\, dq'^2 \,, \tag{105}$$

for the physical form factor itself.

Equation (105) is known as an unsubtracted dispersion relation. It may be wrong because of the assumption used in establishing it, that

$$F(z) \to 0 \qquad \text{as } z \to \infty \,. \tag{106}$$

We can prove that (105) is valid if the charge renormalization constant $(1/Z_3^{\frac{1}{2}} = e_0/e_r)$ is finite. To lowest order in perturbation theory the form factors (79) have the asymptotic behaviours

$$F_2(z) \to 0 \,, \qquad \text{as } z \to \infty,$$

$$F_1(z) \sim \ln z \qquad \text{as } z \to \infty:$$

This indicates that we may need a dispersion relation for $F(z)/z$ instead of $F(z)$

$$\frac{F(z)}{z} = \frac{1}{2\pi i} \oint_C \frac{F(\omega)\, d\omega}{\omega(\omega - z)} + \frac{F(0)}{z} \,. \tag{107}$$

The second term has been added because we must subtract out the pole introduced at the origin by the additional denominator factor $1/\omega$. To perform this subtraction is equivalent to introducing a new parameter $F(0)$ to be *assigned* from experiment, but not calculated from theory. From (107) we deduce as before

$$F(q^2) = F(0) + \frac{q^2}{\pi} \int \frac{\operatorname{Im} F(q'^2)}{q'^2(q'^2 - q^2 - i\varepsilon)}\, dq'^2 \,. \tag{108}$$

9. – Calculation of absorptive amplitudes and the vector resonances.

In either the subtracted or unsubtracted forms it is clear that the dispersion relations do not *themselves* form the basis for a calculation of the form factors. We must investigate the discontinuity across the branch cut $q^2 > 4m_\pi^2$ and learn how to calculate the $\operatorname{Im} F(q^2)$, or absorptive amplitudes.

For a guide in this quest we return to the perturbation expression. It is a nontrivial task to show that the modification of the Feynman amplitude (96) leading to its absorptive part consists of the replacement

$$(109)\qquad \frac{1}{k^2-m_\pi^2+i\varepsilon}\cdot\frac{1}{(k+q)^2-m_\pi^2+i\varepsilon}\to \to 2\pi^2\,\delta(k^2-m_\pi^2)\theta(-k_0)\,\delta\big((k+q)^2-m_\pi^2\big)\,\theta(k_0+q_0)\,.$$

According to the complex conjugate of the identity (101), we obtain the imaginary part of a single Feynman amplitude by the replacement

$$\frac{1}{k^2-m_\pi^2+i\varepsilon}\to-\pi\delta(k^2-m_\pi^2)\,.$$

That the imaginary part of (96) is obtained by the replacement (109) was discussed by MANDELSTAM [23] for special classes of diagrams and proved to all orders by CUTKOSKY [24]. It requires putting the two pions on their mass shells (*) with positive energy being conveyed to the nucleon line. Introducing (109) into (96) gives

$$(110)\qquad \operatorname{Im} F(q^2)\sim\int\frac{d^4k}{(\mathfrak{P}-k)^2-M_p^2+i\varepsilon}\cdot \cdot\,\delta(k^2-m_\pi^2)\theta(-k_0)\,\delta\big((k+q)^2-m_\pi^2\big)\,\theta(k_0+q_0)\,,$$

and shows explicitly that the absorptive part of $F(q^2)$ is nonvanishing only if q^2 is timelike and exceed the threshold $4m_\pi^2$ to produce two real pions on their positive energy mass shells. Similar results are obtained more generally

(*) A meson and a nucleon entering a common vertex in Fig. 18 cannot be put on their mass shells simultaneously. This would correspond to the three particles at a single vertex all being on their mass shells and would imply the possibility of one decaying into the other two.

without resorting to perturbation theory (*) and are written symbolically

$$\operatorname{Im} F(q^2) \sim \sum_n \delta^4(P_n - q)\, \langle \gamma(q^2)\,|\,n\rangle \langle n\,|\,p\bar{p}\rangle\,, \tag{111}$$

where P_n is the four-momentum of one of the possible states $n\rangle$ contributing $\langle\gamma(q^2)|$ is the amplitude for the virtual photon of mass q^2 to form the state; $\langle n|p\bar{p}\rangle$ is the amplitude for the state to be formed by a $p\bar{p}$ system; and $\delta^4(P_n - q)$ insures energy momentum conservation in forming $|n\rangle$.

The result (111) shows the key to the potential usefulness and power of the dispersion approach. In calculating $\operatorname{Im} F(q^2)$ we must consider the *real physical states* which connect the one photon state to the nucleon line. Since the squares of the four-momenta of physical states are timelike and exceed the squares of their rest masses, q^2 is > 0, and the one-photon state must be scattering the nucleon out of a negative energy into a positive energy state according to Fig. 4; in other words *it is producing a nucleon–anti-nucleon pair.* In the calculations of the amplitudes for the photon to form real physical states which then form the $p\bar{p}$ and $n\bar{n}$ pairs we hope to appeal to other *experimental information* about them and thereby avoid some crude approximations in theoretical calculations. We may be optimistic on two points:

i) only a few amplitudes are important, and

ii) these may be related to experiment.

With regard to the first of these points, (111) provides a basis for an expansion in the *masses* of the intermediate states. The denominator in the dispersion integral (105) gives a weak weighting to the low mass end of the spectrum. In the subtracted version (108), the weighting is more substantial. We may hope then that, at least in the calculation of the small q^2 behaviour of the form factors, only the lightest intermediate states $n\rangle$ are essential. At least the expansion in intermediate states according to their masses may provide a more sensible starting point that the futile one of a perturbation expansion in powers of the large pion-nucleon coupling constant, $g^2/4\pi = 14$!

The difference in these approaches is clearly seen by comparing the two diagrams (a) and (b) of Fig. 18. In perturbation theory these both contribute importantly in the same g^2 order. In the dispersion approach, the 2π intermediate state from diagram (a), with threshold at $q^2 = 4m_\pi^2$ is assumed to play a dominant role. However the lower limit for the dispersion integral for the $\mathcal{N}\bar{\mathcal{N}}$ intermediate state (**) in diagram (b) is much higher, since the pair

(*) For further development of eq. (111) and for bibliography, see ref. [3].

(**) Henceforth $\mathcal{N}\bar{\mathcal{N}}$ denotes a nucleon-anti-nucleon pair — *i.e.* either proton or neutron.

can be formed only when $q^2 > 4M_D^2$. Since many other states contribute to the absorptive amplitude for such large q^2, as sketched in Fig. 20, and in addition there are stringent limitations on the nucleon–anti-nucleon scattering amplitude from the requirement that it be unitary [20] which suppress the amplitude for diagram (*b*) below its perturbation theory value, we drop it in the first approximation.

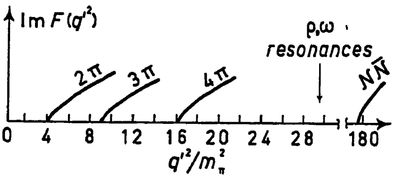


Fig. 20.

In a more pragmatic (and honest!) way we admit that our primary reasons for limiting ourselves to a few of the lightest states are that we can compute them approximately and can correlate their main features with other observations. The 2π intermediate states with $C = -1$, as required to form a photon, must have $I = 1$ and therefore contribute to the isovector form factors only. Recall from the leactures of Professor ROSENFELD that the G parity is related to C and I by

$$G = C(-1)^I, \tag{112}$$

and has eigenvalues $(-1)^n$ for a state of zero total charge formed by n pions. Then (112) shows that states with an even number of pions and no other particles contribute only to the isovector form factors, since G is even and therefore $I = 1$. Conversely the odd pion states contribute to the $I = 0$ isoscalar form factors.

In line with our previous discussion we retain only the 2π and 3π states for the $I = 1$ and $I = 0$ form factors, respectively. If the amplitudes to form these states have resonances strongly enhancing $\operatorname{Im} F(q^2)$ at some value $q'^2 \simeq q_r^2$, then, by (105),

$$F(q^2) = \frac{1}{1 - q^2/q_r^2}, \tag{113}$$

and by (89)

$$\langle r^2 \rangle = \frac{6}{q_r^2}.$$

The normalization to $F(0) = 1$ in (113) is purely for convenience and can be scaled for the appropriate F or G as defined earlier without altering the value of $\langle r^2 \rangle$ according to the usual conventions. The data (90) and ref. [1] indicate need for the peak to occur at $q_r^2 \sim 28 m_\pi^2$ which is low enough so that the neglected 4π contributions to the isovector form factors, with threshold at $16 m_\pi^2$, may not have too strong an influence—we hope! A less stringent

assumption is to collect all of the heavier mass contributions, neglected in (113), into the subtraction constant of the subtracted dispersion relations (108) in an analogue of the effective range expansion. The q^2 variations of these contributions from higher q'^2 values is presumed weaker—so weak that they may be summarized by a constant—and their contribution is fitted as an experimental parameter rather than calculated. We have then

$$F(q^2) = 1 + q^2 \frac{a}{q_r^2 - q^2} = (1 - a) + \frac{a}{1 - q^2/q_r^2}, \tag{114}$$

where constants a and q_r^2 provide a two-parameter fit. Comparing with (94) we see that with the choice of $q_r^2 = m_\rho^2$ (or m_ω^2) we must have $a \sim 1.4$. Our optimistic interpretation of this result is that the observed ρ and ω resonances of the 2- and 3-pion system give rise to a dominant contribution in the analysis of the electromagnetic structure of nucleons. From the detailed fit of HAND *et al.* [1] it is suggested that ρ and ω channels are the major ones, but sizable heavier mass states contribute to a soft core outside the range of our present approximate discussions.

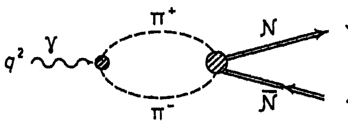


Fig. 21.

In order to develop more familiarity with the dispersion methods of calculation let us look with somewhat more detail into the calculation of the $\pi^-\pi^+$ pair contribution to the isovector form factors. In evaluating the contribution of Fig. 21, which corresponds to the term with $n\rangle$ in the $\pi^+\pi^-$ pair state in (111), we must introduce the amplitude for $\pi^-\pi^+$ pair production by a timelike γ and the amplitude for $\mathcal{N}\overline{\mathcal{N}}$ annihilation into two pions in the p state. This latter quantity cannot however be related completely to the physical amplitude for

$$\mathcal{N}\overline{\mathcal{N}} \leftrightarrow \pi^+\pi^-, \tag{115}$$

since the physical process is confined to the energy region $q'^2 > 4M_p^2$ whereas we must know it according to (111) for all $q'^2 > 4m_\pi^2$ contributing to the sum. This requires an analytic continuation of the amplitude into the unphysical region $4m_\pi^2 < q'^2 < 4M_p^2$. The Mandelstam representation gives a plausible and unique way for carrying out this continuation [23, 25].

The original calculations of the *absorptive* amplitude from diagram (21)

treated the pion as a point charge in interaction at the photon vertex; *i.e.* in analogy with (82) for the point proton, a pion current

$$e\{k + k + q\}^{\mu}, \tag{116}$$

was introduced. It was found [20] for any « reasonable » continuation of the amplitude (115) that the absorptive amplitude was small and smoothly increasing with q'^2 as in the solid line of Fig. 22. The major contributions come from large q'^2 values and the computed electromagnetic radii were an order of magnitude too small. What was needed to explain the experimental facts was an enhancement at low q'^2 of the type shown with the dotted line. Such an enhancement cannot be provided by the higher mass states.

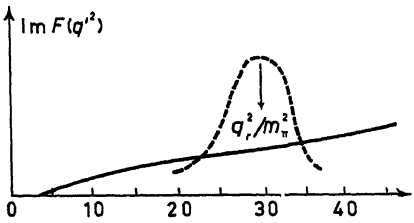


Fig. 22.

In order to remove the discrepancy it seems more natural to enhance the probability for the $\pi^+\pi^-$ pair state to form a low mass state q_r^2 by postulating a strong attractive interaction between them at this mass. This attraction must occur in the P state ($J=1$, $C=-1$) and, if present, introduces a structure into the pion's electromagnetic structure which cannot be considered pointlike. This means replacing (116) by

$$e(2k^{\mu} + q^{\mu})F_{\pi}(q^2), \tag{117}$$

where $F_{\pi}(q^2)$ defines the pion's electromagnetic form factor. It is a scalar function of the invariant momentum transfer q^2 at the vertex, normalized to $F_{\pi}(0)=1$. Since only one conserved four-vector form (117) can be constructed from the variables for a spinless particle, there is only one form factor at the pion vertex.

Calculation of the absorptive amplitude in Fig. 21 requires therefore that we know $F_{\pi}(q'^2)$ for all $q'^2 > 4m_{\pi}^2$. With this we see now that the electromagnetic structure of the pion and nucleon become closely related problems. For the very same reasons given in setting up the dispersion-theory analysis of the nucleon form factors we proceed here by writing a dispersion relation for F_{π}.

$$F_{\pi}(q'^2) = 1 + \frac{q'^2}{\pi}\int_{4m_{\pi}^2}^{\infty} \frac{\operatorname{Im} F_{\pi}(\sigma^2)}{\sigma^2(\sigma^2 - q'^2 - i\varepsilon)}\, d\sigma^2 . \tag{118}$$

The lower limit of integration in (118) is again $4m_{\pi}^2$ since the least massive

physical state is again the 2π system as in Fig. 23. The subtraction constant in (118) is normalized to the pion charge as in (117).

We attempt to evaluate $\operatorname{Im} F_\pi(\sigma^2)$ in (118) in the same approximation being used for the nucleon form factors—*i.e.*, we keep only the least massive intermediate state. This is just the two-pion state as in Fig. 23 and we have the very attractive feature that the $\pi\pi$ scattering in the p-state as occurring at the right-hand vertex of the figure need not be analytically continued since we are never below the 2π threshold of $4m_\pi^2$. Moreover at the left-hand vertex we have again the pion electromagnetic vertex so that eq. (118) becomes an integral equation determining $F_\pi(q'^2)$ in terms of the p-wave $\pi\pi$ scattering phase shift. We have then in (118),

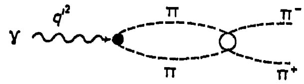


Fig. 23.

$$\operatorname{Im} F_\pi(\sigma^2) = F_\pi^*(\sigma^2)\sin\delta_1(\sigma^2)\exp[i\delta_1(\sigma^2)]\,, \tag{119}$$

where δ_1 is the p-wave $\pi\pi$ scattering phase shift (*).

The integral eq. (118), with (119), can be solved by a method of OMNÈS [26] to give

$$F_\pi(q'^2) = \exp\left[\frac{q'^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_1(\sigma^2)\,\mathrm{d}\sigma^2}{\sigma^2(\sigma^2 - q'^2 - i\varepsilon)}\right]. \tag{120}$$

Rather than repeat the derivation here let us try to understand the properties of the solution. Splitting off the phase in (120) we have

$$F_\pi(q'^2) = \exp[i\delta_1(q'^2)]\exp\left[\frac{q'^2}{\pi}\,P\!\!\int_{4m_\pi^2}^{\infty}\frac{\delta_1(\sigma^2)\,\mathrm{d}\sigma^2}{\sigma^2(\sigma^2 - q'^2)}\right]. \tag{121}$$

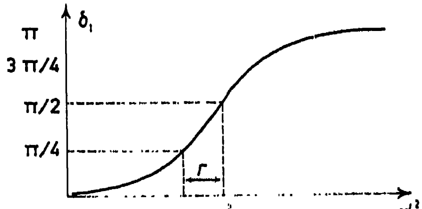


Fig. 24.

If the strong interaction between pions in the p-state is attractive and leads to a narrow resonance of width Γ at mass q_r^2, δ_1 has the behavior illustrated in Fig. 24. At the resonance $\delta_1(q_r^2) = \pi/2$ and the form factor becomes imaginary. The exponential in (121), and hence $|F_\pi|$, reaches a maximum near q_r^2. This can be seen easily

(*) We can check that $F_\pi^*(\sigma^2)$ and not $F_\pi(\sigma^2)$ should appear in (119) by recalling the final state interaction theorem. $F_\pi(\sigma^2)$ is complex for $\sigma^2 > 4m_\pi^2$ and has the phase of the wave function of the emerging $\pi^+\pi^-$ pair — *i.e.* their scattering phase shift. However $\operatorname{Im} F^\pi(\sigma^2)$ is real and, by (119), $F_\pi(\sigma^2) \propto \exp[i\delta_1]$ as desired.

from the figure for a zero-width resonance. For $\sigma^2 < q'^2 = q_r^2$ the integrand has a negative denominator and vanishes. For $\sigma^2 > q_r^2$, the denominator is positive, $\delta_1 = \pi/2$ and one has the maximum positive contribution with no cancellation from the $\sigma^2 < q_r^2$ region.

Just as we have found $F_\pi(q^2)$ to be enhanced at the $\pi\pi$ resonance, if it exists, so the amplitude for $n\bar{n}$ annihilation into the $\pi\pi$ pair at the right-hand vertex of Fig. 23 is enhanced by the same strong attraction [25]. The desired bump in the absorptive amplitude at q_r^2 indicated by the dotted line in Fig. 22 can thus be achieved within the framework of our approximation of retaining the lightest state only if such a $\pi\pi$ resonance occurs. Association of this bump with the ρ resonance, with the subtraction constant in (114) reflecting the contributions of neglected higher mass states is the most attractive interpretation of the form factor problem at present. In effect what we have achieved so far is a trade in of the nucleon form factor problem for the pion one. The peaking of the absorptive amplitude at $\sigma^2 = q_r^2 = m_\rho^2$ in (118) tells us that, if the resonance is narrow, the pion form factor, for small negative q'^2, can be written according to (120) and Fig. 24 as

$$F_\pi(q'^2) = 1 + \frac{q'^2}{\pi}\int_{q_r^2}^{\infty}\frac{\pi\, d\sigma^2}{(\sigma^2)^2} + \ldots \simeq 1 + \frac{q'^2}{q_r^2} = 1 + \frac{q'^2}{m_\rho^2} \qquad \text{as} \quad q'^2 \to 0\,.$$

Thus the pion's electromagnetic radius is

$$\langle r^2\rangle_\pi = \frac{6}{m_\rho^2}\,, \tag{122}$$

and is comparable to the observed nucleon radius if the $\pi^+\pi^-$ state dominates in $\operatorname{Im} F_\pi$ as in $\operatorname{Im} F^V$ in (105).

What have we gained then by all this work?

First of all we have associated the observed nucleon structure with an observed p-wave resonance of the $\pi\pi$ system. Secondly we have predicted in (122) a definite large pion charge radius *which can be checked in the near future* by experiment. Direct measurement of $F_\pi(q^2)$ in an electron-positron colliding beam experiment, $e^- + e^+ \to \pi^+ + \pi^-$ will provide a crucial test of this line of theoretical reasoning [27].

So far we have addressed our remarks to the isovector form factor problem. For the isoscalar form factor the theoretical problems are considerably more formidable because one is faced with the challenge of constructing the ω resonance from a three-pion state. No quantitative progress on this problem can be reported [28]. In fact the « puzzle » of the very small neutron charge radius makes it difficult to look forward to any substantial progress with presently considered ideas and calculational limitations.

This « puzzle » may be stated as follows: Whereas the proton's charge

radius is large in eq. (90), the neutron's is very small, eq. (91). At the same time both the proton and neutron have anomalous magnetic moments of comparable magnitudes, $\varkappa_n = -1.91$ and $\varkappa_p = +1.79$, and structure.

The ρ and ω provide a natural explanation of the first of these observations in terms of the close coincidence of their masses if the couplings to the ρ and ω channels are almost equal. By (92), $F^n = F^S - F^V$; using the form (114) for the low q^2 behaviour of F^n we find

$$F_1^n(q^2) = q^2 \left\{\frac{a_{1S}}{m_\omega^2} - \frac{a_{1V}}{m_\rho^2}\right\} + 0(q^4)\,,$$

where $m_\omega = 785$ MeV is the $I=0$ resonance mass and $m_\rho = 750$ MeV is the $I=1$ resonance mass. Therefore

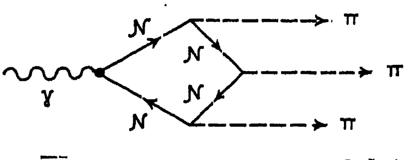
$$\langle r_1^2\rangle_n = \frac{6a_{1S}}{m_\omega^2}\left[1 - \frac{m_\omega^2}{m_\rho^2}\frac{a_{1V}}{a_{1S}}\right] \approx \frac{6a_{1S}}{m_\omega^2}\left[1 - 1.093\,\frac{a_{1V}}{a_{1S}}\right],$$

and is within the experimental limit (91), for $a_1 \sim 1$, provided the coupling parameters a_{1V} and a_{1S} are related by

$$a_{1S} = 1.093 a_{1V}(1 \pm 0.02)\,.$$

This coincidence of coupling strengths must apply for the amplitudes contributing to F_1 and not to F_2, however, since both the magnitude and radius of the neutron's anomalous magnetic moment are large. Stating this difference from another point of view, the ω channel gives a large contribution to F_1^S in order to cancel F_1^V for small q^2. On the other hand it contributes very weakly to the anomalous moment calculation since the isoscalar part is very small; by (92) $F_2^S(0) = \frac{1}{2}[\varkappa_p + \varkappa_n] = -0.06$. This difference between $F_1^{S,V}$ and $F_2^{S,V}$ remains a serious obstacle to any simple intuitive picture (*).

(*) Prior to the advent of dispersion relations and the discovery of the vector ω and ρ resonances one's intuition was even in relatively worse shape. This is because in conventional meson theory one can couple the γ to a 3π state for the isoscalar channel only via a $\mathcal{N}\bar{\mathcal{N}}$ loop as in the diagram



Because of the massive $\mathcal{N}\bar{\mathcal{N}}$ state involved one was led to expect a much smaller radius from this intermediate state. However we now see from the viewpoint of eq. (111) for the absorptive amplitude that this amplitude contributes with a low threshold of $9m_\pi^2$ and can be enhanced at such low mass values by a strong resonance attraction among the pions as discussed in ref. [28].

10. – Electromagnetic structure of neutral particles.

Other neutral particles in addition to the neutron may also have an observable electromagnetic structure. Among the spin-0 particles the π^0 and η^0 mesons have no electromagnetic structure in the sense of Fig. 23. This is because they are self charge-conjugate particles and therefore a γ with $C=-1$ cannot produce a pair of π^0's or η^0's with $C=+1$. This does not preclude all electromagnetic interactions, of course, and indeed decays $\pi^0\to 2\gamma$ and $\eta^0\to 2\gamma$ are observed as allowed by charge conjugation invariance.

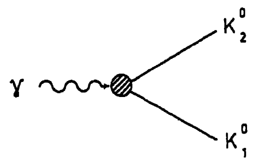


Fig. 25.

In the case of the K^0 mesons there are two neutral mesons, the K_1^0 with $C=+1$ and the K_2^0 with $C=-1$. Therefore C conservation does not rule out a transition vertex as in Fig. 25 which converts a K_1^0 into a K_2^0 or vice versa. Alternatively we may view it as the pair production of a $K^0\overline{K}^0$ pair in a $C=-1$ state. The vertex has the form (117) but with

$$F_K(0)=0\,,$$

here, since the K^0 is neutral. For small q^2 we expand F_K in terms of a mean square radius

$$F_K(q^2)=\tfrac{1}{6}q^2\langle r_K^2\rangle+0(q^4)\,, \tag{123}$$

and compute for the Coulomb scattering in a field of strength Ze,

$$\left(\frac{d\sigma}{d\Omega}\right)_{q\to 0}=\frac{2}{9}Z^2\alpha^2\langle r_K^2\rangle^2 p_K E_K\,, \tag{124}$$

where $E_K=\sqrt{p_K^2+m_K^2}$ is the incident K-meson energy. The dependence of the Rutherford scattering at small momentum transfers is completely cancelled in (124) by the form factor dependence (123). For a neutral particle with spin and a magnetic moment a $1/q^2$ behaviour still survives for small q^2 as we see in (73) with $F_1(0)$ and $F_2(0)=1$. It is of interest to speculate on the size of $\langle r_K^2\rangle$ for the K^0. However the smallness of (124) relative to nuclear cross-sections means that $K_1^0-K_2^0$ conversion in matter, even of high Z as in Xenon bubble chambers, is controlled by the nuclear interactions and not by the Coulomb one. Therefore measurements of $\langle r_K^2\rangle$ must await success with electron-positron colliding beam experiments.

Neutrinos are expected to have a finite electromagnetic structure because

of their coupling to electrons and the possible heavy vector bosons which may mediate the weak interactions. The Feynman diagram with the least massive intermediate state contributing to the absorptive amplitude is shown in Fig. 26. For a massless neutrino there will be only one form factor corresponding to the F_1 in our discussion of the nucleon structure. Evidently there can be no anomalous moment term in (79) for a particle with $M_\nu \to 0$. Moreover $F_1(0) = 0$ since the neutrino is neutral. An approximate perturbation calculation of the neutrinos' charge-radius based on Fig. 26 gives (*)

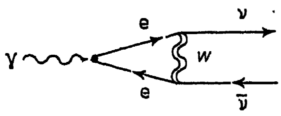


Fig. 26.

$$\langle r_1^2 \rangle_\nu \sim G \ln \frac{M_W^2}{m_e^2} \sim \left(10^{-5} \ln \frac{M_W^2}{m_e^2}\right) \frac{1}{M_p^2} \sim (10^{-16}\ \text{cm})^2, \tag{125}$$

and is hardly observable at this time. The point of the above discussion is only to put you on guard with the warning that form factors show up in strange places.

11. – Validity of electrodynamics.

We have so far not questioned the validity of quantum electrodynamics in our form factor discussions. At this time there is no reason to, and indeed there is impressive experimental confirmation in support of our use of these laws (**). However the precision measurements of the Lamb shift and of the electron's magnetic moment probe quantum electrodynamics at distances measured in units of the electron's Compton wavelength, 3.8×10^{-11} cm and do not exclude the possibility of deviations in large momentum transfer col-

(*) We obtain eq. (125) by repeating the discussion in [29] with appropriate changes of mass from 0 to M_W^2 in the photon propagator and of coupling constants from e^2 to $G\, M_W^2$ where $G \simeq 10^{-5}/M_p^2$ is the weak interaction coupling parameter. For a related discussion see S. BERMAN in «CERN Conference on Very-High-Energy Processes», June 1961.

(**) For the most recent review of this situation see the report of FEYNMAN [30]. Since the Varenna School, D. T. WILKINSON and H. R. CRANE have reported a value of the electron's magnetic moment

$$g_e - 2 = 2\left\{\frac{\alpha}{2\pi} - \frac{\alpha^2}{\pi^2}\,[0.327 \pm 0.005]\right\},$$

in close agreement with the theoretical value of

$$2\left\{\frac{\alpha}{2\pi} - 0.328\, \alpha^2/\pi^2\right\}.$$

See [31].

lisions (*) probing distances down to fractions of 10^{-13} cm. It is not out of the question for example that the factor $C_{\rho,\omega} \sim 1.4$ in (94) reflects not higher mass contributions to the dispersion integrals, but the size of the electron's structure in the electron-proton scattering. The colliding electron beams experiments will settle this point unambigously. Deviations in the photon's propagator will also contribute to any apparent structure but these are quite severely limited by the present measurements on the μ-meson magnetic moment which confirm that $g_\mu - 2 = \alpha/\pi$ to an accuracy (*) of 0.4%. To see how the measurement of the anomalous magnetic moment is related to the photon propagator we adopt the dispersion approach (**). The contribution of order α to the muon's (or electron's) anomalous moment arises from the absorptive amplitude shown in Fig. 27.

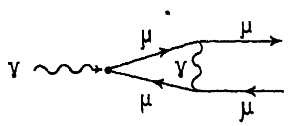


Fig. 27.

The virtual photon produces a $\mu\bar{\mu}$ pair which scatters with the exchange of a photon. The absorptive amplitude is given, as in (111), to this order by $e\bar{u}\gamma^\mu v$, the amplitude to produce the pair, multiplied by $\langle\mu\bar{\mu}|\mu\bar{\mu}\rangle_{^3S_1, ^3D_1}$, the scattering amplitude in the $^3S_1$ and $^3D_1$ states. A correction to this scattering amplitude will, by (105), alter the calculated moment from the Schwinger value of $\alpha/2\pi$.

In view of present interest in Regge trajectories we may cast the proposed modification of the photon propagator in the form suggested for the Reggeized analyses [10] (*,*)

$$(126) \qquad \frac{P_1(\cos\theta_t)}{t} \to \left\{\frac{1-\exp[-i\pi\bar{\alpha}(t)]}{2\sin\pi\bar{\alpha}(t)}\right\}\left[\pi\,\frac{\mathrm{d}\bar{\alpha}(t)}{\mathrm{d}t}\right]_{t=0} \times \\ \times P_{\bar{\alpha}(t)}(-\cos\theta_t) \times \left[\frac{t-4m_\mu^2}{2s_0}\right]^{\bar{\alpha}(t)-1},$$

where t is the square of the invariant momentum transfer in the $\bar{\mu}\mu$ scattering, $P_{\bar{\alpha}}$ is the Legendre polynomial of order $\bar{\alpha}$,

$$\cos\theta_t = 1 - \frac{2q'^2}{t-4m_\mu^2},$$

(*) Recent experiments on the μ meson provide further support of electron-dynamics and as yet fail to uncover differences between the μ-meson and the electron. See reports of A. CITRON on μ-scattering, A. ODIAN on electromagnetic μ-pair production, and A. ZICHICHI on the $g-2$ of the μ-meson tò the *1961 International High Energy Physics Conference at CERN*; also [32]. Further discussion and bibliography are in Chapter 4 of ref. [3]. (Recall the correction in the footnote to page 205).

(**) Alternatively one can regulate the photon propagator as in [33]. Electrodynamic corrections to the photon propagator due to vacuum polarization are included in the usual radiative correction calculations. We follow here ref. [29].

(*,*) Seminars at the Varenna School discussed these points in greater detail. See [34].

is the scattering angle in the t channel center-of-mass, differing from the analogous variable on p. 204 only by the masses, and q'^2 is the squared mass of the photon. The last factor in eq. (126) takes care of the threshold behaviour (*), with the unit s_0 undetermined. For the large energy, $s \equiv q'^2 > 4m_\mu^2$, region where this Regge modification is conjectured to apply, (126) approximates to

$$\frac{1}{t} \to \frac{1}{t} \exp\left[at \ln \frac{s}{s_0}\right] \left[1 + \frac{2m_\mu^2 - t/2}{s}\right]^{at}, \tag{127}$$

with $a \equiv \mathrm{d}\bar{\alpha}/\mathrm{d}t|_{t=0}$ defining the slope of the Regge trajectory. For a choice of $s_0 \sim m_\mu^2$, a straightforward calculation shows that the slope, a, must not exceed

$$a \leqslant 0.08 \text{ GeV}^{-2},$$

in order to keep (105) within the 0.4% experimental error. This is an order of magnitude smaller than the suggested slope for the vacuum trajectory in pp scattering at CERN [34]. Moreover (127) leads to an energy-dependent contribution to the mean square radii of

$$\langle r^2 \rangle_\gamma \simeq 6a \ln \frac{s}{s_0} < \left(0.3 \ln \frac{s}{s_0}\right) \frac{1}{m_\rho^2}. \tag{128}$$

This is an order of magnitude smaller than the correction needed to reduce $C_{\rho,\omega}$ to unity in (94) and shows that a «photon size» of this form is ruled out as a possible explanation of that factor. It remains to be established whether the predicted deviation from the Rosenbluth form (75) is compatible with observations (*).

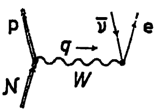
Although the electron's magnetic moment is more accurately measured [31] than the muon's, we have used the latter because the larger mass of the muon means proportionally larger momentum transfers to appear in (127). In other words the energies and momentum transfers in the $\mu\bar{\mu}$ or $e\bar{e}$ scattering in Fig. 27 are measured on a scale determined by the only length appearing in the calculation, $4m_\mu^2$ or $4m_e^2$, which is the lower limit of the dispersion integral (105) for these calculations. Therefore according to (127), the limit on the slope, a, is proportional to the experimental limits of error and inversely proportional to the squares of the masses.

(*) What we have done here is in no way an adequate discussion of the implications of Reggeism for electrodynamics since we have modified the photon propagator only. Additional vertex forms appear as discussed by [35]. Furthermore questions of current conservation are unresolved.

12. – Form factors in the weak interactions and conserved currents (*).

Although it appears to be very different, the β-decay interaction $n \to p + e + \bar{\nu}$ can be related to the isovector nucleon form factors as we now show.

One can *picture* the neutron β-decay as mediated by a massive vector meson, the W introduced in Fig. 26 for the neutrino structure discussion, and as shown in Fig. 28.



The invariant β-decay matrix element established by experiment [36] has the form

Fig. 28.

$$M_{(\beta)} = \frac{1}{\sqrt{2}}\, G\bar{u}_p\gamma^\mu(1-\alpha\gamma_5)u_n\cdot\bar{u}_e\gamma_\mu(1-\gamma_5)v_{\bar{\nu}}\,, \tag{129}$$

which is valid throughout the low-momentum transfer region $q^2/M_p^2 \ll 1$ studied. The β-decay constant is $G = 10^{-5}/M_p^2$ and $\alpha = 1.21$ gives the ratio of Gamow-Teller to Fermi transition amplitudes. The matrix γ^5 is defined by $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\gamma^\mu(1-\gamma_5)$ is the $V-A$ interaction.

For purposes of relating (129) with other weak decays as well as with electromagnetic interactions we have introduced the W line in Fig. 28 and we may think of (129) as

$$M_{(\beta)} = \frac{1}{\sqrt{2}}\,(GM_W^2)\bar{u}_p\gamma^\mu(1-\alpha\gamma_5)u_n\cdot\frac{1}{M_W^2}\cdot\bar{u}_e\gamma_\mu(1-\gamma_5)v_{\bar{\nu}}\,, \tag{130}$$

where (GM_W^2) is the dimensionless coupling constant analogous to charge e^2 in the electron-proton scattering amplitude and $1/M_W^2$ represents the propagator of the massive W boson; its q^2 dependence is neglected in the narrow region $q^2/M_W^2 \ll 1$ experimentally studied. Whether or not an actual W quantum is observed to be produced in reactions is a separate question which does not affect the considerations here. The W line in Fig. 28 is introduced solely for pictorial purposes and to show how the spinor indices are tied together at the vertices in Fig. 28.

Turning next to μ-decay, we expect a similar form of matrix element from the Feynman diagram Fig. 29. The form established by extensive studies of the spectrum and polarizations [33] is

$$M_{(\mu)} = \frac{1}{\sqrt{2}}\,\tilde{G}\big(\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_\mu\big)\big(\bar{u}_e\gamma_\mu(1-\gamma_5)v_{\bar{\nu}}\big)\,. \tag{131}$$

(*) This discussion is adapted from a more complete one in J. D. Bjorken and S. D. Drell «Relativistic Quantum Mechanics» (New York, to appear).

The lifetime of the μ computed by the procedure in Section 4 (see eq. (59)) is

$$\frac{1}{\tau_\mu} = \omega_\mu = \tilde{G}^2 \frac{m_\mu^5}{192\pi^3}, \tag{132}$$

and from the observed $\tau_\mu = 2.2 \times 10^{-6}$ s we find the remarkable result that $\tilde{G} = G$ within 4%.

This close equality of G and $\tilde{G}$ is very surprising. The puzzle is this: In contrast to the proton and neutron which participate in strong interactions the μ-meson has only electromagnetic and weak ones. In the face of this, why does $G \approx \tilde{G}$, indicating equality of the Fermi part of the interactions? In fact the closeness of $\alpha = 1.21$ to unity and, therefore, the approximate equality of the axial parts of the interactions is also unexpected. Viewed in terms of Figs. 28 and 29, why are there no pronounced renormalization effects altering the strength of the interaction current at the nucleon line due to strong interactions?

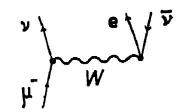


Fig. 29.

It is here that the weak interactions make contact with the electromagnetic form factors. It was pointed out by Gershtein and Zel'dovitch [37] and by Feynman and Gell-Mann [38] that we are familiar with another example of this strange coincidence of interaction strengths; *viz.* the equality of the size of the electric charges of electron and proton. Should we not expect that the physical electron and proton charges should differ due to the strong couplings of the nucleons? Where in the theory of quantum electrodynamics is the clue guaranteeing their equality? It is found in the condition of current conservation, eq. (80). Conservation of current leads to the Ward identity which insures [4] that the strong interaction contributions to the vertex renormalization constant, $1/Z_1$, cancel the wave function renormalization contributions, Z_2, since $Z_1 = Z_2$. Thus the observed electron and proton charges are of equal magnitude, if their bare charges are, since vacuum polarization corrections to the photon propagator affect them in the same way.

With this observation we are motivated to postulate [37, 38] that the vector current part of the weak interactions is also conserved. There will then be no charge renormalization, so that $G = \tilde{G}$ as observed. From this postulate there follow several important predictions which can be tested experimentally.

The first of these is that the effect of the strong interactions at the W-nucleon vertex in Fig. 28 is precisely the same as their effect on the isovector part of the electromagnetic vertex of the nucleon which, by (79) and (92), took

$$e_p \bar{u} \gamma^\mu u,$$

to

$$e_p \bar{u} \left[2F_1^V(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2M_p} q_\nu \cdot 2F_2^V(q^2) \right] u, \tag{133}$$

with $2F_2^V(0) = (1.79 + 1.91) = 3.7$ and $2F_1^V(0) = 1$. Therefore to take account of the strong interaction effects in the conserved vector current theory we replace

$$\frac{1}{\sqrt{2}}\, G\bar{u}_p \gamma^\mu u_n\,,$$

by

$$\frac{1}{\sqrt{2}}\, G\bar{u}_p \left[2F_1^V(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2M_p}\, q_\nu \cdot 2F_2^V(q^2)\right] u_n\,. \tag{134}$$

Pictures help make clear the connection between (133) and (134). We found the form factors from (111), in principle, by evaluating the amplitude to produce the $p\bar{p}$ state starting from a virtual photon which interacted with the conserved current of the proton with a coupling constant e_p. This coupling was then divided into its isovector and isoscalar parts according as the $p\bar{p}$ state was formed with $I=1$ and 0, respectively. For the β-decay interaction we have a similar interaction in the conserved vector coupling theory. The virtual W couples to the same conserved current of the proton with coupling strength $1/\sqrt{2}\; G$ instead of e_p, and since it has one unit of charge and forms a proton-antineutron state with $I=1$, only the isovector channel contributes. However, due to the charge independence of the strong interactions responsible for the nucleon structure there is no change in going from a $p\bar{p}$ system with $I=1$ to a $p\bar{n}$ system. Therefore we adopt (134) which reduces in the low $q^2/M_p^2 \ll 1$ region to

$$\frac{G}{\sqrt{2}}\, \bar{u}_p \left[\gamma^\mu + 3.7\, \frac{i\sigma^{\mu\nu} q_\nu}{2M_p}\right] u_n; \qquad q^\mu \equiv (\mathfrak{P}_p - \mathfrak{P}_n)\,. \tag{135}$$

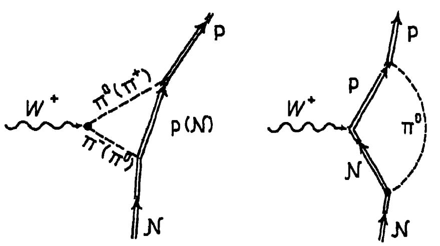


Fig. 30.

In terms of a perturbation expansion in Feynman graphs what we have said can be illustrated by Fig. 18 with appropriate isotopic changes shown below in Fig. 30. The W^+ and γ both interact with the pions and nucleons with a vector current, the only difference being the replacement of charge e by $G/\sqrt{2}$ and the instruction to change the charge by one unit. The remaining interaction vertices in these and in higher-order graphs are all of the charge-independent strong coupling class which do not differ among charged or neutral pions or nucleons and are therefore indifferent to the change of charge at the vector vertex. They therefore lead back to the same form factors for the W as for the γ coupling in the same $I=1$ channel.

The second or «weak magnetism» term in (135) is a unique prediction of this theory. It was first discussed by GELL-MANN [39] and has now been experimentally confirmed [40] in a major success for the conserved-current idea.

A second test of this hypothesis of a conserved vector current is the decay rate for pion β-decay:

$$\begin{cases} \pi^+ \to \pi^0 + e^+ + \nu\,, \\ \pi^- \to \pi^0 + e^- + \bar{\nu}\,. \end{cases} \tag{136}$$

The rates for (136) are related to the strength of the Coulomb scattering of a charged pion

$$\pi^+ + e \to \pi^+ + e\,,$$

which is given in (117) by $eF_\pi(q^2)$. This coupling is entirely in the isovector channel, according to (112) and thus we replace

$$eF_\pi(q^2) \qquad \text{by} \qquad \frac{2G}{\sqrt{2}}\,F_\pi(q^2)\,,$$

in analogy with (136). There is no renormalization of the coupling strength, and approximating $F_\pi(q^2) \approx 1$ for the low q^2 involved in the πβ-decay (136) we have a uniquely determined decay rate in terms of the measured G value for neutron and muon β-decay. The prediction is for the branching ratio

$$\frac{R(\pi^- \to \pi^0 + e^- + \bar{\nu})}{R(\pi^- \to \mu^- + \bar{\nu})} = 1.0 \cdot 10^{-8}\,, \tag{137}$$

which is very small but detectable and confirmed [41].

The constant $\alpha = 1.21$ in (129) appears in the nucleon's weak axial current interaction but is not present for leptons in (129) and (131). α is close enough to unity to make us question whether the axial current may not «almost» satisfy a Ward identity so that the axial vertex renormalization «almost» cancels the wave function renormalization, as on page 233 for the vector vertex. A possible way to achieve this cancellation is to require that the axial current be «almost» or «partially» conserved [42]. The approximate nature of the axial current conservation is reflected in the fact that α is «approximately» unity.

Although the numerical value $\alpha = 1.21$ remains a complete mystery, the idea of approximate conservation at the axial vertex leads to an interesting correlation of the decay rate of the charged pion with the strong coupling pion-nucleon $g^2_{\pi n}/4\pi \sim 14$ which we now review.

The axial current contribution in (129) is

$$-\frac{1}{\sqrt{2}}\, G\alpha \bar{u}_p \gamma^\mu \gamma^5 u_n . \tag{138}$$

The most general form for the axial current including all strong interaction contributions at the W-nucleon vertex is (*)

$$J^{\mu(A)} = -\frac{1}{\sqrt{2}}\, G\bar{u}_p\{\gamma^\mu\gamma^5 \mathscr{F}_1(q^2) + q^\mu \gamma^5 \mathscr{F}_2(q^2)\}u_n; \qquad q^\mu \equiv (\mathfrak{P}_p - \mathfrak{P}_n)^\mu . \tag{139}$$

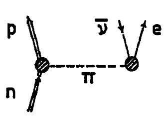


Fig. 31.

Among the many strong coupling terms contributiong to $\mathscr{F}_2(q^2)$ in (139) are those which arise from the graph in Fig. 31. These are denoted the pion pole contribution and include all terms for which one pion couples to the lepton. The easiest way to see that the pion pole term contributes only to $\mathscr{F}_2$ is by direct computation of actual graphs. The general structure of the pion nucleon coupling is

$$g_{\pi n}\sqrt{2}\,\bar{u}_p i\gamma_5 u_n \mathscr{F}(q^2) , \tag{140}$$

which represents the only pseudoscalar which can be formed. $\mathscr{F}(q^2)$ is a scalar function of the invariant momentum transfer and is normalized to

$$\mathscr{F}(m_\pi^2) = 1 , \tag{141}$$

for a physical pion. Assuming a slow variation of $\mathscr{F}(q^2)$ for low q^2 as consistent with the sucess of the effective range plots in low-energy pion nucleon scattering [43] we approximate

$$\mathscr{F}(q^2) \approx \mathscr{F}(0) \approx \mathscr{F}(m_\pi^2) = 1 ,$$

for $q^2 \to 0$. With the normalization (141), $g_{\pi n}^2/4\pi = 14$ is the measured pion-nucleon coupling constant.

(*) The same arguments leading to (79) may be repeated here except that we have not invoked current conservation in writing (139). A third term in (139) of the form $(\mathfrak{P}_p + \mathfrak{P}_n)^\mu \gamma_5 \mathscr{F}_3$ is ruled out on grounds of charge conjugation invariance and isotopic ipin conservation of the strong interactions. This is most easily seen by using charge sndependence according to which (139), like (138), is unchanged if we consider a transition of proton to proton with no exchange of electric charge. Then we appeal to C invariance to require that (139) contains only terms transforming in the same way as (138) when we replace the proton by an anti-proton.

For the $\pi e \nu$ vertex in Fig. 31 we write

$$(141) \qquad \frac{1}{\sqrt{2}} G a(q^2) \bar{u}_e \gamma_\mu (1-\gamma_5) v_{\bar{\nu}} q^\mu ,$$

which is a (pseudo-) scalar of the observed V—A form for the lepton coupling. The form factor $a(q^2)$ is normalized at $q^2 = m_\pi^2$ to the observed $\pi^- \to e^- + \bar{\nu}$ decay rate. From the decay rate, calculated from (59) to be

$$\Gamma_{\pi\to e\nu} = \frac{1}{\tau_{\pi\to e+\nu}} = \frac{G^2 a^2}{8\pi} m_\pi^3 \left(\frac{m_e}{m_\pi}\right)^2 \left[1 - \frac{m_e^2}{m_\pi^2}\right]^2 ,$$

and from the measured lifetime

$$\tau_\pi \sim 2.5 \cdot 10^{-8}\ \text{s} .$$

and branching ratio

$$(142) \qquad \frac{\Gamma_{\pi\to e\nu}}{\Gamma_{\pi\to\mu\nu}} \approx \left(\frac{m_e}{m_\mu}\right)^2 \left\{\frac{1 - m_e^2/m_\pi^2}{1 - m_\mu^2/m_\pi^2}\right\}^2 ,$$

we conclude

$$(143) \qquad a \simeq 0.9\, m_\pi .$$

Using (140) and (141) we separate out the pion pole contribution to $\mathscr{F}_2$ in (139) and write the axial current in the form

$$(144) \qquad J^{\mu(A)} = -\frac{1}{\sqrt{2}} G \bar{u}_p \left\{\gamma^\mu \gamma^5 \mathscr{F}_1(q^2) + q^\mu \gamma^5 \left[\mathscr{F}_2^a(q^2) + \frac{a(q^2) g_{\pi n} \sqrt{2}\, \mathscr{F}(q^2)}{q^2 - m_\pi^2}\right]\right\} u_n ,$$

where $\mathscr{F}_2^a(q^2)$ includes all but the pion pole contribution to $\mathscr{F}_2$ and $1/(q^2 - m_\pi^2)$ is the pion propagator for the exchanged pion (*); by definition $\mathscr{F}_1(0) = \alpha = 1.21$.

Let us now impose the hypothesis of current conservation on (144)—*i.e.*

$$(145) \qquad q_\mu J^{\mu(A)} = 0 .$$

Since

$$\bar{u}_p q_\mu \gamma^\mu \gamma^5 u_n = \bar{u}_p [\mathfrak{P}_p^\mu \gamma_\mu - \mathfrak{P}_n^\mu \gamma_\mu] \gamma^5 u_n = \bar{u}_p \{\mathfrak{P}_p^\mu \gamma_\mu \gamma_5 + \gamma_5 \mathfrak{P}_n^\mu \gamma_\mu\} u_n = 2 M_p \bar{u}_p \gamma_5 u_n ,$$

we find

$$2 M_p \mathscr{F}_1(q^2) + q^2 \left\{\mathscr{F}_2^a(q^2) + \frac{\sqrt{2}\, g_{\pi n} a(q^2) \mathscr{F}(q^2)}{q^2 - m_\pi^2}\right\} = 0 .$$

(*) Vacuum polarization corrections to the pion propagator may be lumped in $\mathscr{F}(q^2)$ but since we are interested in very low q^2 only, $\mathscr{F}(q^2) \approx 1$ and $a(q^2) \approx a$.

An immediate conclusion from (145) for $q^2 \to 0$ is

$$\lim_{q^2\to 0}\left[q^2\left\{\mathscr{F}_2^{(a)}(0)+\frac{\sqrt{2}\,g_{\pi n}\,a}{q^2-m_\pi^2}\right\}\right]=-2M_p\alpha\neq 0\,. \tag{146}$$

The expression between the brackets $\{\}$ must therefore have a pole at $q^2=0$ according to the conserved current assumption. Since all intermediate states connecting the nucleon to the lepton line are massive we have no candidate for explaining this singularity. The lightest state is the one-pion pole term of Fig. 31 which has been isolated into the second term within the brackets $\{\}$. It is tempting to suggest [42], on the basis of (146) that axial current conservation (145) is exact only in the limit of vanishing pion mass, $m_\pi \to 0$ in which limit

$$a\sqrt{2}\,g_{\pi n}=-2M_p\alpha\,. \tag{147}$$

Equation (147) was first derived by GOLDBERGER and TREIMAN [44] on more restrictive assumptions.

Inserting $g_{\pi n}^2/4\pi=14$ and $\alpha=1.21$ gives

$$|a|\sim 0.8\,m_\pi\,,$$

which is close to the experimental value in (143). We end with this remarkable correlation between strong and weak interaction constants!

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Prediction of Pion Phases by Dispersion Relations (*).

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A theoretical method of deriving information about the scattering process $\pi+\pi\to\pi+\pi$ from the experimental knowledge of the process $\pi+n\to\pi+n$ will be developed. The technique depends upon exploiting the analyticity, crossing, and unitarity of the scattering matrix; and these properties will be explained in a heuristic way, in order to facilitate an intuitive grasp of the method.

The scattering processes to be considered are depicted in Fig. 1, where the momenta of the four particles are shown formally ingoing, p_1 is the four-momentum of a nucleon, p_2 that of an antinucleon, whilst p_3, p_4 are those of pions. Six possible reactions are obtained by choosing any two particles to be ingoing, and the other two to be outgoing. If $|\psi_i(p)\rangle$ represents an initial state of the system characterized by momenta p, and $|\psi_f(p)\rangle$ the state formed after scattering, then the scattering matrix is defined in momentum space by

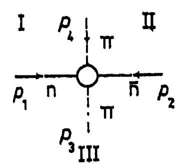

Fig. 1.

$$|\psi_f(p)\rangle = S(p)\,|\psi_i(p)\rangle\,. \tag{1}$$

The matrix element between initial and final states is

$$S_{fi}(p) = \langle\psi_i|\psi_f\rangle = \langle\psi_i|S|\psi_i\rangle\,, \tag{2}$$

and it is this quantity that characterizes the scattering. It is a scalar, and so must be formed out of scalar combinations of the p_n, $n=1,\ldots,4$. It will

(*) A more detailed presentation of the work is to appear in *Physical Review*.

also depend on the spin of the nucleons and on the isospin of the pions and the nucleons. This means that there are four scalar functions instead of one, corresponding to the two spin states of the nucleon, and the two isospin values $\frac{1}{2}$ and $\frac{3}{2}$ of the πn system. However, this spin and isospin-dependence will be neglected in the presentation, although its consideration is necessary in the actual calculation.

Because of momentum conservation

$$\sum_{n=1}^{4} p_n = 0\,, \tag{3}$$

and the mass-shell restriction:

$$p_{1,2}^2 = M^2\,, \qquad p_{3,4}^2 = 1\,, \tag{4}$$

where M is the nucleon mass, and the pion mass is unity, it follows that only two independent scalars can be formed from the p_n, $n = 1, \ldots, 4$. It is convenient to work with three scalars:

$$s_i = (p_i + p_4)^2\,, \qquad i = 1, 2, 3\,, \tag{5}$$

which satisfy the linear relation:

$$\sum_{i=1}^{3} s_i = 2(M^2 + 1)\,. \tag{6}$$

Then S_{fi} can be written as a function of any two of the s_i.

The fundamental properties of analyticity, crossing, and unitarity of S_{fi} will be developed in turn. Three channels of the scattering of Fig. 1 can be considered:

I) p_1 and p_4 incoming, $\pi + n \to \pi + n$;

II) p_1 and p_3 incoming, $\pi + n \to \pi + n$;

III) p_3 and p_4 incoming, $\pi + \pi \to n + \bar{n}$.

The processes obtained by time reversal are also possible. In the centre of-momentum system of channel I,

$$\boldsymbol{p}_1 = -\boldsymbol{p}_4\,, \qquad \text{so} \qquad s_1 = (p_1^0 + p_4^0)^2\,, \tag{7}$$

which is the square of the total energy. In channel I, a possible real intermediate state is the nucleon (Fig. 2). The scattering matrix for this is the propagator

(8) $$S = \frac{G^2}{p^2 - M^2},$$

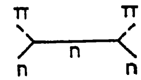

Fig. 2.

where G is a real number, the renormalized πn coupling constant, and p is the four-momentum of the nucleon. In the centre-of-momentum system, $p^2 = p_0^2 = s_1$, so

(9) $$S = \frac{G^2}{s_1 - M^2},$$

that is to say, there is a pole of S in the s_1 plane, at M^2, due to the real nucleon intermediate state. There is possible a continuum of intermediate states, depicted in Fig. 3, of which that of lowest mass is the πn state, which can have any energy above the rest energy, $M+1$.

These continuum states will give rise to a superposition of poles like that in (9), except that the position will vary from $(M+1)^2$ to ∞. This contribution can be written

(10) $$\int\limits_{(M+1)^2}^{\infty} \frac{W(s_1')}{s_1 - s_1'}\, ds_1',$$

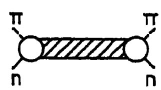

Fig. 3.

where $W(s_1')$ is some real weight function. This means that there is a cut, $(M+1)^2 < s_1 < \infty$, which is caused by the continuum. Similarly, in channel II, where s_2 is the energy squared, there is a pole $s_2 = M^2$ and a cut $(M+1)^2 < s_2 < \infty$. Channel III describes $\pi\pi \to n\bar{n}$, and there is no stable single particle which has the quantum numbers of the system, so there is no pole in s_3. The continuum starts at two-pion intermediate states, giving a cut $4 < s_3 < \infty$. The S-matrix is analytic, apart from these singularities, which are caused by real intermediate states in the three channels. The essential point about this analyticity is that it is the continuation of the same analytic function S that describes the channel I $\pi n \to \pi n$ reaction for $(M+1)^2 < s_1 < \infty$, the channel II for $(M+1)^2 < s_2 < \infty$, and the channel III $\pi\pi \to n\bar{n}$ for $4 < s_3 < \infty$.

The crossing relation states that the scattering matrix is unaffected by the interchange $s_1 \leftrightarrow s_2$. Physically, this means an interchange of channels I and II, and since these are the same reaction, $\pi n \to \pi n$, the invariance is intuitively plausible. At this stage it is useful to replace s_i, $i = 1, 2, 3$, by two independent variables, in order to throw this crossing relation into a very simple form. The first variable is z, the cosine of the scattering angle in the centre-of-momentum system of channel I. This will actually be

fixed at -1: that is, the backward scattering matrix will be considered. The reason for this is that the continuation into the physical regions of the other channels also corresponds to backward scattering in those channels, thus avoiding the continuation in angle which a general z would entail. Phase shifts will be extracted, allowing S to be written down for any angle. The other variable is ν, the square of the three-momentum of the pion in the centre-of-momentum system of channel I.

The relations between the s_i, $i=1, 2, 3$ and ν, with $z=-1$, are

$$\left\{\begin{aligned} s_1 &= [(\nu+M^2)^{\frac{1}{2}}+(\nu+1)^{\frac{1}{2}}]^2\,,\\ s_2 &= [(\nu+M^2)^{\frac{1}{2}}-(\nu+1)^{\frac{1}{2}}]^2\,,\\ s_3 &= -4\nu\,,\end{aligned}\right. \tag{11}$$

s_1 is one-valued on a two-sheeted ν plane cut $-M^2<\nu<-1$. For ν real, $\nu>-1$ on sheet I of ν, one defines

$$s_1 = [\,|(\nu+M^2)^{\frac{1}{2}}|+|(\nu+1)^{\frac{1}{2}}|\,]^2\,. \tag{12}$$

Then the channel I pole and cut map into a pole at $\nu=\nu_0=-1+1/4M^2$ and a cut $0<\nu<\infty$ on sheet I of ν; whilst the channel II singularities map into the same sites on sheet II. The channel III cut becomes the cut $-\infty<\nu<-1$, on both sheets of ν. From eq. (11) it is seen that the crossing interchange, $s_1\leftrightarrow s_2$, corresponds to the interchange of the two sheets of ν. That is, S is identical on the two sheets, so that only one need be considered. This is the reason for the choice of the variable.

A dispersion relation is written for T, which is related to S by

$$S_{fi} = \delta_{fi} - iA\delta\left(\sum p_i\right) T\,, \tag{13}$$

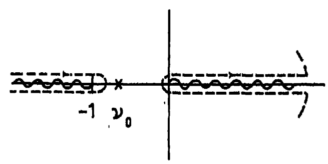

Fig. 4.

where A is a real quantity.

The dispersion relation is simply a Cauchy integral in the ν plane, with the contour shown in Fig. 4, where the singularities of T are also displayed.

$$T(\nu) = \frac{G^2}{\nu-\nu_0} + \frac{1}{2\pi i}\int_0^\infty \frac{\mathrm{d}\nu'}{\nu'-\nu}\,\Delta T(\nu') + \frac{1}{2\pi i}\int_{-\infty}^{-1} \frac{\mathrm{d}\nu'}{\nu'-\nu}\,\Delta T(\nu')\,. \tag{14}$$

ΔT is the discontinuity of T across the cuts. This reproduces the earlier behaviour which was derived in the s_i, $i=1, 2, 3$, variables, if $\Delta T/2\pi i$ is real, in analogy with the reality of G^2. From this it follows that $\Delta T=2i\Im T$,

so that the dispersion relation becomes

$$T(\nu) = \frac{G^2}{\nu - \nu_0} + \frac{1}{\pi}\int_0^\infty \frac{d\nu'}{\nu' - \nu}\, \Im T(\nu') + \frac{1}{\pi}\int_{-\infty}^{-1} \frac{d\nu'}{\nu' - \nu}\, \Im\, T(\nu')\,. \tag{15}$$

In fact, a subtraction is necessary to ensure convergence of the integrals, but this will be omitted. For $0 < \nu < \infty$, $T(\nu)$ is the $\pi n \to \pi n$ amplitude, and is known in terms of the experimental phase shifts. For $-\infty < \nu < -1$, $T(\nu)$ is the required $\pi\pi \to n\bar{n}$ amplitude, and it is necessary to solve eq. (15) as an integral equation.

The first step in this solution is the definition of a function, Ξ, with only the left-hand cut, and no other singularities, the discrepancy

$$\Xi(\nu) = T(\nu) - \frac{G^2}{\nu - \nu_0} - \frac{1}{\pi}\int_0^\infty \frac{d\nu'}{\nu' - \nu}\, \Im T(\nu')\,. \tag{16}$$

$\Xi(\nu)$ is real for $0 < \nu < \infty$, and can be calculated from $T(\nu)$, which is given in terms of the $\pi n \to \pi n$ phase shifts by

$$T(\nu) = \sqrt{\frac{s_1}{\nu}} \sum_{l=0}^{\infty} (2l + 1) P_l\,(-1) \exp[i\delta_l] \sin \delta_l\,. \tag{17}$$

In practice, only S, P, and D waves were inserted into this expansion.

The next stage is the extrapolation of $\Xi(\nu)$ from $0 < \nu < \infty$ on to its cut, $-\infty < \nu < -1$. This is done by a conformal mapping:

$$\eta = \frac{(\nu + 1)^{\frac{1}{2}} - 1}{(\nu + 1)^{\frac{1}{2}} + 1}\,, \qquad \xi(\eta) = \Xi(\nu)\,, \tag{18}$$

which transforms the region $-1 < \nu < \infty$ into the interval $-1 < \eta < 1$, and the cut $-\infty < \nu < -1$ into the unit circle $|\eta| = 1$. It is now useful to expand $\xi(\eta)$ in a Maclaurin series in the η plane:

$$\xi(\eta) = \sum_{n=0}^{\infty} a_n \eta^n\,, \tag{19}$$

for this converges at all points inside the circle $|\eta| = 1$, and also at all points on the circle, except the inelastic branch points. The coefficients a_n can be determined because $\xi(\eta)$ is known on the real axis: and the value on the circle is then

$$\xi(\exp[i\varphi]) = \sum a \exp[in\varphi]\,, \qquad 0 < \varphi < \pi\,, \tag{20}$$

whence the value of $\Xi(\nu)$ on its cut follows by transformation back to the ν plane. Finally, $N(\nu)$ is given in the $\pi\pi \to n\bar{n}$ region by rewriting eq. (16):

$$T(\nu) = \Xi(\nu) + \frac{G^2}{\nu - \nu_0} + \frac{1}{\pi}\int_0^\infty \frac{d\nu'}{\nu' - \nu}\,\Im T(\nu')\,, \qquad -\infty < \nu < -1\,. \tag{21}$$

Lastly, explicit use of the unitarity of the S-matrix is made, in order to connect the $\pi\pi \to \pi\pi$ phase shifts with the $\pi\pi \to n\bar{n}$ amplitude $T(\nu)$, which has just been calculated. Since

$$|\psi_f\rangle = S\,|\psi_i\rangle\,,$$

then

$$\langle\psi_i|\tilde{S}S|\psi_i\rangle = \langle\psi_f|\psi_f\rangle = 1\,,$$

so that

$$\tilde{S}S = 1\,. \tag{22}$$

Since $|\psi_i\rangle$ is a complete set of states. In particular,

$$\sum_m \int dp_m \langle\pi\pi|\tilde{S}|m\rangle\langle m|S|n\bar{n}\rangle = 1\,, \tag{23}$$

where the unit operator $\sum_m \int dp_m |m\rangle\langle m|$ has been inserted between $\tilde{S}$ and S. The approximation is made of neglecting all intermediate states $|m\rangle$ except two-pion states. Then, if S is replaced by T, according to eq. (13), and partial waves are projected out by the formula

$$T_l(\nu) = \tfrac{1}{2}\int_{-1}^{1} dz\,P_l(z)\,T(\nu, z)\,, \tag{24}$$

then the unitarity relation (23) becomes

$$\Im T_l(\pi\pi \to n\bar{n}) = \frac{\pi A}{2}\,T_l^*(\pi\pi \to \pi\pi)\,T_l(\pi\pi \to n\bar{n})\,. \tag{25}$$

Since A is a real normalization function, it follows that

$$\arg T_l(\pi\pi \to n\bar{n}) = \arg T_l(\pi\pi \to \pi\pi)\,, \tag{26}$$

and this is the required $\pi\pi \to \pi\pi$ phase shift. The total $\pi\pi \to \mathrm{n}\bar{\mathrm{n}}$ amplitude has been calculated, and the S-wave approximation is made:

$$T(\pi\pi \to \mathrm{n}\bar{\mathrm{n}}) \fallingdotseq T_{l=0}(\pi\pi \to \mathrm{n}\bar{\mathrm{n}})\,, \tag{27}$$

so that eqs. (26) and (27) together give

$$\delta_{l=0}(\pi\pi \to \pi\pi) = \arg T(\pi\pi \to \mathrm{n}\bar{\mathrm{n}})\,. \tag{28}$$

Hence, by a combination of analyticity, crossing and unitarity, the reaction $\pi\pi \to \pi\pi$ has been related to the reaction $\pi\mathrm{n} \to \pi\mathrm{n}$. In fact, because there are actually four invariant amplitudes, two of which only have even partial waves, and two of which only odd, the unitarity approximation is not as crude as eq. (27), and S, P, and D wave $\pi\pi \to \pi\pi$ phase shifts can be deduced.

The S-wave is found to be predominantly attractive, but the low-energy region is very sensitive to alterations of the $\pi\mathrm{n} \to \pi\mathrm{n}$ P-waves. This suggests that any attempt to calculate the 33 resonance must allow for the $\pi\pi \to \pi\pi$ S-wave. The $\pi\pi \to \pi\pi$ P-wave is found to resonate at $s_3 = 26 \pm 4$, corresponding to the ρ-meson, with a half-width of the order of 100 MeV, and the D-wave is found to be quite large, agreeing quantitatively with the assumption of the dominance of the Regge vacuum pole on this wave.

Determination of the μ-Neutrino Helicity.

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1. – Introduction.

Prior to proposal [1,2] and the experimental discovery [3] of the existence of two different kinds of neutrinos, the *V—A* theory prediction [4-6] was that all leptons emitted in weak processes should have negative helicity while antileptons would have positive helicity. This agrees with the experimental finding for the β-decay process [7,8].

The same prediction makes the processes

$$\pi^- \to \mu^- + \bar{\nu}\,, \tag{1}$$

$$\pi^- \to e^- + \bar{\nu}\,, \tag{2}$$

forbidden, because the helicity of the antineutrino being +1 and that of the electron —1, the two spins add to give a total angular momentum of 1, while the initial state has a total angular momentum equal to zero (see Fig. 1*a*).

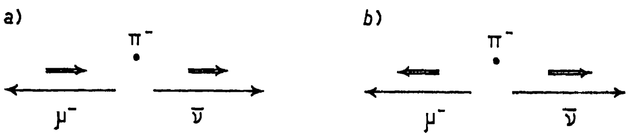

Fig. 1. – Decay of a negative pion in its centre-of-mass. Case *a*) prediction of *V—A* theory. Case *b*) prediction of *V—A* theory, taking into account angular momentum conservation. (Single arrows represent momenta, double arrows represent spins.)

However, the μ being rather massive, reaction (2) can go with the usual speed of weak processes producing negative muons with positive helicity (Fig. 1*b*).

The proposals that the neutrino associated with muons could be different from the one involved in β-decay have led to renewed interest in the helicity of μ-neutrinos, the experimental finding of the opposite sign being a proof of the existence of two different kinds of neutrinos.

With this aim in mind an experiment was undertaken at the Nevis Cyclotron Laboratory of Columbia University by M. BARDON, J. LEE, and myself to measure the sign of the helicity of negative μ-mesons produced in the decay of negative pions.

The experimental result [9,10], in agreement with other experiments [12,13], is that the helicity of negative muons is positive as predicted by V—A theory thus bearing no information on the identity of the two neutrinos.

In the following I will explain the physical ideas involved in the experiment. The experimental details are already in the literature [9,11], so I will avoid a tedious explanation of how the experiment was designed, and will forget about such details as the electronic logic, the beam preparation, and especially the severe background that had to be overcome in order to perform the experiment.

2. – Experimental method.

As mentioned before, the quantity in which we are really interested is the μ-neutrino helicity. However, this determines the muon helicity which can be directly measured.

The two methods which have been extensively used to measure the electron polarization in β-decay, that is Mott scattering and Møller scattering [14], both apply also to muons in so far as they can be considered Dirac particles with e.m. interaction only.

The energy at which the two methods apply is quite different.

Møller scattering, *i.e.* scattering of longitudinally polarized muons on an electron target polarized parallel or antiparallel to the muon spin, is effective for negative muons of a few GeV/c momentum.

This method has been used with cosmic-ray μ-mesons by a Russian group [12], and with machine-produced μ-mesons at the CERN Proton Synchrotron [13].

Mott scattering means scattering of transversely polarized muons in the Coulomb field of a nucleus. For negative muons the most effective energy is around a few tens MeV, and the analysing power is adequate for heavy nuclei like, for instance, lead.

In such a scattering the spin-orbit coupling produces a left-to-right asymmetry where the meaning of left and right, as well as the reasons for such an asymmetry, will be explained in what follows.

The electromagnetic interaction responsible for the scattering of charged particles in a Coulomb field is parity-conserving, hence we can write the scat-

tering matrix as

$$M = f(p,\vartheta) + g(p,\vartheta)\boldsymbol{\sigma}\cdot\boldsymbol{n}\,, \tag{3}$$

where $\boldsymbol{\sigma}\cdot\boldsymbol{n}$ has to be invariant under reflection. In eq. (3) $f(p,\vartheta)$ and $g(p,\vartheta)$ are two scalar functions of the magnitude of the incident momentum and of the scattering angle, $\boldsymbol{\sigma}$ is the (vector) Pauli spin operator.

In order for $\boldsymbol{\sigma}\cdot\boldsymbol{n}$ to be scalar, $\boldsymbol{n}$ has to be an axial vector, and the only one we can construct with the vectors defining the scattering process is

$$\frac{\boldsymbol{p}_1\times\boldsymbol{p}_2}{|\boldsymbol{p}_1\times\boldsymbol{p}_2|}\,,$$

where $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the muon momenta before and after scattering; in other words, $\boldsymbol{n}$ is the normal to the scattering plane.

From eq. (3) it follows that the scattering cross-section for muons with a polarization $\boldsymbol{P}$ is given by

$$\frac{\mathrm{d}\sigma(p,\vartheta)}{\mathrm{d}\Omega} = F(p,\vartheta) + G(p,\vartheta)\boldsymbol{P}\cdot\boldsymbol{n}\,, \tag{4}$$

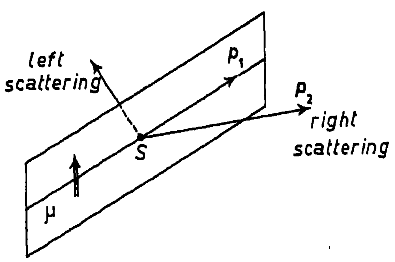

Fig. 2. – Definitions of right and left scattering for a transversely polarized beam of muons. S represents the scatter.

where F and G are two functions of the same variables as before.

We notice first that in order to have $\mathrm{d}\sigma/\mathrm{d}\Omega$ depending upon the polarization, $\boldsymbol{P}$ must have a nonvanishing component along $\boldsymbol{n}$.

If we then have a beam of completely transversely polarized muons ($|\boldsymbol{P}|=1$) and count the particles scattered with the same value of ϑ to the right and left of the plane containing the muon momentum and spin (left and right are defined as in Fig. 2, *i.e.* looking along the direction of motion of the muons), we will find

$$L = \varphi(F+G)\,, \tag{5}$$

$$R = \varphi(F-G)\,, \tag{6}$$

where L is the number of muons scattered to the left, R the number of those scattered to the right, and φ a proportionality factor.

It is usual to introduce the asymmetry

$$A = \frac{L-R}{L+R} = \frac{G}{F}\,. \tag{7}$$

For a beam with polarization P_n along the normal to the scattering plane, eq. (7) becomes

$$A = P_n \frac{G}{F}\,. \tag{8}$$

We have seen previously that muons coming from pion decays are polarized in the direction of motion. But consider a pion moving in the laboratory and decaying in its centre of mass into a muon at some angle ϑ with respect to the trasformation velocity (see Fig. 3). In the laboratory the muon will appear projected forward and flying out at some angle ϑ' smaller than ϑ, while its spin direction is hardly affected by the Lorentz transformation, at least for small velocities of the parent pion. This effect was computed exactly in ref. [10].

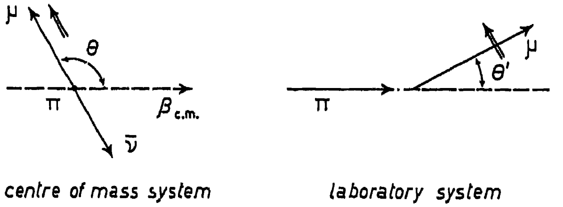

Fig. 3. – Transformation of momenta and spin from the centre-of-mass system to the laboratory system.

We can now describe the experimental set-up. A beam of 28 MeV negative pions was obtained from the Nevis cyclotron. A target T, Fig. 4, was placed so as to collect muons flying off at about 20 degrees from the pion beam corresponding to 100% transverse polarization. The muons scattered backward from the target to the left and to the right were dected by the counters C_L and C_R, respectively.

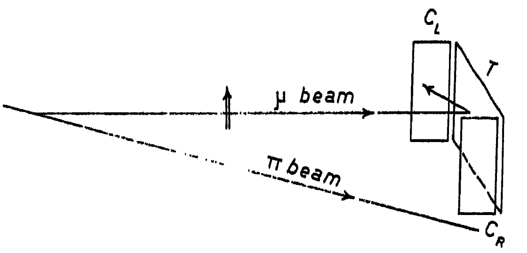

Fig. 4. – Geometry for measuring left-to-right asymmetry. T is the scattering target, C_L the left counter, C_R the right one.

Naturally, not all the muons reaching the target have 100% polarization; however, the minimum decay angle accepted, because of the geometry and the maximum decay angle kinematically possible, allowed the preparation of a muon beam with 90% polarization.

It is clear that the arrangement of Fig. 4 has an axial symmetry around the pion beam, so in the actual experiment we had 10 scattering targets placed symmetrically around the beam and between each of them a counter which was at the same time a left and a right counter for two adjacent targets. In front of each target, very close to them, there was another counter to detect the arrival of a muon.

Although it is necessary to integrate the Dirac equation [15,16] to relate

the magnitude of the asymmetry to the polarization, the following simple picture gives the correct sign relationship.

Consider a negative muon and a nucleus passing each other in the two cases shown in Fig. 5. The spin is shown up in both cases (positive helicity) and thus the magnetic moment is down for a negative muon, corresponding to a clockwise current viewed from above. In *a*) the muon is passing to the right of the nucleus, in *b*) to the left. The situation is clearly analogous to a muon at rest — a clockwise current — and the positively charged nucleus moving in the direction opposite to the original direction of the muon. Then case *a*) is seen to correspond to two antiparallel currents, or a repulsion of the muon towards the right; and *b*) shows two parallel currents, an attraction towards the right. It follows that an enhancement to the right is expected for positive helicity or that the asymmetry as defined above is negative.

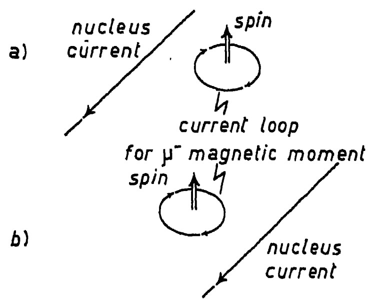

Fig. 5. – Spin-orbit coupling in Mott scattering.

3. – Results.

The expected asymmetry was

$$A = \mp 0.08\,, \qquad \text{for } H_\mu = \pm 1.$$

Thus the experimental result

$$A = -0.09 \pm 0.03\,,$$

leads to the conclusion that negative muons from π-decay have positive helicity.

Moreover, since we have only to distinguish between the two possible signs we can say that the measured value for the asymmetry is 5.6 times the standard deviation of our measurement away from the value of the asymmetry corresponding to the other helicity assignment.

The above-mentioned result is thus another proof that the muons behave, as far as weak interactions are concerned, just as a heavy electron.

Other physically significant information is obtained from this experiment.

The total number of scattering events was calculated to be

$$N = 1\,210 \pm 220\,,$$

and we have counted

$$N = 1\,133 \pm 60\,.$$

The agreement between the expected and measured yield and asymmetry is a proof that the muon is indeed a Dirac particle having only e.m. interaction.

The result quoted above for the yield can be used to obtain information about the muon structure.

Inserting a form factor [17]

$$f(q) = [1 + \tfrac{1}{6}\langle R_\mu^2\rangle q^2]^{-1} ,$$

in the expression for the expected yield, an upper limit to the muon r.m.s. radius is obtained

$$[\langle R_\mu^2\rangle]^{\frac{1}{2}} < 1.9 \cdot 10^{-13}\ \text{cm} ,$$

in agreement with other results [18,20].

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The Two Neutrinos.

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Introduction.

I shall talk essentially on the following subjects:

1. *High-energy neutrinos.*

   1·1. Elastic cross-sections in the simplest hypothesis.

   1·2. Elastic cross-sections in more general hypotheses.

   1·3. Cross-sections on complex nuclei.

   1·4. Inelastic cross-sections.

2. μ-e *symmetry and its consequences.*

   2·1. Two neutrinos.

   2·2. A multiplicative μ-e selection rule.

3. *Additive selection rule*: *additive vs. multiplicative selection rule.*

4. *Further symmetries.*

   4·1. Pauli-Gürsey transformation.

   4·2. SU(2) or else (?).

1. – High-energy neutrinos.

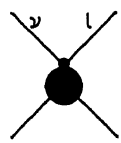
1·1. *Elastic cross-sections in the simplest hypothesis.*

1·1.1. Assuming lepton conservation, the two elastic reactions (the term elastic is here only used for convenience) are

$$\bar{\nu}+p \rightarrow l^{+}+n\,, \tag{1}$$

$$\nu+n \rightarrow l^{-}+p\,. \tag{2}$$

We describe a reaction such as (1) in terms of a graph of the kind



The ν-l pair is coupled at a single space-time point through a current

$$l_{\mu}=(\bar{\nu}\gamma_{\mu}al)\,, \tag{3}$$

where $a=\frac{1}{2}(1+\gamma_5)$, and the other notations are obvious. The matrix element for (1) is given by

$$\sqrt{2}\,G\,\frac{i}{(2\pi)^2}\,\delta_4(\bar{\nu}+p-l^{+}-n)j_{\mu}l_{\mu}\,, \tag{4}$$

where G is the weak coupling constant

$$G\cong 10^{-5}M^{-2}, \tag{5}$$

(M= nucleon mass), $\bar{\nu}$, p, l^{+}, n are used also to denote the four-momenta and

$$j_{\mu}=(2\pi)^3\,\langle n\,|\,j_{\mu}^{(V)}(0)+j_{\mu}^{(A)}(0)\,|\,p\rangle\,. \tag{6}$$

We have split the weak (strangeness-conserving) current into a sum of a vector part and an axial part.

1·1.2. A simplest set of hypotheses to determine j_{μ} is provided by:

a) The non-renormalization hypothesis for the vector current [1], *i.e.*

$$\frac{\partial}{\partial x_{\mu}}\,j_{\mu}^{(V)}(x)=0\,. \tag{7}$$

b) $j_{\mu}^{(A)}$ behaves as the appropriate component of an isotopic spin vector.

I shall illustrate later the meaning of such hypotheses. Under *a*) and *b*) (and, of course, also assuming invariance under the Lorentz group including time reversal) one can write a simple form for j_μ, valid at sufficiently high energies such that the mass of l can be neglected:

$$j_\mu = u(\bar{n})\left[H_1(K^2)\gamma_\mu\gamma_5 + F_1(K^2)\gamma_\mu + \frac{\mu}{2M}F_2(K^2)\sigma_{\mu\nu}K_\nu\right]u(p)\,. \tag{8}$$

In eq. (8), $u(\bar{n})$ and $u(p)$ are Dirac spinors, $H_1(K^2)$, $F_1(K^2)$ and $F_2(K^2)$ are form factors depending on the invariant momentum transfer K^2, where

$$K_\mu = (p-n)_\mu\,. \tag{9}$$

They are normalized according to

$$H_1(0) = \frac{G_A}{G} = 1.2\,, \qquad F_1(0) = F_2(0) = 1\,, \tag{10}$$

and, as it directly follows from *a*), F_1 and F_2 are differences of the corresponding proton and neutron electromagnetic form factors, and $\mu = \mu_p - \mu_n \cong 3.7$ in nucleon Bohr magnetons. In the limit $K^2 \to 0$

$$j_\mu \to \bar{n}\gamma_\mu\left(1 + \frac{G_A}{G}\gamma_5\right)p\,. \tag{11}$$

The differential cross-section [2] derived from eqs. (4), (3), (6) and (8) is reported in Appendix I.

1'1.3. Now what do we expect in general for the behaviour of the cross-section? If E is the centre-of-mass energy, then phase space goes as E^2 and the amplitude is proportional to G. On this ground you would have

$$\sigma \propto E^2\,G^2 = \frac{1}{M^2}\left(\frac{E}{M}\right)^2 10^{-10} \propto \mathscr{E}_\nu\,G^2\,. \tag{12}$$

However there are form factors present. What do they do? Suppose they are essentially equivalent to a cut-off at $K^2 = a^2$, in the sense that momentum transfers $> a^2$ are excluded. Then I have to multiply eq. (12) by a factor $\Delta\omega$ expressing the «allowed» solid angle. But $K^2 < a^2$ means

$$4|\boldsymbol{p}|^2 \sin^2\frac{\Theta}{2} < a^2\,, \tag{13}$$

where $\boldsymbol{p}$ is the c.m. proton momentum. Therefore the maximum allowed solid

angle is

$$\Delta\omega = 4\pi \sin^2 \frac{\Theta_{\max}}{2} = \pi \frac{a^2}{|\boldsymbol{p}|^2}, \tag{14}$$

and for large energies $\Delta\omega \propto E^{-2}$, so that

$$\sigma \propto E^2 G^2 \Delta\omega \to G^2 a^2 \tag{15}$$

in the high-energy limit, *i.e.* it goes to a constant. To elaborate a little better on this point let me write down the expression that you get for σ_{total} from the expression in Appendix I, in the high-energy limit. You find, in the high-energy limit, for both (1) and (2)

$$\sigma_{\text{tot}} \to G^2(h_1 + f_1 + f_2), \tag{16}$$

where

$$h_1 = \frac{1}{2\pi}\int_0^\infty \mathrm{d}K^2 H_1^2(K^2), \tag{17}$$

$$f_1 = \frac{1}{2\pi}\int_0^\infty \mathrm{d}K^2 F_1^2(K^2), \tag{17$'$}$$

$$f_2 = \frac{1}{2\pi}\left(\frac{\mu}{2M}\right)^2\int_0^\infty K^2\,\mathrm{d}K^2 F_2^2(K^2). \tag{17$''$}$$

If we now choose for the form factors

$$F_1(K^2) = \frac{1}{(1 + K^2/a_1^2)^2}, \tag{18}$$

$$F_2(K^2) = \frac{1}{(1 + K^2/a_2^2)^2}, \tag{18$'$}$$

$$H_1(K^2) = \frac{G_A}{G_V}\frac{1}{(1 + K^2/b^2)^2}, \tag{18$''$}$$

we find, in the high-energy limit,

$$\sigma_{\text{total}} = \frac{1}{6\pi}\left[G^2 a_1^2 + \frac{\mu^2}{8M^2} G^2 a_2^2 + G_A^2 b^2\right], \tag{19}$$

which is precisely of the form (15). If you put $a_1^2 = a_2^2 = b^2 = 37.5\, m_\pi^2$ (a value suggested by the Stanford experiments) you find

$$\sigma_{\text{total}} \cong 0.75 \cdot 10^{-38}\ \text{cm}^2. \tag{20}$$

The statement (15) that $\sigma_{tot} \to$ constant is only to be intended in a practical sense. For instance, if the form factors are written in the more fashionable form

$$F = (1 - V) + \frac{V}{1 + K^2/\varrho^2},$$

where ϱ is some mass, then according to eq. (16)

$$\sigma \to \infty .$$

If they are written as

$$F = \frac{a}{1 + K^2/\varrho^2} + \frac{b}{1 + K^2/x^2},$$

where x is another mass, then σ increases logarithmically. And so on.

1'1.4. In the high-energy limit the differential cross-section of Appendix I takes a simple form. Always in the laboratory frame call $\mathscr{E}_\nu$, $\mathscr{E}_1$ the energies of ν and l, and T the kinetic energy of the recoil proton. Then

$$K^2 = \frac{\mathscr{E}_\nu - \mathscr{E}_1}{2M} = \frac{T}{2M}, \tag{21}$$

and introduce

$$x^2 = \frac{K^2}{(2M)^2}. \tag{22}$$

Then, for both (1) and (2), in the high-energy limit

$$\frac{d\sigma}{d\mathscr{E}_1} \to \frac{1}{\pi} MG^2(H_1^2 + F_1^2 + \mu^2 x^2 F_2^2), \tag{23}$$

where the form factors are taken at K^2.

1'1.5. It may be instructive also to talk of the polarization. In a scattering process you expect a polarization normal to the scattering plane even starting by unpolarized particles. However, the amplitude in our case is real (we are assuming time-reversal invariance) and there is no such polarization normal to the plane. The recoil nucleon can, however, have a longitudinal polarization, just originating from the fact that the process is parity-nonconserving. For reaction (1), under the simplest hypotheses a) and b) described above, the longitudinal polarization of the final n becomes

$$P_L \to \frac{x}{\sqrt{1 + x^2}} \frac{2H_1(F_1 + \mu F_2)}{H_1^2 + F_1^2 + \mu^2 F_2^2 x^2}. \tag{24}$$

For reaction (2) the final p has a longitudinal polarization given by $-P_L$. Under the rough assumption we have already used, of all form factors equal, you get

$$P_L \to \frac{x}{\sqrt{1+x^2}}\,\frac{(4.5)}{1+(5.6)x^2}\,. \tag{25}$$

1·2. *Elastic cross-sections in more general hypotheses.*

1·2.1. If I relax the hypotheses *a*) and *b*) and rely only on Lorentz invariance (and time reversal) I would write for the general form of j_μ

$$j_\mu = \bar{n}\left|F_1\gamma_\mu + \frac{\mu}{2M}F_2\sigma_{\mu\nu}K_\nu + iF_3K_\mu + \right. \\ \left. + H_2\gamma_\mu\gamma_5 + \frac{\mu'}{2M}H_3\sigma_{\mu\nu}\gamma_5K_\nu + iH_2K_\mu\gamma_5\right] p\,, \tag{26}$$

where the F's are form factors for the vector part and the H's are form factors for the axial part. Equation (26) is a rigorous but perhaps too general framework for analysing the experiments. Thus, next to the simplest hypothesis for j_μ discussed before, we will consider a possible situation where *a*) (*i.e.* nonrenormalization for the vector part) still holds, but *b*) (*i.e.* behaviour of $j_\mu^{(A)}$ as a component of an isospin vector) does not necessarily hold. In fact, one has a theoretical feeling that the nonrenormalization hypothesis for the vector strangeness-conserving current is true (*). By the way, this implies that such a current transforms as a vector in isospin space (it is actually assumed to be the isospin current itself). On the other hand, the behaviour of the axial strangeness-conserving current under isospin rotations has to be postulated. It is true that in a possible scheme where you first construct the vector current as the isospin current out of the known particles, and then you generate the axial current by inserting a γ_5 for each fermion pair, you get a current that transforms as a vector in isospin space. However, everybody would feel that this procedure is too much model-dependent to be completely reliable. Thus, although we like the hypothesis *b*), it may be opportune to allow for its possible nonvalidity. Now from *a*) you have directly

$$K_\mu j_\mu = 0 \qquad \text{or equivalently} \qquad F_3 = 0\,. \tag{27}$$

Furthermore you can safely neglect the term proportional to H_2 since

$$K_\mu l_\mu \propto m_e\,, \tag{28}$$

(*) A recent experiment by LEE, MO and WU (*Phys. Rev. Lett.*, **10**, 253 (1963)) confirms the hypothesis of conserved vector current.

so you can neglect it at high energies, assuming H_2 does not become pathologically big. In conclusion, the step next to the simplest hypothesis, discussed in 1'1), will be to assume

$$j_\mu = \bar{n}\left[F_1\gamma_\mu + \frac{\mu}{2M}F_2\sigma_{\mu\nu}K_\nu + H_1\gamma_\mu\gamma_5 + \frac{\mu'}{2M}H_3\sigma_{\mu\nu}\gamma_5 K_\nu\right]p\,,$$

where F_1 and F_2 are still differences of the corresponding proton and neutron electromagnetic form factors.

1'2.2. The new term is that containing $\sigma_{\mu\nu}K_\nu\gamma_5$. The high-energy limit (16) now becomes

(29) $$\sigma_{\text{tot}} \to G^2(h_1 + h_3 + f_1 + f_2)\,,$$

where h_1, f_1 and f_2 are still given by eqs. (17), (17′) and (17″), and

(30) $$h_3 = \frac{1}{2M}\left(\frac{\mu'}{2M}\right)^2\int_0^\infty K^2\,\mathrm{d}K^2\,H_3^2(K^2)\,.$$

Similarly, eq. (23) becomes

(31) $$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathscr{E}_1} \to \frac{1}{\pi}MG^2[H_1^2 + F_1^2 + (\mu^2F_2^2 + \mu'^2H_3^2)x^2]\,,$$

and eq. (24) becomes

(32) $$P_L \to \frac{x}{\sqrt{1+x^2}}\,\frac{2[(H_1 - \mu'H_3)(F_1 + \mu F_2) + \mu\mu' F_2H_3(1+x^2)]}{H_1^2 + F_1^2 + (\mu^2F_2^2 + \mu'^2H_3^2)x^2}\,.$$

All the results derived in this Section are valid also if an intermediate vector meson exists provided one multiplies each form factor by the (spinless) propagator of a boson with the mass of the vector meson.

1'3. *Cross-section on complex nuclei.*

1'3.1. We shall briefly deal here with the effect of the Pauli principle on the cross-section when a reaction such as (1) or (2) occurs on complex nuclei. Consider, for instance, reaction (1)

(1) $$\nu + p \to l^+ + n\,.$$

The individual protons of the nucleus act incoherently so that at large momentum transfers the cross-section on the nucleus is Z times the cross-section

on a proton. However, in an independent particle language at small momentum transfers the recoil n may not find a free final state where to jump and the reaction may therefore become forbidden. Let us see roughly what happens. In the laboratory system the momentum transfer K^2 is given by

$$K^2 = (p - n)^2 = |\boldsymbol{n}|^2 - (\mathscr{E}_n - M)^2 = 2MT\,, \tag{33}$$

where we have denoted by $\boldsymbol{n}$ and $\mathscr{E}_n$ the laboratory neutron momentum and total energy (T is the neutron kinetic energy). Now consider a Fermi model. In order for the reaction to occur we can say that T must be larger than the Fermi energy T_F so that the neutron can escape from the Fermi sphere. This implies

$$K^2 = 2MT > 2MT_F = 2M\,\frac{p_F^2}{2M} = p_F^2\,, \tag{34}$$

where p_F is the Fermi momentum. Still in the laboratory system

$$K^2 = (p - n)^2 = (l - \nu)^2 \cong 4\mathscr{E}_1\mathscr{E}_\nu \sin^2\frac{\Theta}{2}\,, \tag{35}$$

neglecting the mass of l. From eqs. (35) and (34)

$$4\mathscr{E}_1\mathscr{E}_\nu \sin^2\frac{\Theta}{2} > p_F^2\,, \tag{36}$$

$$2\sin\frac{\Theta}{2} > \frac{p_F}{\sqrt{\mathscr{E}_1\mathscr{E}_\nu}}\,.$$

Since $\mathscr{E}_\nu - T = \mathscr{E}_1$, we can neglect T (here always $< T_F$) and write eq. (36) in the form

$$\Theta > \sim \frac{p_F}{\mathscr{E}_\nu}\,. \tag{37}$$

If $\mathscr{E}_\nu \cong 0.7$ GeV and $p_F = 0.2$ GeV you find $\Theta > \sim 10°$. Reactions occurring at an angle Θ less than $\sim p_F/\mathscr{E}_\nu$ are thus cut off by the exclusion principle. For a rough picture of the process on a complex nucleus we thus have to take into account two cut-off angles. One for small angles, less than $\sim p_F/\mathscr{E}_\nu$ due to the Pauli principle, the other for large angles greater than Θ_{max} (see eq. (14)) due to the nucleon form factors.

1·3.2. A better estimate can be made, as usual in similar cases, by calculating the fraction of volume of the proton Fermi sphere that contains those protons which, for a given momentum transfer $\Delta = \boldsymbol{n} - \boldsymbol{p}$, change into neutrons out-

side the neutron Fermi sphere. Then consider the proton Fermi sphere of radius P_F (Fig. 1).

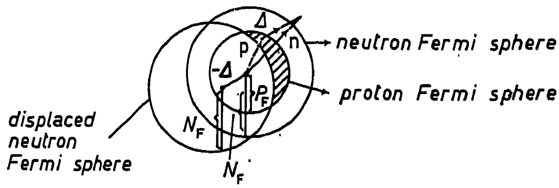


Fig. 1. – Fermi sphere description of ν scattering on a nucleus.

Draw the neutron Fermi sphere of radius N_F with a centre displaced by $-\mathbf{\Delta}$ from the centre of the proton sphere. A proton lying in the shaded volume satisfies the condition that $|\mathbf{p}+\mathbf{\Delta}|>N_F$. The fraction of the « effective » protons is then given by the ratio of the volume of this region to the total volume. Defining

$$\eta=\frac{\Delta}{N_F}\qquad \eta_0=1-\frac{P_F}{N_F}, \tag{38}$$

one finds [3] for the fraction δ_z of the « effective » protons at a momentum transfer Δ

$$\delta_z=0\,,\qquad\qquad \text{for } \eta<\eta_0, \tag{39}$$

$$\delta_z=\frac{N}{Z}F(\eta)-\left(\frac{N}{Z}-1\right),\quad \text{for } \eta>\eta_0\,, \tag{39'}$$

until, at large Δ, $\delta_z=1$. In eq. (39′), $F(\eta)$ is given by

$$F(\eta)=\frac{3}{4}\eta-\frac{1}{16}\eta^3+\frac{1}{2}\left(1-\frac{Z}{N}\right)+\frac{3}{16\eta}\left[1-\left(\frac{Z}{N}\right)^{\frac{2}{3}}\right]\left[1-\left(\frac{Z}{N}\right)^{\frac{2}{3}}-2\eta^2\right].$$

Similarly, for reaction (2)

$$\nu+\mathrm{n}\to \mathrm{l}^-+\mathrm{p}\,, \tag{2}$$

the fraction δ_N of the « effective » neutrons is given by

$$\delta_N=1-\frac{Z}{N}\qquad \text{for } \eta<\eta_0\,, \tag{40}$$

$$\delta_N=F(\eta)\,,\qquad \text{for } \eta>\eta_0, \tag{40'}$$

until it also reaches the value one. With this more accurate treatment you

get, instead of the rigorous cut-off for $\Theta < p_F/\mathscr{E}_\nu$, a smooth depression for the small angles that satisfy the above inequality. It should however be said that, apart from the exclusion principle limitation discussed here, other more complicated nuclear effects will presumably make precise quantitative estimates of the interaction on a complex nucleus rather uncertain.

1·4. *Inelastic cross-sections.*

1·4.1. Some of the inelastic cross-sections are still two-body reactions

$$\bar{\nu}+p \to l^+ + \Lambda^0\,, \tag{41}$$

$$\bar{\nu}+p \to l^+ + \Sigma^0\,, \tag{42}$$

$$\bar{\nu}+n \to l^+ + \Sigma^-\,, \tag{43}$$

$$\bar{\nu}+n \to l^- + \Sigma^+\,, \tag{44}$$

and similarly starting from ν. Note that the last reaction (44) goes through a current with $\Delta S = -\Delta Q$, which for a long time has been assumed not to exist. But now there seems to be good experimental evidence in favour of such a current.

Other inelastic processes are

$$\bar{\nu}+p \to l^+ + n + \pi^0\,, \tag{45}$$

$$\bar{\nu}+p \to l^+ + p + \pi^-\,, \tag{46}$$

$$\bar{\nu}+n \to l^+ + n + \pi^-\,, \tag{47}$$

and similar processes starting from ν.

One has also processes

$$\bar{\nu}+p \to l^+ + p + K^-\,, \tag{48}$$

$$\bar{\nu}+p \to l^+ + n + K^0\,, \tag{49}$$

$$\bar{\nu}+p \to l^+ + n + \bar{K}^0\,, \tag{50}$$

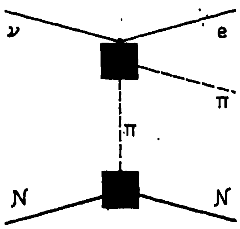
and similarly from ν.

1·4.2. The reactions (41)-(44) are inverse hyperon β-decay processes. One knows that their experimental rates are lower than universal Fermi theory would predict. On this basis one would also expect that (41)-(44) have smaller cross-sections (say, by a factor of 10 or more) than the elastic cross-sections

we considered before. However, this statement is far from being rigorous. The momentum transfer K^2 equal, say, to $(Y-p)^2$, takes on generally different values in the decay and in the high-energy production process. Knowledge of the decay only gives very partial information on the production.

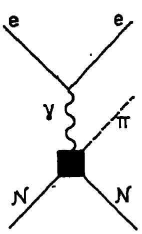
1'4.3. As to reactions (45)-(47) [4] estimates have been given for the vector part by:

i) Estimating the contribution from the one-pion pole



In this graph the vertex (ππeν) is expressed in terms of the electromagnetic pion form factor (on which theorists think they already know a lot) by use of the nonrenormalization hypothesis. For an energy $\mathscr{E}_\nu \sim 1$ GeV one finds $\sigma \sim 10^{-39}$ cm².

ii) Comparing with electropion production, as shown by the graphs



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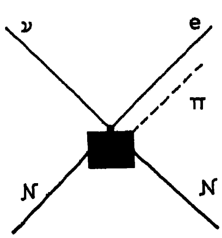


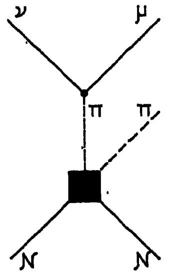
.

Again the connection is provided by the nonrenormalization hypothesis. One would roughly have

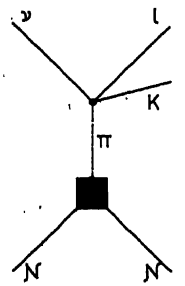
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1\,\mathrm{d}E_1}\right)_{\text{neutrinopion}}=\frac{1}{4}\left(\frac{GK^2}{4\pi\alpha}\right)^2\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1\,\mathrm{d}E_1}\right)_{\text{electropion}},\tag{51}$$

where K is the momentum transfer and $\alpha=1/137$. Extrapolating existing electropion data at a lower energy one finds in this way a cross-section of the order of 10^{-39} cm².

As to the axial part, an estimate can be made of the one-pion pole (when the final lepton is a muon, otherwise this pole is negligible). The graph is



The estimate leads to a cross-section $\sim 10^{-40}\,\text{cm}^2$. A one-pion approximation for (48)-(50) uses the graph



and therefore connects the process to the decay

$$K \to \pi + l + \nu\,.$$

The cross-section is again small $\sim 10^{-40}\,\text{cm}^2$ [5].

It is hard to decide how reliable are the above estimates.

2. – μ-e symmetry and its consequences.

2'1. *Two neutrinos.*

2'1.1. In this Section I shall describe an approach that we proposed about two years ago which led us to postulate a property of μ-e symmetry [6]. From such a μ-e symmetry it followed in a natural way that

i) there are two neutrinos;

ii) there is a multiplicative selection rule forbidding transformations of μ into e.

Two neutrinos and the existence of a selection rule are now verified experimentally. The selection rule may be an additive, instead of multiplicative selection rule. We shall discuss later in this section the experimental differences between the two kinds of selection rules. The formal argument that we shall give runs roughly as follows. Let us assume that μ and e have identical interactions and they only differ in their rest mass. It can then be seen that, if electromagnetic interactions satisfy the requirement of minimality and if weak interactions are neglected, there exists a symmetry property of the theory, that we call μ-e symmetry, related to an exchange of the two lepton fields that are necessary to describe μ and e. We then find that in order to maintain the μ-e symmetry also in a complete theory including weak interactions one has to postulate two neutrinos. This possibility of having two neutrinos has been considered since a long time by different authors on various grounds. Experimentally it seems to be suggested from the absence of $\mu \to e+\gamma$, which should almost certainly be present if in $\mu \to e+\nu+\bar{\nu}$ there is only one kind of neutrino. Now it is known that to a symmetry property of the theory there corresponds, in general, a physical conservation law. The conservation law corresponding to μ-e symmetry is a multiplicative conservation law that forbids transformations of μ into e.

More specifically, one assigns to each particle a multiplicative muonic quantum number K, according, for instance, to the assignment shown in Table I, and one obtains that the only reactions that are permitted are those

TABLE I.

Particles	Quantum number K	Quantum number M
μ^-, ν_μ, e^+, $\bar{\nu}_e$	$-i$	-1
μ^+, $\bar{\nu}_\mu$, e^-, ν_e	$+i$	$+1$
mesons, baryons, γ	1	0

for which the product of the initial K values is equal to the product of the final K values. To the mesons, baryons and photon we have assigned $K=1$, but we could have chosen instead $K=\exp[i(\pi/2)N]$ where N is the nucleonic quantum number of the particle, without altering any physical consequence, but only with a slight alteration of the formal argument leading to the postulated law. In Table I there is also a column defining the quantum number M. This is the additive quantum number and we shall talk of it later.

2'1.2. Let us now develop the formal argument in some more detail. If we neglect weak interactions the total Lagrangian looks like

$$\mathscr{L} = -\bar{e}(\gamma\partial + m_e)e - \bar{\mu}(\gamma\partial + m_\mu)\mu + \text{other terms}\,. \tag{52}$$

The « other terms » do not contain e or μ. The electromagnetic interaction of μ and e is included in eq. (52)—following our requirement of minimality—through the definition of ∂

$$\partial = \frac{\partial}{\partial x} + ieA\ . \tag{53}$$

The way it is written, eq. (52) is not symmetric under the exchange $\mu \leftrightarrow e$ However, nobody told us what to call a μ and what to call an e. If I introduce new fields e' and μ' through

$$e = \frac{1}{\sqrt{2}}(e' + \mu')\ , \tag{54}$$

$$\mu = \frac{1}{\sqrt{2}}(e' - \mu')\ , \tag{54'}$$

$\mathscr{L}$ takes the form

$$\mathscr{L} = -\bar{e}'(\gamma\partial + m_+)e' - \bar{\mu}'(\gamma\partial + m_+)\mu' + m_-[(\bar{e}'\mu') + (\bar{\mu}'e')] + \text{other terms.} \tag{55}$$

In this form $\mathscr{L}$ is symmetric under $e' \leftrightarrow \mu'$.

The situation here is not really quite different from what one does with the « physical » particles K_1^0 and K_2^0. For reasons of symmetry one prefers to use in strong interactions K^0 and $\bar{K}^0$ that are related by charge conjugation. At some point there is CP conservation, however, that tells you that the « physical » particles are K_1^0 and K_2^0. Similarly the hope here is that at some point there will be a conservation law (it will be muonic number conservation) that will tell you that the « physical » particles are just μ and e.

2·2. *A multiplicative* μ-e *selection rule.*

2·2.1. It is convenient now to introduce a fictitious L space (lepton space). In L space, e and μ form a doublet

$$\psi = \begin{bmatrix} e \\ \mu \end{bmatrix}. \tag{56}$$

The transformation (54) and (54') can then be generalized to

$$\psi = T^{-1}\psi'\ , \tag{57}$$

where T is a nonsingular matrix. Then $\mathscr{L}$ takes the general form

$$\mathscr{L} = -\bar{\psi}'[\gamma\partial(A + B\gamma_5) + (C + iD\gamma_5)]\psi' + \text{other terms}\ . \tag{58}$$

in eq. (58) A, B, C and D are hermitian matrices in L space (*i.e.* they are 2×2 matrices acting on the spinors ψ), and furthermore $A+B$ and $A-B$ are both positive definite. The matrix T can be decomposed as

(59) $$T = aR + \bar{a}S ,$$

where $a = \frac{1}{2}(1+\gamma_5)$, $\bar{a} = \frac{1}{2}(1-\gamma_5)$, and R and S are matrices in L space. If we define the mass matrix M as

(60) $$M = m_+ + m_-\sigma_3 ,$$

the conditions that R and S must satisfy to bring eq. (52) into eq. (58) are

(61) $$R^\dagger(A+B)R = 1 ,$$

(61′) $$S^\dagger(A-B)S = 1 ,$$

(61″) $$S^\dagger(C+iD)R = M .$$

This is shown in Appendix II.

2·2.2. Now μ-e symmetry implies that eq. (58) be invariant when

(62) $$\psi' \to \sigma_1\psi' ,$$

as this amounts to the substitution of the two components of the doublet ($\psi'_1 \leftrightarrow \psi'_2$). This condition requires that A, B, C, D all commute with σ_1 so that each of them can be written in a form

(63) $$\alpha P_+ + \beta P_- ,$$

where α and β are coefficients and

(64) $$P_\pm = \tfrac{1}{2}(1 \pm \sigma_1) .$$

In fact eq. (63) is the most general function of the unit matrix 1 and σ_1. There are infinite choices of R and S that satisfy eqs. (61), (61′) and (61″) with A, B, C, D of the form (63).

2·2.3. So far we have neglected weak interactions. Now we require that μ-e symmetry [as expressed by eq. (62)] be valid also when weak interactions are present. We shall see that this is impossible if there is only one neutrino.

A term of weak interactions is

(65) $$G[\bar{\nu}\gamma_\mu(ae + a\mu)]J_\mu ,$$

where J_μ is a current not containing leptons, but only baryons and mesons. In doublet notation eq. (59) is

$$G(\bar{\nu}\gamma_\mu a\psi)J_\mu \,. \tag{65'}$$

In terms of ψ'

$$G(\bar{\nu}\gamma_\mu a\psi)\,J_\mu = G(\bar{\nu}\gamma_\mu aT^{-1}\psi')\,, \tag{66}$$

and if we want μ-e symmetry

$$(66) = G(\bar{\nu}\gamma_\mu aT^{-1}\sigma_1\psi') = G(\bar{\nu}\gamma_\mu aT^{-1}\sigma_1 T\psi)\,. \tag{67}$$

So we must have

$$a\psi = aT^{-1}\sigma_1 T\psi\,. \tag{68}$$

In Appendix III it is shown that eq. (68) is incompatible with eqs. (61), (61′) and (61″).

2'2.4. Thus we have to introduce two neutrinos. Then the Lagrangian

$$\begin{aligned}\mathscr{L}' = -\,\bar{\psi}'(\gamma\partial + m_+)\psi' - m_-\bar{\psi}'\sigma_1\psi' - \bar{\nu}'\gamma\partial\nu' + {}\\ + G[\bar{\psi}'\gamma a\nu' + \ldots][\bar{\nu}'\gamma a\psi' + \ldots] + \text{other terms}\,,\end{aligned} \tag{69}$$

where

$$\nu' = \begin{bmatrix}\nu'_e\\ \nu'_\mu\end{bmatrix}, \tag{70}$$

is also a doublet in L space, and is now invariant under μ-e symmetry defined by

$$\psi' \to \sigma_1\psi'\,, \qquad \nu' \to \sigma_1\nu'\,. \tag{71}$$

In fact eq. (69) is the Lagrangian one usually adopts. However, eq. (69) is *not* the most general Lagrangian compatible with eq. (71). To end up with eq. (69) there must be more in the story. But first let us go back to the μ-e selection rule ensuing from invariance under eq. (71). To develop a physical feeling for such a selection rule we want to know what eq. (71) means in terms of the good old e and μ. Now

$$\psi' \to \sigma_1\psi'$$

means

$$T\psi \to \sigma_1 T\psi ,$$

or

$$\psi \to T^{-1}\sigma_1 T\psi . \tag{72}$$

What is $T^{-1}\sigma_1 T$? It must be traceless, of unit square, commute with σ_3 (because it must commute with M). So it is $\pm\sigma_3$. And then it is obvious which is the selection rule: it is a multiplicative (corresponding to the discreteness of the operation (71)) selection rule that forbids a μ to go into an e. How this selection rules operates we have already illustrated before.

2'2.5. Now let us state that there is a reason to believe that this framework of μ-e symmetry, although it is sufficient to imply i) two neutrinos; ii) a selection rule, is perhaps too wide. First, the selection rule may very well be additive (see below). Secondly, take an example: add to the weak current in eq. (69) the term $\lambda\bar{\psi}\gamma a\sigma_1\nu'$, allowed by eq. (71). Then, in the μ-e representation, μ is coupled proportionally to $(1-\lambda)$, and e proportionally to $(1+\lambda)$. But, as we know, all the evidence is instead for equal couplings. So we learn that there must be more into the story if one wants to get finally to eq. (69).

3. – Additive selection rule: additive *vs.* multiplicative selection rule.

3'1. Besides the multiplicative conservation law that we have discussed one can consider the more stringent additive conservation law: to each particle there corresponds an additive muonic quantum number M, such that, for instance,

$$K = \exp\left[i\frac{\pi}{2}M\right],$$

and only those reactions are allowed for which the sum of the initial M values is equal to the sum of the final M values. Corresponding to our previous assignments of K one can assign values of M as in Table I.

It is obvious that if the additive muonic conservation law is verified, the multiplicative law is also verified.

However, if the multiplicative muonic conservation law is verified it does not follow that the additive conservation law is also verified. It only follows that the difference ΔM between the sum of the initial M values and the sum of the final M values can only take the values 0, ± 4, ± 8 (*i.e.*, $\Delta M = 0$ mod. 4).

Odd ΔM are always forbidden because of lepton conservation. $\Delta M = \pm 2$, ± 6, etc., are forbidden for both types of conservation laws, and this is suf-

ficient to prevent reactions such as

$$\mu \to e+\gamma\,,$$

$$\mu \to e+e+e\,,$$

$$\mu+(\text{nucleus}) \to e+(\text{nucleus})\,,$$

$$\mu \to e+\nu_e+\bar{\nu}_e\,, \qquad \pi \to \mu+\nu_e\,, \quad \text{etc.}$$

Reactions with $\Delta M = \pm 4$, ± 8, etc., are forbidden by the additive law, but they are allowed by the multiplicative law. Thus evidence for reactions like, for instance,

$$e^+ + \mu^- \to e^- + \mu^+\,, \tag{73}$$

$$e^+ + e^+ \to \mu^+ + \mu^+\,, \tag{74}$$

$$\nu_\mu + (\text{nucleus}) \to \nu_e + e^- + \mu^+ + (\text{nucleus})\,, \tag{75}$$

would exclude the additive conservation law and be consistent with the multiplicative conservation law. None of these reactions can occur by electromagnetic interactions alone, and thus any evidence for them would be an evidence for some new mechanism, for instance of the kind we are considering. The reaction (73) is very interesting: it can occur as a charge-exchange reaction in muonium

$$(e^- + \mu^+)_{\text{bound}} \to (e^+ + \mu^-)_{\text{bound}}\,.$$

It has been studied theoretically by PONTECORVO [7], and by FEINBERG and WEINBERG [8]. We refer to the work of these authors for further discussion.

The reaction (74) might, in principle, be studied by colliding beam experiments of the type considered at Stanford, but carried out at a much higher energy or with higher intensity. A possible interaction of the type

$$\mathscr{L}' = f(\bar{\psi}^{(\mu)}\gamma_\lambda a \psi^{(e)})(\bar{\psi}^{(\mu)}\gamma_\lambda a \psi^{(e)})\,,$$

where

$$a = \tfrac{1}{2}(1+\gamma_5) \qquad \text{and} \qquad \bar{a} = \tfrac{1}{2}(1-\gamma_5)\,,$$

through Fierz re-ordering can be written in the form (we use the Majorana representation)

$$\mathscr{L}' = 2f(\psi^{(e)}\gamma_4 \bar{a} \psi^{(e)})(\psi^{(\mu)*}\gamma_4 \bar{a} \psi^{(\mu)*})\,.$$

The cross-section is then clearly isotropic in the c.m. system. The total cross-section is given by

$$\sigma_{\text{total}} = \frac{f^2 E^2}{\pi}\, \beta(1+\beta^2)\,,$$

where E is the c.m. energy of each colliding electron and β is the muon velocity in the c.m. system. If, tentatively, we identify f with $\sqrt{8}\,G$, where G is the weak coupling constant, the cross-section turns out to be

$$\sigma_{\text{total}} = 1.6 \cdot 10^{-37} (E/M)^2 \text{ cm}^2\,,$$

where M is the nucleon mass.

The final muons are polarized longitudinally. The value of the polarization is

$$P^{\pm} = \pm \frac{2\beta}{1+\beta^2}\,,$$

where the upper sign holds for μ^+, the lower sign for μ^-.

The reaction (75) can occur through an interaction of the type

$$\mathscr{L}' = f(\bar\psi^{(\mu)}\gamma_\lambda a \psi^{(\nu_e)})\,(\bar\psi^{(\nu_\mu)}\gamma_\lambda a \psi^{(e)})\,,$$

together with the absorption of a virtual nuclear γ-ray by one of the final charged leptons. An estimate of its cross-section on lead gives $0.5 \cdot 10^{-39}$ (E_ν in GeV) cm^2 where E_ν is the incident-neutrino energy in the laboratory system. The rapid increase of the cross-section with energy will perhaps make this process one of the most convenient to decide between the two possibilities of an additive or of a multiplicative muon-electron selection rule (*).

3·2. The formal expression of lepton conservation by a gauge invariance property (quite similar to that which expresses conservation of charge) is, in terms of the fields $e(x)$, $\mu(x)$, $\nu_e(x)$, $\nu_\mu(x)$

$$(76)\qquad \begin{cases} e \to \exp[i\lambda]e\,, & \bar e \to \exp[-i\lambda]\bar e\,,\\ \mu \to \exp[i\lambda]\mu\,, & \bar\mu \to \exp[-i\lambda]\bar\mu\,,\\ \nu_e \to \exp[i\lambda]\nu_e\,, & \bar\nu_e \to \exp[-i\lambda]\,\bar\nu_e\,,\\ \nu_\mu \to \exp[i\lambda]\nu_\mu\,, & \bar\nu_\mu \to \exp[-i\lambda]\bar\nu_\mu\,, \end{cases}$$

(*) One has to find an upper limit to the ratio of $e^-\mu^+$ pairs to $e^+\mu^-$ pairs in the reactions $\nu_\mu + (\text{nucleus}) \to (\text{nucleus}) + \nu + e + \mu$, initiated by muon-neutrinos. The $e^-\mu^+$ pairs from a possible contamination of muon-antineutrinos in the beam must be subtracted (muon-antineutrinos can produce $e^-\mu^+$ pairs without violating the additive section rule). The existence of intermediate bosons is irrelevant to this problem.

with the same real number λ for all fields. Similarly if we want a continuous gauge group also for muonic conservation (implying the additive selection rule), muonic conservation is expressed by

$$\left|\begin{array}{ll} e \to \exp[-im]e\,, & \bar{e} \to \exp[im]\bar{e}\,, \\ \mu \to \exp[im]\mu\,, & \bar{\mu} \to \exp[-im]\bar{\mu}\,, \\ \nu_e \to \exp[-im]\nu_e\,, & \bar{\nu}_e \to \exp[im]\,\bar{\nu}_e\,, \\ \nu_\mu \to \exp[im]\nu_\mu\,, & \bar{\nu}_\mu \to \exp[-im]\bar{\nu}_\mu\,. \end{array}\right. \tag{77}$$

3'3. We now write eqs. (76) and (77) in a formalism in which the two 2-component spinors ν_e and ν_μ are joined to form a single 4-component spinor. In such a formalism you can still go on saying that there is only one neutrino.

I shall work in a representation with γ_5 diagonal. If you take

$$\gamma_K = \begin{bmatrix} 0 & -i\sigma_K \\ i\sigma_K & 0 \end{bmatrix}, \quad \gamma_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{78}$$

$$\left(\text{with } \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right),$$

you find that the γ's are hermitian, satisfy $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$ and, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. (In fact, the representation you are accustomed to is one in which one has simply replaced $\gamma_4 \to -\gamma_5$ $\gamma_5 \to \gamma_4$, leaving the commutation relations and $\gamma_5\gamma_4$ $\gamma_1\gamma_2\gamma_3$ unchanged.) The equation for a two-component left-handed neutrino is obtained from

$$\gamma\,\partial\psi = 0\,, \tag{79}$$

by projecting out with

$$\tfrac{1}{2}(1-\gamma_5)\psi\,. \tag{80}$$

Using eq. (78), eq. (80) gives

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = 0\,, \tag{81}$$

where R, L mean right, left-handed. Thus $\psi_R = 0$ and eq. (79) is

$$\begin{bmatrix} 0 & -i\sigma_K\partial_K + \partial_4 \\ i\sigma_K\partial_K + \partial_4 & 0 \end{bmatrix}\begin{bmatrix} \psi_L \\ 0 \end{bmatrix} = 0\,,$$

or

$$[i\sigma_K\,\partial_K + \partial_4]\psi_L = 0\,. \tag{82}$$

With $\psi_L = u_L \exp[ipx]$, eq. (82) tells

$$(\boldsymbol{\sigma}\cdot\boldsymbol{p})\,u_L = -|\boldsymbol{p}|\,u_L\,,$$

confirming that u_L is indeed left-handed. Take, then, two 2-component neutrinos ν_e and ν_μ, both satisfying eq. (82) (*i.e.* both left-handed)

$$(i\sigma_K\,\partial_K + \partial_4)\nu_{\substack{e\\ \mu}} = 0\,. \tag{83}$$

Form the 4-component neutrino

$$\nu = \begin{bmatrix} \nu_e \\ i\sigma_2\nu_\mu^* \end{bmatrix}. \tag{84}$$

It can then be shown that ν satisfies a 4-component neutrino equation

$$\gamma\partial\nu = 0\,. \tag{85}$$

In fact eq. (85) is

$$\begin{bmatrix} 0 & -i\sigma_K\,\partial_K + \partial_4 \\ i\sigma_K\,\partial_K + \partial_4 & 0 \end{bmatrix}\begin{bmatrix} \nu_e \\ i\sigma_2\nu_\mu^* \end{bmatrix} = 0\,,$$

or, equivalently,

$$(i\sigma_K\,\partial_K + \partial_4)\nu_e = 0\,,$$

and

$$(-i\sigma_K\,\partial_K + \partial_4)\,i\sigma_2\nu_\mu^* = 0\,.$$

The first equation is (83), the second becomes (83) after complex conjugation and multiplication by σ_2. Thus eq. (85), with eq. (84), is equivalent to eq. (83).

3·4. Now, what form does lepton conservation (76) take in this representation? The transformations (76) become

$$\begin{cases} e \to \exp[i\lambda]e\,, & \bar{e} \to \bar{e}\exp[-i\lambda]\,, \\ \mu \to \exp[i\lambda]\mu\,, & \bar{\mu} \to \bar{\mu}\exp[-i\lambda]\,, \\ \nu \to \exp[i\lambda\gamma_5]\nu\,, & \bar{\nu} \to \bar{\nu}\exp[-i\lambda\gamma_5]\,. \end{cases} \tag{86}$$

In fact

$$\exp[i\lambda\gamma_5] = \begin{bmatrix} \exp[i\lambda] & 0 \\ 0 & \exp[-i\lambda] \end{bmatrix},$$

and

$$\exp[i\lambda\gamma_5]\nu = \exp[i\lambda\gamma_5]\begin{bmatrix}\nu_e \\ i\sigma_2\nu_\mu^*\end{bmatrix} = \begin{bmatrix}\exp[i\lambda]\nu_e \\ i\sigma_2(\exp[+i\lambda]\nu_\mu)^*\end{bmatrix},$$

so that $\nu_e \to \exp[i\lambda]\nu_e$ and $\nu_\mu \to \exp[i\lambda]\nu_\mu$. And next, what about muon number conservation (77)? It is easy to see that it becomes

$$\text{(87)}\qquad \begin{cases} e \to \exp[-im]e\,, & \bar{e} \to \exp[im]\bar{e}\,, \\ \mu \to \exp[im]\mu\,, & \bar{\mu} \to \exp[-im]\bar{\mu}\,, \\ \nu \to \exp[-im]\nu\,, & \bar{\nu} \to \exp[im]\bar{\nu}\,. \end{cases}$$

In fact

$$\exp[-im]\nu = \exp[-im]\begin{bmatrix}\nu_e \\ i\sigma_2\gamma_\mu^*\end{bmatrix} = \begin{bmatrix}\exp[-im]\nu_e \\ i\sigma_2(\exp[im]\nu_\mu)^*\end{bmatrix},$$

so that $\nu_e \to \exp[-im]\nu_e$ and $\nu_\mu \to \exp[im]\nu_\mu$.

3'5. In order to write the Lagrangian in terms of the 4-component ν, we have to invert eq. (84). To this purpose we first have to find out what the charge conjugate spinor ψ^c of a spinor ψ is, in our representation with γ_5 diagonal. Take a Dirac equation with an external electromagnetic field

$$[\gamma(\partial - ieA) + m]\psi = 0\,.$$

If now you complex-conjugate it and multiply by γ_2 you find

$$[\gamma(\partial + ieA) + m](\gamma_2\psi^*) = 0\,.$$

The reality conditions for the γ in our particular representation have been used at this step. We thus see that in our representation with γ_5 diagonal

$$\text{(88)}\qquad \psi^c = \gamma_2\psi^*\,,$$

since such a ψ^c satisfies an equation with charge opposite to that in the equation satisfied by ψ. Now, from eqs. (84) and (78)

$$\text{(89)}\qquad \nu^c = \gamma_2\nu^* = \begin{bmatrix}0 & -i\sigma_2 \\ i\sigma_2 & 0\end{bmatrix}\begin{bmatrix}\nu_e^* \\ i\sigma_2\nu_\mu\end{bmatrix} = \begin{bmatrix}\nu_\mu \\ i\sigma_2\nu_e^*\end{bmatrix},$$

and we can now invert eq. (84) by projecting out the upper components from both ν and ν^c by use of the projection $\frac{1}{2}(1+\gamma_5) = a$, which applied to a

spinor, in our representation, projects out the upper component. Thus

$$\nu_e = a\nu\,, \qquad \nu_\mu = a\nu^c\,. \tag{90}$$

The Lagrangian takes the form

$$\begin{aligned}\mathscr{L} = &-\bar{\nu}\gamma\partial\nu - \bar{\nu}^c\gamma\partial\nu^c - \bar{e}(\gamma\partial + m_e)e - \bar{\mu}(\gamma\partial + m_\mu)\mu + \\ &+ G[(\bar{e}\gamma a\nu) + (\bar{\mu}\gamma a\nu^c) + \ldots][(\bar{\nu}\gamma a e) + (\bar{\nu}^c\gamma a\mu) + \ldots] + \text{other terms}\,.\end{aligned} \tag{91}$$

4. – Further symmetries.

4·1. *Pauli-Gürsey trasformation.*

4·1.1. For Lagrangians such as eq. (91), with its characteristic appearance of both ν and ν^c, Pauli and Gürsey had considered the group of transformation (the Pauli-Gürsey group) given by

$$\begin{cases}\nu \to d\nu + b\gamma_5\nu^c \\ \nu^c \to d^*\nu^c - b^*\gamma_5\nu\end{cases} \qquad \begin{cases}\bar{\nu} \to d^*\bar{\nu} - b^*\bar{\nu}^c\gamma_5 \\ \bar{\nu}^c \to d\bar{\nu}^c + b\bar{\nu}\gamma_5\end{cases} \tag{92}$$

with d and b complex numbers satisfying

$$|d|^2 + |b|^2 = 1\,. \tag{93}$$

The free neutrino Lagrangian is invariant under eq. (92). In fact, using eqs. (92) and (93)

$$\bar{\nu}\gamma\partial\nu + \bar{\nu}^c\gamma\partial\nu^c \to (|d|^2 + |b|^2)(\bar{\nu}\gamma\partial\nu + \bar{\nu}^c\gamma\partial\nu^c)\,.$$

However the current

$$\bar{e}\gamma a\nu + \bar{\mu}\gamma a\nu^c \to (d\bar{e} - b^*\bar{\mu})\gamma a\nu + (d^*\bar{\mu} + b\bar{e})\gamma a\nu^c\,, \tag{94}$$

and it is not invariant, unless $b = 0$. In this case we can write, from eq. (93)

$$d = \exp[-im]\,, \qquad b = 0\,, \tag{95}$$

where m is a real number and we have

$$\bar{e}\gamma a\nu + \bar{\mu}\gamma a\nu^c \to \exp[-im](\bar{e}\gamma a\nu) + \exp[im](\bar{\mu}\gamma a\nu^c)\,. \tag{96}$$

If we now « complete » our transformation on the neutrino field, by also acting

on e and μ with

$$\left\{\begin{array}{l} e \to \exp[-im]e\,, \\ \mu \to \exp[im]\mu\,, \end{array}\right. \tag{97}$$

we can reabsorb the phase factors in eq. (96) and get full invariance. But eq. (97), together with eq. (95) inserted into eq. (92), taken altogheter are nothing else than eq. (87). Therefore this subgroup of the Pauli-Gürsey group is nothing else than our gauge group (87) expressing muon conservation.

4·1.2. Equation (94) also teaches you that you might have invariance of the current under the full group provided you extend the transformation also to e and μ, by transforming according to

$$\left\{\begin{array}{l} d\bar{e} - b^*\bar{\mu} \to \bar{e}\,, \\ d^*\bar{\mu} + b\bar{e} \to \bar{\mu}\,. \end{array}\right. \tag{98}$$

In matrix notations this is

$$U\begin{bmatrix} e \\ \mu \end{bmatrix} = \begin{bmatrix} d^* & -b \\ b^* & d \end{bmatrix}\begin{bmatrix} e \\ \mu \end{bmatrix} \to \begin{bmatrix} e \\ \mu \end{bmatrix}, \tag{99}$$

where U is a unitary matrix

$$U^{+}U = 1\,, \tag{100}$$

with determinant unity

$$\det U = |d|^2 + |b|^2 = 1\,. \tag{101}$$

The matrices U satisfying eqs. (100) and (101) form the subgroup $SU(2)$ of the unitary group in two dimensions $U(2)$. However, the $\mu - e$ mass difference does not allow eq. (99) to be a symmetry operation for the whole Lagrangian. This is a situation already met with various times in theoretical physics. We also note that in terms of $\nu_e = a\nu$ and $\nu_\mu = a\nu^c$ the Pauli-Gürsey transformation is given by

$$\left\{\begin{array}{l} \nu \;\, = a\nu \;\to d(a\nu) \;\;\, + b(a\nu^c) = d\nu_e \;\, + b\nu_\mu\,, \\ \nu_\mu = a\nu^c \to d^*(a\nu^c) - b^*(a\nu) = d^*\nu_\mu - b^*\nu_e \end{array}\right. \tag{102}$$

or in doublet notation

$$\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} \to \begin{bmatrix} d & b \\ -b^* & d^* \end{bmatrix}\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = U^{-1}\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}.$$

Therefore, in this description, the transformation on e, μ, and that on ν_e, ν_μ are both realized by a unitary unimodular matrix U. Lepton conservation together with this group $SU(2)$ give as a direct product the full unitary group $U(2)$.

4·1.3. I shall add, in this section, a summary of some further speculations on higher symmetry schemes for leptons, in which I have been involved lastly. I shall adopt the four-component description of the neutrino. I shall also define as leptons: the positive muon, the neutrino, and the negative electron. The negative muon, the antineutrino, and the positive electron are antileptons. Furthermore lepton number is conserved (additively). It follows from lepton conservation and from our definition of leptons that processes such as

$$\mu \to e + \gamma \,,$$

$$\mu \to 3e \,,$$

$$\mu^- + \text{nucleus} \to e^- + \text{nucleus} \,,$$

$$e^- + \mu^+ \to \mu^- + e^+ \,,$$

$$e^- + e^- \to \mu^- + \mu^+ \,,$$

are forbidden, since they would imply a change of lepton number (for a multiplicative lepton number rule the last two processes would be allowed). It also follows directly that double β-decay is forbidden, that a neutrino-antineutrino pair is emitted in μ-decay, etc. Our leptonic world contains three basic leptons: the muon, the neutrino, and the electron. It is then natural to try a classification of the leptons and of the leptonic currents according to their behaviour under the group $U(3)$ of unitary transformations on three variables and under its subgroups. The group $U(3)$ can be split into $U(1)\times SU(3)$, where $SU(3)$ is the unimodular group in three dimensions and $U(1)$ can be represented by the phase transformation corresponding to the lepton gauge. Before sketching how the argument runs we summarize the main results of such a classification:

1) *Leptonic currents*: in Tables II*a* and II*b* we report the possible independent sets of currents. They divide into two groups. The currents of the sets of the first group have a definite behaviour under parity. Sets of the first group must be excluded because they would not allow for parity nonconservation in muon decay. The sets of the second group have the chiral character of the AV theory. The charged current

$$\tfrac{1}{2}(j_1 - ij_2) = -\tfrac{1}{2}(\pm \bar{e}\gamma a\nu + \bar{\nu}\gamma a\mu) \,,$$

TABLE II*a*. – *Sets of the first group: positive-helicity leptons and negative-helicity leptons transform according to equivalent representations of* $SU(3)$.

$\frac{1}{2}(j_1+ij_2)$	$-\frac{i}{2}\bar{\mu}\gamma\nu$	$-\frac{i}{2}\bar{\mu}\gamma\nu$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5\nu$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5\nu$
j_3	$-\frac{i}{2}(\bar{\mu}\gamma\mu-\bar{\nu}\gamma\nu)$			
$\frac{1}{2}(j_4+ij_5)$	$-\frac{i}{2}\bar{\mu}\gamma e$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5 e$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5 e$	$-\frac{i}{2}\bar{\mu}\gamma e$
$\frac{1}{2}(j_6+ij_7)$	$-\frac{i}{2}\bar{\nu}\gamma e$	$\frac{i}{2}\bar{\nu}\gamma\gamma_5 e$	$-\frac{i}{2}\bar{\nu}\gamma e$	$\frac{i}{2}\bar{\nu}\gamma\gamma_5 e$
j_8	$-\frac{i}{2}\frac{1}{\sqrt{3}}(\bar{\mu}\gamma\mu+\bar{\nu}\gamma\nu-2\bar{e}\gamma e)$			

TABLE II*b*. – *Sets of the second group: positive-helicity leptons and negative-helicity leptons transform according to inequivalent representations of* $SU(3)$.

$\frac{1}{2}(j_1+ij_2)$	$-\frac{i}{2}(\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-\frac{i}{2}(\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-\frac{i}{2}(-\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-\frac{i}{2}(-\bar{\nu}\gamma ae-\bar{\mu}\gamma\bar{a}\nu)$
j_3	$-\frac{i}{2}(\bar{\nu}\gamma a\nu-\bar{e}\gamma ae+\bar{\mu}\gamma\bar{a}\mu-\bar{\nu}\gamma\bar{a}\nu)$			
$\frac{1}{2}(j_4+ij_5)$	$-\frac{i}{2}(-\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-\frac{i}{2}(\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-\frac{i}{2}(\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-\frac{i}{2}(-\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$
$\frac{1}{2}(j_6+ij_7)$	$-\frac{i}{2}(\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-\frac{i}{2}(-\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-\frac{i}{2}(\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-\frac{i}{2}(-\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$
j_8	$-\frac{i}{2}\frac{1}{\sqrt{5}}(2\bar{\mu}\gamma a\mu-\bar{\nu}\gamma a\nu-\bar{e}\gamma ae+\bar{\mu}\gamma\bar{a}\mu+\bar{\nu}\gamma\bar{a}\nu-2\bar{e}\gamma\bar{a}e)$			

for such sets is also equal to

$$=-\tfrac{1}{2}(\pm\bar{e}\gamma\nu_e-\bar{\mu}^c\gamma\nu_\mu)\,, \tag{103}$$

with the position (90). One sees that (103) is the well-known expression for the charged lepton current (with $\Delta Q=1$). We conclude that only sets of the second group can be physically acceptable. Furthermore we are led to choose for a further classification that subgroup $SU(2)$, whose generators are space integrals of the fourth components of $\frac{1}{2}(j_1\pm ij_2)$ and j_3.

2) *Lepton classification*: the choice, from experiment, of sets of the second group implies that the positive-helicity leptons (that we call: μ_+, ν_+ and e_+) and the negative-helicity leptons (that we call: μ_-, ν_- and e_-) transform, under $SU(3)$, according to inequivalent three-dimensional representations. This is illustrated in Fig. 2 where the weight-diagrams of the representations $D^3(1,0)$

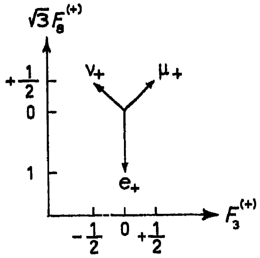


Fig. 2*a*. – Weight diagram for the positive-helicity leptons.

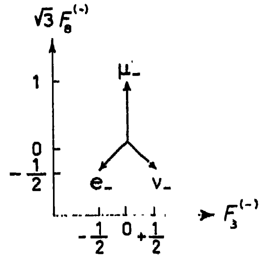


Fig. 2*b*. – Weight diagram for the negative-helicity leptons.

and $D^3(0,1)$ are reported. We have chosen the positive-helicity leptons to transform according to $D^3(1,0)$ and the negative-helicity leptons to transform according to $D^3(0,1)$.

3) *Lepton isospin and lepton strangeness*: in the weight diagrams of Fig. 2, $(F_3^{(+)}, F_8^{(+)})$ and $(F_3^{(-)}, F_8^{(-)})$ are commuting group generators. We introduce

$$I_3^{(+)} = F_3^{(+)}, \tag{104}$$

$$S^{(+)} = 2\sqrt{3}\,F_8^{(+)} - L, \tag{104'}$$

(where L is the lepton number) and, similarly, $I_3^{(-)}$ and $S^{(-)}$. We can write for the charge Q:

$$Q = I_3^{(+)} + \frac{L + S^{(+)}}{2} = I_3^{(-)} + \frac{L + S^{(-)}}{2},$$

and we can label particles and currents by their right isospin $I^{(+)}$, left isospin $I^{(-)}$, right strangeness $S^{(+)}$ and left strangeness $S^{(-)}$ (see Tables III*a*, III*b*, and IV).

4) *Baryon-lepton symmetry*: in the last columns of the Tables III*a*, III*b*, and IV we have reported the « corresponding baryon » and the « corresponding meson » for each particle and current, *i.e.* the baryon or boson with corresponding quantum numbers (the correspondence is: lepton number $\leftrightarrow$ nucleon number: $Q \leftrightarrow Q$; $S^{(+)}$, $S^{(-)} \leftrightarrow S$; $I^{(+)}$, $I^{(-)} \leftrightarrow I$). The baryons Z^- (isotopic spin $I = 0$, $S = -3$) and X^+ ($I = 0$, $S = +1$), and the mesons φ ($I = \frac{1}{2}$, $S = +3$) have not been found so far. The baryon-lepton correspondence rules of Tables III*a*, III*b* and IV replace the GAMBA, MARSHAK, OKUBO correspondence rule $\mu \leftrightarrow \nu$, $n \leftrightarrow e^-$, $\Lambda \leftrightarrow \mu^-$ [11].

TABLE IIIa. – *Quantum number assignements to the leptons with positive helicity.* The corresponding baryons are indicated in last column. The baryon Z^- has not yet been discovered.

Particle	Lepton number L	Charge Q	$S^{(+)}$	$\lvert I^{(+)}\rvert$	$I_3^{(+)}$	Corresponding baryon
μ_+	+1	+1	0	$\frac{1}{2}$	$\frac{1}{2}$	p
ν_+	+1	0	0		$-\frac{1}{2}$	n
e_+	+1	−1	−3	0	0	Z^-

TABLE IIIb. – *Quantum number assignements to the leptons with negative helicity.* The corresponding baryons are indicated in the last column. The baryon X^+ has not yet been discovered.

Particle	Lepton number L	Charge Q	$S^{(-)}$	$\lvert I^{(-)}\rvert$	$I_3^{(-)}$	Corresponding baryon
μ_-	+1	+1	1	0	0	X^+
ν_-	+1	0	−2	$\frac{1}{2}$	$\frac{1}{2}$	Ξ^0
e_-	+1	−1	−2		$-\frac{1}{2}$	Ξ^-

TABLE IV. – *Quantum number assignments to the currents.* The corresponding (vector) mesons are indicated in the last column. The mesons φ have not yet been discovered.

Current	Lepton number L	Charge Q	$S^{(-)}=S^{(+)}$	$\lvert I^{(-)}\rvert-\lvert I^{(+)}\rvert$	$I_3^{(-)}=I_3^{(+)}$	Corresponding meson
$\frac{1}{2}(j_1 \pm ij_2)$	0	±1	0	1	±1	ρ^+, ρ^-
j_3	0	0	0	1	0	ρ^0
$\frac{1}{2}(j_4 \pm ij_5)$	0	±2	±3	$\frac{1}{2}$	$\pm\frac{1}{2}$	φ^{++}, φ^{--}
$\frac{1}{2}(j_6 \pm ij_7)$	0	±1	±3	$\frac{1}{2}$	$\pm\frac{1}{2}$	φ^+, φ^-
j_8	0	0	0	0	0	ω^0

5) *Weak four-lepton coupling*: invariance of the weak four-lepton coupling under the full unitary group can be excluded: it would lead to a parity con-

serving interaction. We generate the weak four-lepton Lagrangian, l self-coupling of the currents of Table IIb (for each of the chosen sets o second group). We write

$$L' = gL_1 + fL_2 + hL_3\,, \tag{105}$$

where L_1, L_2 and L_3 are invariant under the lepton-isospin subgroup S. L_1 arises from the self-coupling of j_1, j_2, j_3; L_2 from the self-coupling of j j_6, j_7; and L_3 from the self-coupling of j_8. In the limit of unitary symm $g=f=h$. From the measured value of the ξ parameter in μ-decay we show that

$$f < 0.2\,g\,. \tag{106}$$

We shall now sketch briefly the main lines of the argument. The g ators of the unitary group are called $F_0 = L, F_1, F_2, \ldots, F_8$, and are assu to be integrals of partially-conserved local currents

$$F_i = -i\int j_i^\mu(x)\,\mathrm{d}\sigma_\mu\,. \tag{107}$$

The current j_i, because of its vector character, can be decomposed a contribution from positive-helicity particles and one from negative-he particles. Correspondingly

$$F_i = F_i^{(+)} + F_i^{(-)}\,, \tag{108}$$

and $F_i^{(+)}$, $F_i^{(-)}$ satisfy the commutation relations

$$[F_i^{(+)}, F_j^{(+)}] = i f_{ijk} F_k^{(+)}\,, \tag{109}$$

$$[F_i^{(-)}, F_j^{(-)}] = i f_{ijk} F_k^{(-)}\,. \tag{109'}$$

The commutation relations with the charge operator Q must be of the

$$[Q, F_i^{(+)}] = C_{ik} F_k^{(-)}\,, \tag{110}$$

$$[Q, F_i^{(-)}] = C_{ik} F_k^{(-)}\,. \tag{111}$$

We do not need to specify C_{ik}.

We now construct the 3-dimensional representations f_i of F_i. We tinguish two cases:

1) $f_i^{(-)}$ and $f_i^{(+)}$ are related by a similarity transformation

$$f_i^{(-)} = w f_i^{(+)} w^{-1}\,. \tag{112}$$

2) $f_i^{(-)}$ and $f_i^{(+)}$ are not related by a similarity transformation. In such a case

$$f_i^{(-)} = w\tilde{f}_i^{(+)}w^{-1} \tag{113}$$

must hold, where $\tilde{f}_i^{(+)}$ is the representations contragradient to $f_i^{(+)}$. Using (110) and (111) we can show that:
in case 1)

$$[q, w] = 0 \, , \tag{114}$$

while in case 2)

$$\{q, w\} = 0 \, , \tag{115}$$

where $\{...\}$ denotes the anticommutator and q is the 3×3 representation of Q. In both cases, (114) or (115) define w in terms of two real parameters. Having constructed $f_i^{(-)}$ we demand for it the same reality properties of $f_i^{(+)}$ (time-reversal invariance). Such a procedure leads directly to the currents of Tables IIa and IIb.

The Lagrangians L_1, L_3 are given by

$$\left\{\begin{aligned} L_1 = {} & + (\bar{\nu}\gamma ae)(\bar{\nu}\gamma a\mu) + (\bar{\mu}\gamma\ddot{a}\nu)(\bar{e}\gamma a\nu) + \tfrac{1}{4}(\bar{\nu}\gamma\nu)(\bar{e}\gamma ae) + \tfrac{1}{4}(\bar{\nu}\gamma\nu)(\bar{\mu}\gamma\ddot{a}\mu) - \\ & - \tfrac{1}{2}(\bar{e}\gamma ae)(\bar{\mu}\gamma\dot{a}\mu) + \tfrac{1}{4}(\bar{e}\gamma ae)(\bar{e}\gamma ae) + \tfrac{1}{4}(\bar{\mu}\gamma a\mu)(\bar{\mu}\gamma\dot{a}\mu) + \tfrac{1}{4}(\bar{\nu}\gamma\gamma_5\nu)(\bar{\nu}\gamma\gamma_5\nu) \, , \\ L_3 = {} & \tfrac{1}{12}[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma a\mu)][2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)] + \\ & + \tfrac{1}{12}[2(\bar{e}\gamma\dot{a}e) + (\bar{e}\gamma ae)][2(\bar{e}\gamma\dot{a}e) + (\bar{e}\gamma ae)] + \tfrac{1}{12}(\bar{\nu}\gamma\gamma_5\nu)(\bar{\nu}\gamma\gamma_5\nu) + \\ & + \tfrac{1}{6}[2(\bar{e}\gamma\ddot{a}e) + (\bar{e}\gamma ae)](\nu\gamma\gamma_5\bar{\nu}) - \tfrac{1}{6}[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\ddot{a}\mu)](\bar{\nu}\gamma\gamma_5\nu) - \\ & - \tfrac{1}{6}[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\dot{a}\mu)][2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)] \, . \end{aligned}\right. \tag{116}$$

We do not report the form of L_2, since as we have indicated it is presumably absent. Both L_1 and L_2 would contribute to μ-decay. The total contribution would be of the form

$$(\bar{e}\gamma(q + p\gamma_5)\mu^c)(\bar{\nu}^c\gamma\gamma_5\nu) \, . \tag{118}$$

If L_2 is absent, $p = q$ and (118) becomes the known μ-decay Lagrangian. The physical consequences of (118) can be simply read off from a paper on Pauli-Gürsey invariants in μ-decay [10]. It gives for the muon-decay parameters $\varrho = \frac{3}{4}$, $\sigma = \frac{3}{4}$ and $\xi = -2pq/(p^2 + q^2)$. Using Steinberger's figures [13] we have obtained (106).

The result (106) indicates that the self-coupling of j_4, j_5, j_6, j_7 is presumably absent. Such coupling, if mediated by vector bosons, would have required

doubly charged bosons. The coupling of j_6 and j_7 to stronly interacting currents (both strangeness conserving and strangeness nonconserving) seems to be experimentally excluded.

Invariance under $SU(2)$ of the weak four-lepton interaction can directly by checked by measurement of the cross-section for scattering of ν_e on e (with neutrinos from nuclear reactors). It also allows to predict scattering of ν_μ on e and weak effects in scattering and μ-pair production in high-energy electron-positron colliding beams.

Appendix I

The differential cross-section in the laboratory system for $\bar{\nu}+p \to l^+ + n$ derived from formulae (4), (3), (6) and (8) of the text is

$$\sigma(\Theta)\,\mathrm{d}(\cos\Theta) = \frac{G^2}{\pi}\,\mathscr{E}_\nu^2 \sum(\Theta, \mathscr{E}_\nu)\,\mathrm{d}(\cos\Theta)\,,$$

where $\mathscr{E}_\nu$=laboratory neutrino energy, Θ=laboratory scattering angle, and

$$\sum(\Theta, \mathscr{E}_\nu) = \left(1 + 2\xi\sin^2\frac{\Theta}{2}\right)^{-3}\left\{\cos^2\frac{\Theta}{2}\left[F_1^2 + \frac{K^2}{4M^2}\left(2(F_1+\mu F_2)^2\,\mathrm{tg}^2\frac{\Theta}{2} + \mu^2F_2^2\right)\right] + \right.$$
$$+ H_1^2\left[1 + \sin^2\frac{\Theta}{2} + 2\xi^2\frac{\sin^4(\Theta/2)}{1+2\xi\sin^2(\Theta/2)}\right] +$$
$$\left. - H_1(F_1+\mu F_2)4M^2\xi\,\frac{\sin^4(\Theta/2)}{1+2\xi\sin^2(\Theta/2)}\left(1+\xi\sin^2\frac{\Theta}{2}\right)\right\},$$

with $\xi=\mathscr{E}_\nu/M$. For $\nu+n \to p+l^-$ the $A-V$ interference term [that proportional to $H_1(f_1+\mu F_2)$] changes sign, and the rest is identical.

Appendix II

Write

$$A + B\gamma_5 = (A+B)a + (A-B)\bar{a}$$
$$C + iD\gamma_5 = (C+iD)a + (C-iD)\bar{a}\,.$$

Now substitute into eq. (58) together with

$$\psi' = T\psi = (aR + \bar{a}S)\psi$$
$$\bar{\psi}' = (\overline{T\psi}) = \bar{\psi}(\bar{a}R^\dagger + aS^\dagger)\,.$$

Comparing with eq. (52) we find

$$aR^\dagger(A-B)R + aS^\dagger(A+B)S = 1 = a + \bar a$$
$$aR^\dagger(C-iD)S + aS^\dagger(C+iD)R = M = M(a+\bar a)\,,$$

that are equivalent to eqs. (61), (61′) and (61″).

Appendix III

Write

$$T^{-1}\sigma_1 T = aX + \bar a Y\,.$$

Then, because of eq. (68)

$$a\psi = a(aX + \bar a Y)\psi = aX\psi\,.$$

Therefore $X = 1$, Y arbitrary; or

$$T^{-1}\sigma_1 T = a + \bar a Y$$
$$\sigma_1 T = Ta + T\bar a Y\,.$$

Inserting eq. (59)

$$a\sigma_1 R + \bar a\sigma_1 S = aR + \bar a S Y\,,$$

implying the condition

$$\sigma_1 R = R\,.$$

Then take eq. (61) and what you get by eliminating S from eqs. (61′) and (61″), namely multiplying according to

[hermitian conjugate of eq. (61′)]·[inverse of eq. (61″)] [eq. (61″)],

which reads

$$R^\dagger(C^2+D^2)(A-B)^{-1}R = M^2\,.$$

Write

$$A + B = aP_+ + bP_-$$
$$(C^2+D^2)(A-B)^{-1} = pP_+ + qP_-\,.$$

Then eq. (61) and this last equation give

$$R^\dagger(aP_+ + bP_-)R = aR^\dagger P_+ R = 1$$
$$R^\dagger(pP_+ + qP_-)R = pR^\dagger P_+ R = M^2\,,$$

where we have used the above condition $\sigma_1 R = R$ to obtain

$$R^\dagger P_- R = R^\dagger \sigma_1 P_- R = - R^\dagger \tfrac{1}{2}(1-\sigma_1)R = - R^\dagger P_- R = 0 .$$

But the equation

$$aM^2 = p$$

has no nontrivial solutions because a and p are numbers, while M^2 contains a term proportional to σ_3 (it is compatible only if $m_\mu = m_e$).

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The CERN Proton Synchrotron, 1954-1962: Forecasts and Reality Compared.

An informal talk by

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Introduction.

During the Varenna Summer School in 1954, several sessions were held on the accelerators and experimental facilities proposed for the CERN Laboratory in Geneva, which had just been founded. I gave at one of these sessions a talk [1] on the possibilities of the Proton Synchrotron, whose preliminary design and layout was then being completed. Eight years later, now that the PS has been running for experiments for two years, it seems a good moment to compare what we were then thinking with what has actually happened, the more so as there is today much talk of building a much larger accelerator and of what its uses and properties should be. In discussions of this sort it is good to know what kind of questions it is useful to ask and what kind essentially cannot be answered, and our experience with the PS may be helpful here.

I shall try to compare our ideas and predictions of 1954 with the reality of 1962 under three headings:

1) The general design and performance of the accelerator.

2) Methods of using the machine, *i.e.* targets, beams, layout, general facilities.

3) Experimental technique and the problems of physics the machine is used for.

This is the order in which things come naturally to me, as a machine man, but I think it will become clear that the usefulness and reliability of predictions about these topics fall also in this order, and that this will be even more true in discussing a 300 GeV machine today.

1. – General machine design and performance.

In 1953-1954 we had established a certain design philosophy in the course of the work, which, as applied to the possible use of the machine, I tried to set out at the beginning of my 1954 talk here, and which I will quote now for reference:

« When planning for a machine which will not be in full use for seven years or more, and for an energy ten times greater than that obtainable now, the main difficulty is, of course, one's ignorance of the course of physics in the intervening time, and of what will be the focus of interest when the machine actually works. There are, however, a few general considerations one can find from past experience which help in deciding what to include or leave out, and which we have tried to satisfy in our plans.

a) If a machine is to have a useful life of, say, ten years, the vast majority of the experiments done on it will not be of the kind which stimulated the desire for the machine before it was built, the qualitative, exciting work which discovers new particles and wins Nobel Prizes; the machine must also be designed for the hundreds of experiments which are done to explore systematically the whole range of energies and nuclear properties and which are essential for proper testing of theories.

b) Past experience shows that machines usually fail in these latter types of experiments because of lack of intensity, which may, for example, rule out multiple scattering experiments, or which may prevent adequate analysis and collimation of the wanted particles to reduce backgrounds; because of lack of facilities to observe some energies or kinds of particles at all; and because of lack of experimental space round the machine, both close to the orbit and behind shielding walls.

c) With a good machine, the demand to use it can be so high that the machine should be running most of the time, so the shielding must be adequate for some people to be preparing their apparatus near the machine while other people are doing experiments, and also the targets must be arranged so that as many experiments as possible can run simultaneously, even if requiring different energies and intensities ».

In reading point *a*) one must remember that high-energy physics was then in a state where, more than today, discovery of new particles rather than successful understanding or even measurement of their properties was the point of advance, especially in Europe. Points *b*) and *c*) refer to the way in which the first generation of big machines had been designed and laid out: having designed the PS to be as flexible and powerful as possible in these respects, we find in 1962 that the cost of taking full advantage of it is appar-

ently more than Europe is willing to support, but this is hardly the fault of the design.

With these reservations, I think these criteria have proved to be good ones and are applicable today to the 300 GeV machine.

On the machine itself, there was not much in my earlier lecture, which was mainly about experimental facilities. Most people know that the PS was built inside the specified time (prediction in 1954: « operation in 1960 », and in fact it ran first in November 1959 and was doing physics in January 1960) and that it has surpassed our design figures in nearly all respects.

In particular, the accelerated current, which was given in 1954 as « a reasonable hope of $6\cdot10^{9}$ protons/pulse, 12 pulses/min », is now $5\cdot10^{11}$ protons/pulse, 20 pulses/min, *i.e.* more than 100 times up in intensity.

The staff and budget estimates of January 1954, of $\sim$140 peak construction staff and 84 million Swiss francs, compare with $\sim$180 staff and 110 million Swiss francs in January 1960, when construction was essentially finished.

The increase in current is due to two facts: i) present-day A.G. machines have a very well understood theory in all regions where space-charge is not important, and if accurately designed and engineered they work according to prediction, with very small loss of beam during acceleration. ii) Ion sources for protons, which are still to some degree mystery boxes, have improved by natural selection by a factor of 10 or more in performance over the past decade. Thus we now have 30 mA of protons at 50 MeV, compared with 1 mA foreseen, and the overall acceleration efficiency from 500 keV to 25 GeV is $\sim(30\div40)\%$, the losses being largely for known reasons.

Today a 25 or 50 GeV machine could be confidently designed for $(1\div2)\cdot$ $\cdot10^{12}$ protons/pulse, near the space-charge limit, but of course without much hope of gaining a further factor of 100 by magic, since that factor has already been used up in the design.

For a 300 GeV machine, the same attitude is adopted by the more enthusiastic designers; I myself am a little more cautious, because a scaling factor of ten in size introduces many new engineering problems in realizing even a well understood design, and there may also be some points where the theory of a really big machine is not so simple as for the CERN PS. Thus I would not use a design figure for current so near to the calculated upper limit, and I would not be surprised if cost, manpower and time-scale turned out later to have been underestimated more than for the CERN PS. For these factors, for medium-sized machines the CERN PS figures show that good estimates are possible, provided they are honestly made and not tailored to a preconceived budget to suit politicians (physicists and other kinds!).

2. - Use of the PS: targets, beams, layout, facilities.

In the 1954 lecture, I gave a discussion of how targets could be used in an A.G. proton synchrotron, and also our estimates on what secondary particle yields one could expect from such targets.

We pointed out that in an A.G. synchrotron at ~ 20 GeV or more the multiple Coulomb scattering and the fractional energy loss of protons in a target 1 nuclear mean free path thick were so small that protons traversing it would still stay inside the vacuum chamber of the machine and not hit the wall and be lost. Thus any small object would act as an efficient multiple traversal target, consuming all the beam that was not lost by single scattering.

This led us to predict the use of thin foils or wires to give long-burst targets, with up to $\frac{1}{10}$ second burst length, and about 80% efficiency with light elements. This is borne out in practice, except that the elastic single scattering is larger than we thought and leads to 60% efficiency rather than 80%.

We suggested small flip targets, like those we now use with bubble chambers, and the use of several targets simultaneously with relative intensities determined by the relative masses of the targets. We also mentioned the possibility of a liquid hydrogen target in a small thin tube; so far this has not been made.

The particles for which we predicted yields were:

— elastically scattered protons;

— « charge-exchange » neutrons;

— pions of all energies;

— γ-rays from π^0 decay;

— low-energy pions, protons and neutrons.

The figures were based on cosmic-ray emulsion data [2] and the first results from the Cosmotron. In 1962 we have direct measurements from CERN and Brookhaven of the fluxes of these particles in most of the more interesting regions, and the comparison is as follows:

i) *Scattered protons*

1954: About 10% of protons scattered into a cone of $\sim 5°$ half-width at half intensity.
Flux at small angles $= \sim 2$ p/sr/p
Diffraction scattering at very small angles mentioned.

1962: Distribution more sharply peaked forward.
Flux at $\sim 1° = \sim 15$ p/sr/p.

The agreement between expectation and reality in most cases is quite good at angles of $(5 \div 10)°$, but real distributions are found now to be rather sharply peaked forward for 25 GeV processes, and so our predicted forward yields were too low by a factor of ~ 5 and large-angle yields were too high.

Beams and apparatus used in them were less clearly foreseen. I mentioned earlier that the style of experimenting imagined in 1954 was one of discovery rather than of measurement, and the source of the particles was also thought of primarily as the target in the machine rather than externally. Thus we were led to a layout involving the possibility of large angles of emission, for observing new particles just above the creation threshold, clear of unwanted faster particles. Such a layout, with economical buildings, leads to the use of targets in many straight sections and to beams crossing on the experimental floor. If we had used exactly this layout, it would have been rather disastrous with the big semi-permanent beams we now use: luckily, we woke up at the last moment and increased the possibilities of getting several low-angle beams into the South hall off the main target, but we are still stuck with a big concrete pillar which obscures part of the useful angle. This is, I think, the biggest mistake we made in the general design of the machine.

The idea of using a permanent shielding roof, in the form of a bridge across the experimental area, and only moving blocks at beam level has worked rather well and seems to lead to rather quick changes of layout during short shut-down periods.

The points in our original criteria about use of several experimental areas have also been justified in practice. In fact we are now adding a new hall larger than the South hall in the East area.

In the design of shielding we were—luckily—rather conservative: we assumed about 10^9 p/s, but added a factor 10 for unknown biological effects, only part of which is used up in 1962, and we assumed the full proton beam hitting the shielding wall. In addition we took 180 g/cm² for the absorption length in concrete, and we now find ~ 150 g/cm². With about 10^{11} p/s on internal targets, we are in 1962 getting just above the 40 h/week tolerance level close to the outer shielding wall, but this is so far not very troublesome because access to this region has to be limited by the presence of the beams themselves.

Still in the field of machine facilities, we saw that beams could be made much more intense to satisfy criterion *b*) by using quadrupole focusing channels, and my early lecture contains an analysis of the acceptance of such channels and suggests that 1 million Swiss francs worth of magnets will make all the difference between successful and unsuccessful beams. In fact, in our 1962 layout, we have typically about 1 million Swiss francs of magnets per beam and a total stock worth some 10 million Swiss francs.

The last machine facility we planned for in 1954 was ejection, in fact an

adaptation of the Piccioni type ejector, using two energy-loss targets in series, and an estimated efficiency of $(10 \div 20)\%$, with a 10 m burst length. We now think that a variant of the Tuck and Teng resonant ejector used in cyclotrons can also be used and can have as much as 70 % efficiency. This will not be tried until next year.

In general, we can say for these questions of beams, layout and targets that we were qualitatively right in most of our predictions, but as usual understimated the general scale and complexity of operations and cost, and also that we did not foresee the need or possibility of separated beams.

3. – Physics and experiments.

Here we enter a field where the unknowns and the unpredictables in fact are determining. I have mentioned before that the planning of experiments in the minds of physicists was in terms of the search for new particles rather than the detailed quantitative comparison of measurements with theory, and we as machine designers had to fight to get agreement to spend money on two experimental areas or on provision for all conceivable uses of the machine, when the only physics directive we had was « find out if antinucleons exist, this is the crucial experiment for physics ».

Remember that at the beginning of 1954 antiprotons had not been found; K-mesons and some hyperons were only just known to exist; no one knew if a hydrogen bubble chamber could be made; separators, spark chambers, transistorized electronics had not been heard of—nor had reliable accelerators or any powerful beam design techniques. No one had any good reason for going from 10 to 30 GeV for the machine energy; 25 GeV was fixed for the CERN PS because with A.G. focusing such a machine was expected not to cost much more than the amount talked of in the first days of CERN for a 10 GeV weak-focusing machine.

This is a situation which is again with us because of the possibility of a 300 GeV machine: should it be 100 GeV or 300 GeV, or should it be some other kind of machine entirely? The conclusion I draw from our experience with the overall design of the CERN PS is that detailed physics predictions are a waste of time and should not be considered very seriously in finally deciding on a machine: they are unfortunately a necessary part of the process of raising money, but that is another story. In fact, the important thing in designing a new accelerator is to get one or preferably more of its basic performance parameters at least ten times better than in any current machine, and then to trust to the generosity of Nature to provide new fields of research and exciting results. In the past, She has never let us down, which is more than can be said of most theoreticians.

There is an extension of this philosophy, to the design of experimental apparatus, which is becoming obvious just now. More and more experiments are made using beams and detectors built not for one experiment, or to solve any particular problem in physics, but because they make measurements possible in a certain field of phenomena; it is then up to the ingenuity of physicists to extract the physics with the means at hand. Both the generosity of Nature and the soundness of this doctrine are, I think, displayed by the success of the Berkeley hydrogen bubble chamber programme, where nothing of what has now been found could have been foreseen in any detail when the crucial decisions were taken.

Today we are in a more difficult situation than ever before in planning new accelerators: there are now three time-scales—a decade for accelerators, $(3 \div 5)$ years for major experimental techniques, and that of the individual physicist. He, as we know to our cost, is always wrapped up in his current work, and can hardly ever be induced to make a serious contribution to the choice of new accelerators, or even to the design of major pieces of apparatus to be used with them. Thus the construction of a new accelerator must be started with only very general ideas for its future use, and none as to how experiments will in fact be carried out with it. It may help a little if these facts are recognized, but it is still going to be very difficult to make decisions for spending hundreds of millions of Swiss francs on such tenuous grounds.

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